

## Section 0:

## 'Dialectical Meta-Axiomatics' --

## a Methodology

for
Marxian, 'Dialectical Science'

## Meta-System-atic Dialectics:

Complex Unity of 'Internal' \& 'External' Systematic Views of Dialectical Systems-Progressions


# 'Dialectical Meta-Axiomatics' 

"'Historical-Dialectical Moment"' of 'Meta-Systematic Dialectics' -
Human-Social System Self-Reproduction Turns Into System Self-Dissolution / System Self-Supercession
"The fundamental condition of property based on tribalism ... is to be a member of the tribe. Consequently a tribe conquered and subjugated by another becomes propertyless and part of the inorganic conditions of the conquering tribe's reproduction, which that community regards as its own. Slavery and serfdom are therefore simply further developments of property based on tribalism.

But this also clearlymeans that these conditions change. What makes a region of the earth into a hunting-ground, is being hunter over by tribes; what turns the soil into a prolongation of the body of the individual is agriculture. Once the city of Rome had been built and its surrounding land cultivated by its citizens, the conditions of the community were different from what they had been before. The object of all of these communities is preservation, i.e. the production of the individuals which constitute them as proprietors, i.e. in the same objective mode of existence, which also forms the relationship of the members to each other, and therefore forms the community itself. But this reproduction is at the same time necessarily new production and the destruction of the old form. ...

The act of reproduction itself changes not only the objective conditions - e.g. transforming village into town, the wilderness into agricultural clearings, etc. - but the producers change with it, by the emergence of new qualities, by transforming and developing themselves in production, forming new powers and new conceptions, new modes of intercourse, new needs, and new speech. ...

The community itself appears as the first great force of production. ..
In the last instance the community and the property resting upon it can be reduced to a specific stage in the development of the forces of production of the labouring subjects -- to which correspond specific relations of these subjects with each other and with nature. Up to a certain point, reproduction. Thereafter, it turns into dissolution. ...

These forms are of course more or less naturally evolved, but at the same time also the results of a historic process. The evolution of the forces of production dissolves them, and their dissolution is itself an evolution of the human forces of production."

Karl Marx, [《Grundrisse»»:] Pre-Capitalist Economic Formations, Internat'l. Publishers [NY: 1965], pp. 92-95, [emphasis added by F.E.D.]

# ‘Dialectical Meta-Axiomatics' 

"'Systematic-Dialectical Moment"' of 'M eta-Systematic Dialectics' -

The Order of Presentation of the Categories of the Present Human-Social System of Relations of Production should be that of their
'Anatomical' Importance in the Self-Reproduction of that System, Not Necessarily that of their Historical Appearance
"Money can exist and has existed in history before capital, banks, wage labour, etc., came into being. In this respect it can be said, therefore, that the simpler category can express relations predominating in a less developed whole or subordinate relations in a more developed whole, relations which already existed historically before the whole had developed the aspect expressed in a more concrete category. To that extent, the course of abstract thinking, which advances from the elementary to the combined, corresponds to the actual historical process.
Bourgeois society is the most developed and many-faceted historical organization of production. The categories which express its relations, and understanding of its structure, therefore, provide, at the same time, an insight into the structure and relations of all previous forms of society the ruins and components of which were used in the creation of bourgeois society. Some of these remains are still dragged along within bourgeois society unassimilated, while elements which previously were barely indicated have developed and attained their full significance, etc. The anatomy of man is a key to the anatomy of the ape. On the other hand, indications of higher forms in the lower species of animals can only be understood when the higher forms themselves are already known. Bourgeois economy thus provides a key to that of antiquity, etc. But by no means in the manner of those [socio-ontological reductionist] economists who obliterate all historical differences and see in all forms of society the bourgeois forms. One can understand tribute, lithe, etc., if one knows rent. But they must not be treated as identical.

Just as generally in the case of any historical, social science, so also in examining the development of economic categories it is always necessary to remember that the subject, in this context modern bourgeois society, is given, both in reality and in the mind, and that therefore the categories express forms of being, determinations of existence - and sometimes only individual aspects -of this particular society, of this subject, and that even from the scientific standpoint it therefore by no means begins at the moment when it is first discussed as such. This has to be remembered because it provides the decisive criteria for the arrangement [of the material].

For example, nothing seems more natural than to begin with rent, with landed property, since it is bound up with the earth, the source of all production and all life, and with agricuiture, the first form of production in all more or less established societies. But nothing could be more erroneous. In every form of society there is a particular [branch of] production which determines the position and importance of all the others, and the relations obtaining in this branch accordingly determine those in all other branches. It is the general light tingeing all other colours and modifying them in their specific quality; it is a special ether determining the specific gravity of everything found in it. ... Among peoples with settled agriculture - this setting is already a greal advance where agriculture predominates, as in antiquity and the feudal period, even industry, its organization and the forms of property corresponding thereto, have more or less the character of landed property. Industry is either completely dependent on it, as with the ancient Romans, or, as in the Middle Ages, it copies in the lown and in its conditions the organization of the countryside. In the Middle Ages even capital - unless it was purely money-capital - capital as traditional lools, etc., has this character of landed property.
The reverse is the case in bourgeois society. Agriculture to an increasing extent becomes merely a branch of industry and is completely dominated by capital. The same applies to rent. In all forms in which landed property rules supreme, the nature relationship still predominates; in the forms in which capital rules supreme, the social, hislorically-evolved element predominates. Rent cannot be understood without capital, but capital can be understood without rent. Capital is the economic power that dominates everything in bourgeois society. It must form both the point of departure and the conclusion and must be analyzed before landed property. After each has been considered separately, their interconnection must be examined.

It would therefore be inexpedient and wrong to present the economic categories successively in the order in which they played a determining role in history. Their order of succession is determined rather by their mutual relation in modern bourgeois society, and this is quite the reverse of what appears to be their natural relation or corresponds to the sequence of historical development. The point at issue is not the place the economic relations took relative to each other in the succession of various forms of society in the course of history ... but their position within modern bourgeois society."

Karl Marx, Frederick Engels, Collected Works, vol. 28, Karl Marx: 1857-1861 [‘Grundrisse»]], Internat'l. Publishers [NY: 1986], pp. 39-44 [colored-text, [inserted], and underscore emphasis added by F.E.D.]

## «Aufheben» Diagram: 'Intra-Duality' of 'Dialectical Meta-Axiomatics'

## 'Dialectical Meta-Axiomatics':

Axioms-Systems' Dialectical, 'Qualo-Peanic Consecuum' Progressions as Axioms 'Meta-Systems'

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Axiomatic
'Ideo-Systems' [e.g., Math. Systems] as models of Intuitive
'Idea-Systems'; TheoremsExpansion, leading to '"Gödelian'"
Supercession, i.e., to Expanded Axioms-System
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> Axiomatic
> 'Ideo-Systems' as models of Natural-Historical Systems' Phenomenologies, via Principles derived inductively from Phenomena-Data, then shown to deductively reproduce that Data / History, up to System singularity/ "'Meta-System Transition""

## 'Dialectical Meta-Axiomatics'

...Nevertheless, even the Synchronic Systematics of a Systematic-Dialectical Method of Exposition of the Self-Reproduction Process of the Present Human System Points 'Meta System-atically' to both its Diachronic Predecessor System \& its Diachronic Successor System...
"It must be kept in mind that the new forces of production and relations of production do not develop out of nothing, nor drop from the sky, nor from the womb of the self-positing Idea; but from within and in antithesis to the existing development of production, and the inherited, traditional relations of property. While in the completed bourgeois system every economic relation presupposes every other in its bourgeois economic form, and everything posited is thus also a presupposition, this is the case in every organic system. The organic system itself, as a totality, has its presuppositions, and its development to its totality consists precisely in subordinating all elements of society to itself, or in creating out of it the organs which it still lacks. This is historically how it becomes a totality. The process of becoming this totality forms a moment of its process, of its development. ...
... Our method indicates the points at which historical investigation must enter in, or where bourgeois economy as a merely historical form of the production process points beyond itself to earlier historical modes of production. In order to develop the laws of bourgeois economy, therefore, it is not necessary to write the real history of the relations of production. But the correct observation and deduction of these laws, as having themselves become in history, always leads to primary equations - like the empirical numbers e.g. in natural science - which point towards a past lying behind this system. These indications, together with a correct grasp of the present, then also offer the key to the understanding of the past - a work in its own right which, it is to be hoped, we shall be able to undertake as well. This correct view likewise leads at the same time to the points at which the suspension of the present form of production relations gives signs of its becoming - foreshadowings of the future. Just as, on one side the pre-bourgeois phases appear as merely historical, i.e. suspended presuppositions, so do the contemporary conditions of production likewise appear to be engaged in suspending themselves and hence in positing the historic presuppositions for a new state of society."

Karl Marx, Grundrisse: Foundations of the Critique of Political Economy (Rough Draft), Penguin Books Ltd., [Baltimore, Maryland: 1973], pp. 278; 460-461 [emphasis added by F.E.D.]

Visualization / Specimen of a 'Meta-Systematic' Dialectical,
'Diachronico-Synchronic Dialectical' '"Model of History'" for the recent human-social history of European 'Mediterranea', with foreshadowings of two of its possible futures
"pre-historic", rura[- fiefdom] feudalist, tyrannical, serfdom-based 'agrico-manufactural' [in-/sub-]human-social system
"pre-historic", city[-state ]imperialist, city[-state ]imperialist,
tyrannical, slavery-based 'agrico-manufactural' [in-/sub-]human-social system


## 'Dialectical Meta-Axiomatics'

"The mathematico-scientific methodology that we recommend as the standard for Marxian, dialectical science, including for the 'human-collective-mind $/$ '"memepool"' intro-empirical' science of mathematics, is that which we term 'Dialectical Meta-Axiomatics'.
'Dialectical Meta-Axiomatics' "aufheben»-conserves, without apology - indeed, with unequivocal affirmation - the full logical rigor of the formal-logical / mathematical-logical deductive proof of the "verstand") or 'dianoesis' moment of human rationality, within each axioms-system of an 'intra-duality'-driven, e.g., a Gödel-incompleteness-driv en, immanent-critique self-propelled, self-progression of axioms-systems. That self-progression of axioms-systems constitutes the expositionally-diachronic 'axioms meta-system' for that connected, 'qualo-Peanic consecuum/cumulum' of axioms-systems.
'Dialectical Meta-Axiomatics' also applies dialectical reason -- the «vernunft") or "(dialektiké") moment of human rationality - in the trans-deductive realm of the necessarily non-deductive derivation / determination of the possible axioms [cf. Plato]; in the rational justification of the choice / selection of axioms from those possibilities, especially for the "arché", or initiating, axioms-system of each such axioms-systems-progression, or 'meta-system'.

In particular, 'Dialectical Meta-Axiomatics' applies dialectical logic to the «aufheben", transitions between pairs of axioms-systems, that is, to the transition from each predecessor axioms-system to its immediate / consecutive successor axioms-system. Each such transition both "aufheben" conserves, and "aufheben» conforms/adapts/adjusts, the axioms of the predecessor axioms-system into those of the successor axioms-system. It also adds, via «aufheben", "transformation"' / '"elevation"', the new "'comprehension axioms'" [cf. Gödel], addressing the new 'physio-ontology', and/or the new 'ideoontology', whose emergence/irruption drives this "'meta-system transition"' [cf. Turchin]. Such "meta-system transitions"' typically involve a 'metamonadological' [cf. Leibniz] '[self-]involution' [cf. Chardin], a 'real [self-]subsumption' [cf. Marx], and a '[self-]internalization' of the "umonads") of the axiomatically-modeled predecessor system/«anithmos", whereby the predecessor «monads» self-construct the neo-«monads» of the axiomatically-modeled successor sy stem/《arithmos».
"'Diachronically"', "inter-system-ically"' - between each predecessor/successor pair of axioms-systems, the methodology which we term 'Dialectical MetaAxiomatics', as a "'method of presentation'" [cf. Marx], practices an expository, pedagogical discipline, which utilizes an heuristic, intuition-involving, ""intensional"' derivation of the current step of the self-«aufheben") self-progression - that from this predecessor to its successor axioms-system - i.e., of the current element of the "'diachronic'" 'axioms meta-system'.
"'Synchronically"", "intra-system-ically"" - within each, progressive, 'intra-duaf", successor/predecessor axioms-system [which is successor to its predecessor, but also predecessor to its successor], 'Dialectical Meta-Axiomatics' justifies the theorems implied by that axioms-system's own axioms«arithmos», via rigorous deductive logic. Those theorems are both deductively justified, and also explained, conceptually and intuitiv ely [«segrifflichkeit»] -without apology for the employ ment of either the deductive logic or the intuitive explication.

Indeed, the main expository narrative, in a work of 'Dialectical Meta-Axiomatics', should ty pically be the intuitive / conceptual exposition. It must also include, as a parallel stream of discourse, the formal-logical, algorithmic/mechanical, proof-fromthe-axioms exposition - which, if offered by itself, might merely compel assent to successive propositions, without comprehension. The latter exposition, so as not to interrupt the flow of the intuitive / conceptual exposition, might be provided as a facing-page, end-notes, or technical-appendix dual explication, supplying a necessary verification-check upon the conceptual/intuitive narrative's flow of claims, or cumulative progression of assertions, and with bridging, interconnecting commentaries - "'transversals"' and asides -- linking from the deductive proofs stream to the intensional-heuristic / intuitive narrative stream, and from the intensional-heuristic / intuitive narrative stream to the deductive proofs stream, wherever such interconnexions can be 'explicitized' with pedagogical / cognitive gain. This format minimizes interruptions in the flow of either discourse by its other, while also actualizing the cognitive benef its of 'explicitizing' their interconnexions.

Karl Seldon, Encyclopedia Dialectica, Prolegomena [emphasis added]

«Aufheben» Diagram: The 'Meta-Axiomatic' 'Gödelian Dialectic' as a whole


"If we imagine that the system Z [a formal, logical, propositional-/-predicate-calculus axiomatic system inclusive of "Natural" Numbers' Arithmetic, not the full system of the positive and negative Integers, and zero [which is both, [or neither] positive and [nor] negative], also standardly denoted by $\mathbf{Z}-\mathbf{F} . \underline{E} . \underline{D}$.] is successively enlarged by the introduction of variables for classes of numbers, classes of classes of numbers, and so forth, together with the corresponding comprehension axioms, we obtain a sequence (continuable into the transfinite) of formal systems that satisfy the assumptions mentioned above, and it turns out that the consistency ( $\omega$-consistency) of any of these systems is provable in all subsequent systems. Also, the undecidable propositions constructed for the proof of Theorem 1 [Gödel's "First Incompleteness Theorem'" - F.E.D.] become decidable by the adjunction of higher [logical - F.E.D.] types and the corresponding axioms; however, in the higher systems we can construct other undecidable propositions by the same procedure. ...To be sure, all the propositions thus constructed are expressible in $\mathbf{Z}$ (hence are number-theoretic propositions); they are, however, not decidable in Z, but only in higher systems..."

Numbers-Spaces Progression Ingredient in the "'Self-Reflexion"' / Self-Confrontation / Immanent [Self-]Critique of the $\mathbf{A x}$ xiom-Sy stems of those Numbers-Spaces,

«Aufheben» Diagram: Axioms-Systems «Aufheben»-Processes of the 'Gödelian Dialectic'

 whose «arché" numbers-Space, or number-Set, is $\mathbf{S}_{0}=\mathbf{N} \equiv\{\mathbf{I}, \mathrm{II}, \mathrm{III}, \ldots\}$. The first $\underline{\text { axiom of }} \underline{\boldsymbol{A x i o m}} \underline{\underline{s}}$-Set $\underline{\mathbf{A}}_{0}$ is denoted by $\underline{\mathbf{a}}_{0,1,}$, the second by $\underline{\mathbf{a}}_{0,2}, \ldots$, the last by $\underline{a}_{0, \mathrm{~N}_{0}}$ that is, the number of axioms for $\underline{A} x i o m \underline{s}-\operatorname{Set} \underline{A}_{0}$ is denoted by $\mathrm{N}_{0}$.

Caveat: it might be difficult to locate that precise allocation of the basic ideograms of symbolic formal logic to prime "Natural Numbers", for an implementation of a Gödel numbering scheme that would induce the Gödel formulae to "'deformalize"' to the precise diophantine equations that tie to the numbers-spaces as given herein. Therefore, the sequence of numbers-spaces set forth herein should be seen as illustrative only.

Kex: The ideogram 'Ф' denotes a 'generalized addition' operation, one that encompasses ""inhomogeneous'", "'nonamalgamative'" addition. Expressions of the form ' $\Delta \mathbf{A}_{\mathrm{k}}$ ', involving the operator $\Delta$. denote qualitative increments i innovations, i.e., 'ideo-ontological' gains upon ' $\underline{\mathbf{A}}_{k}$ ', not the "'"purely quantitative"' increments associated with the standard finite difference operator, $\boldsymbol{\Delta}$.
«Aufheben» Diagram: Numbers-Systems «Aufheben»-Processes of the 'Gödelian Dialectic'

"... The Gödel sentence $\varphi . .$. asserts its own undeducibility from the postulates....Deformalizing $\varphi$... we see that under the standard interpretation it expresses a fact of the form [for every $\boldsymbol{n}$-ary list of number-components of $\boldsymbol{x}$ such that each number-component is a member of the set of 'diophantine numbers', or "Natural" Numbers, in use - F.E.D.] ...fx $\boldsymbol{f} \boldsymbol{g x}$..., where $f$ and $\boldsymbol{g}$ are $\mathbf{n}$-ary polynomials....An equation $\boldsymbol{f} \boldsymbol{x}=\boldsymbol{g} \boldsymbol{x}$, where $\boldsymbol{f}$ and $\boldsymbol{g}$ are two such polynomials, is called diophantine... By a solution of the equation we mean an n-tuple $\alpha$ of natural numbers such that $f \alpha=g \alpha \ldots$ So $\varphi \ldots$ asserts the unsolvability of the...equation $\boldsymbol{f x}=\boldsymbol{g} \boldsymbol{x}$, and the proof of [Gödel's "First Incompleteness Theorem" - F.E.E.] produces...a particular diophantine equation that is really unsolvable, but whose unsolvability cannot be deduced from the postulates..." [ emphasis added by F.E.․․ ]

Moshé Machover, Set Theory, Logic, and their Limitations, 1996

## «Aufheben» Diagram: The 'Meta-Axiomatic' 'Gödelian Dialectic' as a whole



## Section 2:

$A n_{N} \mathbf{Q}$ Heuristic Model of the Dialectic of The Historical Progression of

Albert Einstein's Scientific Work
"Aufheben» Diagram: An $\mathbf{N} \mathbf{Q}$ Heuristic Model of the "'Dialectic"' of The Historical Progression of Einstein's Scientific Work


Axis of Progressive [Self-]Opposition / Partial [Self-]Refutation / Immanent [Self-]Critique / Self-Confrontation of 'Ideo-Ontological Categories / Systems-of-Ideas'

«Aufheben» Diagram: Dialectical-Arithmetical Model of the 'Meta-System-atic' Dialectic of the Systems of Dialectical Arithmetic,
Formulated via the 'Purely-qualitative' Arithmetic of the $\mathbf{N}_{\mathbf{N}} \mathbf{Q}$, the First Fully-Dialectical Arithmetic to Emerge in that Dial ectical Progression of Arithmetics, to Epoch $\tau=4$



The "Non-Addible Numbers" of Plato's Dialectical «(IdE<<>-Arithmetic of the «Arithmoi Eide-tikoi»», and the 'Dialector Meta-Numbers' of the $\mathbf{N Q}^{\mathbf{Q}}$ Dialectical Arithmetic
"arithmos: number; arithmêtikê; the science of number. Zero was unknown as a number and one also was not counted as a number, the first number being the duas - two. From the Pythagoreans, ton arithmon nomizontes arkhên einai - who consider number to be the first principle (Ar. Met. 986a15) - number played a great part in metaphysics, especially in Plato's unwritten doctrines, involving obscure distinctions of e.g. sumblêtoi and assumblêtoi - addible and non-addible numbers."
J. O. Urmson, The Greek Philosophical Vocabulary, Gerald Duckworth \& Co., Ltd. [London: 1990], pp. 31-32, [emphasis added by F.E.․․]

The Dialectic of the Dialectical Arithmetics：The Rules of the First of these Dialectical Arithmetics－
The Arithmetical Rules－System for the＇Dialector Meta－Numbers＇of the $\mathbf{N}_{\mathbf{N}} \mathbf{Q}$ Meta－Number Space／＂Set＂


Note：The ${ }_{\mathbf{N}} \mathbf{Q}$ space is a potentially infinite－dimensional＇dialector space＇．These diagrams depict only two of ${ }_{\mathbf{N}} \mathbf{Q}$＇s dimensions at a time．
［ § $\mathbf{0} \mathbf{0}]$ Rule 0．Inheritance［＇the $\mathbf{N}$ heritage of $\mathbf{N}_{\mathbf{N}} \mathbf{Q}$＇］：for every $\mathbf{n}$ in $\mathbf{N}, \mathbf{q}_{\mathbf{n}}$ is in $\mathbf{N} \mathbf{Q}$ ，or，$\left[[\forall \mathbf{n} \in \mathbf{N}]\left[\mathbf{q}_{\mathbf{n}} \in_{\mathbf{N}} \mathbf{Q}\right.\right.$ ］］．
［ §1］Rule 1．Ontological Diversity［＇qualifier heterogeneity＇］：for every $\mathbf{k} \& \boldsymbol{\ell}$ in $\mathbf{N}, \mathbf{k} \geqslant \ell$ implies $\underline{\mathbf{q}}_{\mathbf{k}}$ 半 $\mathbf{q}_{c}$

$$
\text { or, }\left[[\forall k, \ell \in \mathbf{N}]:[k \geqslant \ell] \Leftrightarrow\left[\underline{q}_{k} \frac{⿻ 丷 ⿻ 二 丨}{} \mathbf{q}_{\ell}\right]\right] .
$$

The Dialectic of the Dialectical Arithmetics: The Rules of the First of these Dialectical Arithmetics -
The Arithmetical Rules-System for the 'Dialector Meta-Numbers' of the $\mathbf{N}_{\mathbf{N}} \mathbf{Q}$ Meta-Number Space / "Set"
[ §2] Rule 2. Ontological Parsimony [ "Additive Idempotency", or 'Super-Amalgamative Additivity of Likes' ]: for every $\mathbf{n}$ in $\mathbf{N}, \underline{q}_{\mathbf{n}}+\underline{\mathbf{q}}_{\mathbf{n}}=\underline{q}_{\mathbf{n}}$, or, $\left[[\forall \mathbf{n} \in \mathbf{N}]\left[\underline{\mathbf{q}}_{\mathbf{n}}+\underline{\mathbf{q}}_{\mathbf{n}}=\underline{q}_{\mathbf{n}}\right]\right]$.

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\square_"Non-Redundancy"; 'Unquantifiability'; first half of 'Unaddability'
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[ §3] Rule 3. Ontological Irreducibility [ "Non-Amalgamative Additivity of Unlikes"" ]:
for every $\mathbf{k}, \ell, \& \min \mathbf{N}, \mathbf{k} \neq \ell$ implies that $\underline{\mathbf{q}}_{\mathbf{k}}+\underline{\mathbf{q}}_{\ell} \neq \underline{\mathbf{q}}_{\mathrm{m}}$, or, $\left[[\forall \mathbf{k}, \ell, \mathbf{m} \in \mathbf{N}]:[\mathbf{k} \neq \ell] \Leftrightarrow\left[\underline{\mathbf{q}}_{\mathbf{k}}+\underline{\mathbf{q}}_{\ell} \neq \underline{\mathbf{q}}_{\mathrm{m}}\right]\right]$.


The Dialectic of the Dialectical Arithmetics: The Rules of the First of these Dialectical Arithmetics -
The Arithmetical Rules-System for the 'Dialector Meta-Numbers' of the $\mathbf{N}_{\mathbf{N}} \mathbf{Q}$ Meta-Number Space / "Set"
[ §4 ] Rule 4. Ontological Innovation [ "'Contra-Boolean, «Aufheben») Multiplication"" ];
the '《aufheben»> evolute product rule' --
for every $\mathbf{k}, \ell$ in $\mathbf{N}, \underline{q}_{\mathbf{k}} \times \underline{q}_{\iota} \equiv \underline{q}_{\iota}+\underline{q}_{\mathbf{k}+\iota}$ or, $\left[[\forall \mathbf{k}, \iota \in \mathbf{N}]:\left[\underline{q}_{\mathbf{k}} \times \underline{q}_{\iota} \equiv \underline{q}_{\iota}+\underline{q}_{\mathrm{k}+\ell}\right]\right]$.

special case: $\underline{s e l f}$-reflexive self-multiplication/self-operation/self-«<aufheben»> of 'dialector ontological qualifiers' --
for every $\mathbf{k}$ in $\mathbf{N}, \mathbf{q}_{\mathbf{k}} \times \underline{q}_{\mathbf{k}} \equiv \underline{q}_{\mathbf{k}}{ }^{2} \equiv \underline{q}_{\mathbf{k}}+\mathbf{q}_{\mathbf{k}+\mathbf{k}}$ or, $\left[[\forall \mathbf{k} \in \mathbf{N}]:\left[\underline{q}_{\mathbf{k}} \times \underline{q}_{\mathbf{k}} \equiv \underline{q}_{\mathbf{k}}+\underline{q}_{2 k}\right]\right]$.

Together, these Rules, especially Rules $\mathbf{2 , 3 , \& 4}$, converge to make the Seldon Function of $\underline{q}_{1}$ work as follows:
$\left[\underline{q}_{1}\right]^{2^{\tau}}=\underline{\boldsymbol{\Sigma}}_{\mathbf{k}=1,2^{\tau}}\left[\mathbf{q}_{\mathbf{k}}\right]=\left[\underline{q}_{1}+\ldots+\underline{\mathbf{q}}_{2}\right]$, i.e, $\underline{\mathbf{q}}_{1}$ raised to an $\mathbf{N}$ power $=$ the sum of all of the $\mathbf{q}_{\mathbf{k}}$ up to that power,
wherein the underscored sigma symbol, $\underline{\underline{\boldsymbol{\Sigma}}}$, denotes the summation operator / operation for 'pure-qualifier' 'meta-number' summands.

# The Dialectic of the Dialectical Arithmetics: Specimen of the Higher Arithmetics - 

Applied to the 'Self-Meta-Evolution' of a Hypothetical, Collision-Singularity-Destined Triple-Star System

[ Formulated as the 'Meta-State' 'Meta-Dynamics' of a 'Super'-System' in its 'State/Control Meta-Space' ]

## "'Meta-Model"' Scenario:

The $\mathbf{t} \mathbf{= 0} \mathbf{~ s y n c h r o n i c ~ ' s u p e r - s y s t e m ' ~ i n ~ t h i s ~ e x a m p l e ~ i s ~ a ~ g r a v i t i c a l l y - b o u n d , ~ p l a n e t l e s s , ~ 3 - s t a r ~ s t e l / a r ~ [ s u p e r - ] s y s t e m , ~ g r a s p e d ~ a s ~ o n e ~}$ constituted out of 3 "'systems" - the "'system"' denoted star ${ }_{1}$, the "'system"" denoted star $_{2}$, and the '"system'" denoted star ${ }_{3}$.

Each star is represented, in this idealization, by a "mass-point" located at its center of mass. We assume that star ${ }_{1}$ \& star ${ }_{2}$ are destined to collide. Per this idealization, that collision is modeled via a finite time-point, $t$. , for which $\mathbf{r}_{12}(\mathrm{t})=.\mathbf{0}, \mathrm{w}$ here $\mathbf{r}_{\mathbf{1 2}}(\mathrm{t})$ denotes the radial distance betw een the mass-point of $\mathbf{s t a r}_{1}$ and the mass-point of $\mathbf{s t a r}_{2}$ as of time $\mathbf{t}$. Thus, the collision is represented as a condition in which the mass-point of $\boldsymbol{s t a r}_{1}$ and the mass-point of $\mathbf{s t a r}_{2}$ coincide in physical-spatial position.

This collision is further idealized, for this example, as a resulting in a pure coalescence of star ${ }_{1}$ and star $_{2}$, with no fragmentation of their bodies, or loss to their masses, so that they merge completely into a new star, denoted star ${ }_{4}$, which did not exist before $t$., and whose irruption thus coincides with the disappearance, or 'dis ex[is]tent[iat]ion,' of both star ${ }_{1}$ and $\mathbf{s t a r}_{2}$.

This synchronic 'super-system' is thus also represented as a 'diachronic meta-system', i.e., as a temporal / historical progression of tw o distinct "'dynamical systems', involving not only '"dynamical evolution'" as merely quantitative change, within a single state-space \& control-space, of fixed dimensionality \& dimensional content, but also involving ontological, qualitative change, from a predecessor, triple-star system, to a successor, double-star system - the latter consisting of only star ${ }_{3}$ \& star ${ }_{4}$ - mediated through the 'metaevolutionary meta-dynamic' of a collision singularity. That event qualitatively transforms the dimensionality \& the dimensional content of both the state-space and the control-parameter-space ['mass(es)-space]. It does so because the post-singularity successor system is a qualitatively, ontologically different '"dynamical system"' vis-à-vis its predecessor system, the one that existed prior to the $t$. "'selfrevolution"' that irrupts inside the purely-quantitative, dynamical "'self-evolution'" of that predecessor system. That predecessor system is itself the very agent-of-action, or "subject"', which transforms itself into its successor system.

The state-space state-variables for the triple-star super-system as a whole consists of its classical "'phase-space"' variables, i.e., the "timevarying'" position 3 -vector, denoted $\underline{r}_{k}(\mathbf{t})$ for $\boldsymbol{s t a r}_{k}$, \& the "'time-varying'" momentum 3 -vector, denoted $\mathbf{m}_{\mathbf{k}} \mathbf{r}_{\mathbf{k}}(\mathbf{t})$ for $\mathbf{s t a r}{ }_{k}$.
The control-parameter-space is a space consisting of 1-D [ Dimension ]/ axis for each mass in the stellar-[super-]system, so that the operating-/control-point coordinates are $\left[m_{1}, m_{2}, m_{3}\right]$, for the predecessor system, \& $\left[m_{3}, m_{4}=m_{1}+m_{2}\right]$ for the successor system; a state-space of, initially, 18-D, \& a control-parameter-[mass-]space of 3-D, for a 'state / control meta-space' of 21-D.

## The Dialectic of the Dialectical Arithmetics: Specimen of a Higher 'Dialector' Arithmetic -

"Aufheben") Diagram for Ideographic Formula: "'Self-M eta-Evolution"" of Hypothetical, Collision-Destined Triple-Star System
[ Depicted as an 'intra-Coordinated' 'Meta-State' 'Meta-Dynamics' of a Multi-Star "'System"' and of its Individual-Star '"Sub-Systems'"]
Gravitational Force Formula Zero-Division, 'Self-Bifurcation', Collision-Singularity. Self-Transformation of a 3-Star Stellar-System into a 2-Star Stellar-System


Diachronic Direction / Dimension of 'Systems-Progression' - of "'Self-Evolution"' to "'Self-Meta-Evolution'" / "'Self-Revolution'" of Multi-Star System-Unit as '"Meta-Super-System"'

## [Nonlinear] Dynamical Systems Theory

in the Nonlinear-Differential-Equation-Unsolvability-Driven, False-Infinity- / Infinite-Residual-of-Singularity-Driven Transition to

> the Theory of the 'Meta-Dynamics' of 'Meta-System Meta-Evolution' via 'Singularity Self-Bifurcation' as Model of System «(Auto-Meta-Kinesis»", and of HISTORICAL DIALECTIC / DIALECTICS OF NATURE


#### Abstract

'SINGULARITY SELF-BIFURCA TIONS' involve a "coupling", or interconnexion, of one or more state-variables, each one forming a dimension \& axis of the system's state-space, with one or more control-parameters, each one forming an axis and a dimension of the system's "control-parameter-space"" The values of one or more control-parameters change - thus becoming dynamical 'control-variables' - driven by "time-driven" [i.e., by 'self-driven'] changes in the values of 1 or more state-variables. This "coupling" unifies state-space and contro-space into a single space: 'state'control meta-space', which itself becomes also a dynamical object, a dynamical variable. It changes both quantitatively and qualitatively, dimensionally, with each system self-induced 'singularity self-bifurcation', i.e., with each progressive advance in the resource-base of the system. Such advance thus constitutes a 'system self-revolution', and a 'meta-system transition', from one "'meta-stable'" system 'meta-state', to a qualitatively different, advanced, progressed '"meta-stable"' system 'meta-state', i.e., from one dynamical system to a new, distinct dynamical system in this 'self-bifurcation singularitydriven', resource-base / negentropy-source progression-driven, "meta-system"', or 'diachronic self-progression from old system to new system.



'SINGULARITY SELF-BIFURCATIONS' \& 'Change of Space' Model the key System Intra-action: internally-induced shifts -internal-system self-induced shifts -- in the value(s) of one or more dynamical system control-parameters that change the very dimensional, axial, state-variables and control-parameters content of the state-space, and of the control-parameter-space. Such "'self-bifurcations"' typically involve the draw-down, to zero, at a finite value of the time-parameter, of the value a dynamical function, situated in the denominator of the dynamical differential equation modeling the system, 8 Jor in the denominator of the solution-function for that differential equation, for a dynamical function which meters the conversion of an internalized, finite, basic resource of the system, one whose conversion energizes and drives that system's evolution. Completion of conversion of that internalized resource base is thus modeled by a zero-division singularity. A stellar example would be the completion of the fusion-conversion of stellar-core Hydrogen into Helium, which drives the star off of the "main sequence" of that 2-dimensioal state-space known as the "Hertzsprung-Russell Diagram". "'Self-bifurcations"' model system self-determination, or ""selfcontrol'". as opposed to the 'external control', or other-[system-]determination' that is modeled by "bifurcations", as conceived conventionally, in ""standard'" dynamical systems theory.

The state-space is conceived, in conventional dynamical systems theory, as a non-dynamical, statica/ structure, with a fixed number and content of state-variables as its axes and dimensions. Likewise, the control-parameter-space is conceived as non-dynamical, statical structure, with a fixed number and content of control-parameters as its axes and dimensions. Moreover, these control parameters' values, unlike the values of the state-variables, are considered to be fixed constants, corresponding to the constant "coefficients", and other constant "parameters", of the dynamical-system-modeling differential equation(s): time invariant-except when an external controller "shifts" those values, thereby inducing a "bifurcation" in the stale-space trajectory, and in the "attractor(s)", of the system.

## Control-ParameterSpace

"BIFURCATIONS' Model Systems interaction: externally-induced shifts - external-system -induced shifts -- in the value(s) of one or more dynamical system control-parameters, change the dynamics - the "flow" or "vector-field"; the basin / separatrix / attractor / repellor structure -- of the state-space. Such parameter-shifts depict as a change in position of the dynamical system's "'control-point" in its "'controt-parameter-space",", due to a change in value of one or more control-parameter "coordinales" of that '"control-position'", or '"control-point"'.

# The Dialectic of the Dialectical Arithmetics: Specimen of a Higher Arithmetic - 

Applied to the 'Self-Meta-Evolution' of a Hypothetical, Collision-Singularity-Destined Triple-Star System<br>[ Formulated as the 'Meta-State' 'Meta-Dynamics' of a 'Super'-System' in its 'State/Control Meta-Space' ]

## '"M eta-M odel"' Design Principles:

- "Meta-Model": We term this ideographical-linguistic construct a " $m$ meta-model"' because it is "' made up out of'" two "models" in the standard sense, the first, a model of the 3-body system, before $\mathbf{t}=\mathbf{t}$., the other a model of the $\mathbf{2}$-body system, after $\mathbf{t}=\mathbf{t}$.. We call the systemsprogression from the three-body system to the two-body system a "diachronic meta-system"'. Since we are view ing each of the two multi-star systems in that systems-progression as "synchronic super${ }^{1}$-systems", given that the two multi-stellar-systems are each made up out of stars which are themselves view ed here as systems, i.e., as "super ${ }^{0}$-systems"'. This "meta-model"', as a whole, thus addresses a 'diachronico-synchronic meta-super ${ }^{1}$-system"'.
- "'Holistic Notations"' and "'Non-Reductionist Mathematics / Mathematical Modeling"': The dialectical-ideographical representation of this "'meta-super'-system"' explicitly represents both the dynamical state of the super'-system as a dynamical system, and the dynamical state of each of the individual stars - as the dynamical systems which constitute that higher-level dynamical super-system-and with "'equal billing"' to both levels of the two-level, two-"'scale"' 'meta-fractal', 'meta-monadic' structure of this "'meta-super ${ }^{1}$-system'". That is, the super ${ }^{1}$-system is not "'reduced" to its constituent system-units, nor is the super ${ }^{1}$-system unit, as an abstraction from the greater complexity and concreteness of the constituent system-level, represented by the super-system level, allow ed to stand on its ow n. Via these "'syntactics"', causal 'intra-actions' - those operating from the system-level to the super-system level, as w ell as from the super-system level to the system level -- may be expressed explicitly, via 'inter-argument-ation' of the state-variable functions [ $\&$, generally, in 'self-bifurcation' scenarios, also of the control-parameter functions]. That is, the functions $T_{k} \& L_{k}$, in their explicit forms, will contain $\underline{r}_{k}(t) \& \underline{p}_{k}(t)$ function-values in their arguments, or operands, \&/or the functions $\underline{\underline{r}}_{\mathbf{k}}$ \& $\mathbf{p}_{k}$, in their explicit forms, will contain $\mathbf{T}_{\mathbf{k}}(\mathbf{t})$ \& $\mathbf{L}_{\boldsymbol{k}}(\mathbf{t})$ function-values in their arguments, or operands.
- The state-variables of the initially 3 -star super-system are those of the classical "phase-space" version of "state-space". There are 3 physical-spatial-directional components for position for each star, one for each of the 3 mutually-perpendicular physical-space dimensions defined for 3 -dimensional space coordinate systems - generically ${ }_{k} r_{x}(t),{ }_{k} r_{y}(t), \&_{k}{ }_{r}(t)$ for $s t a r_{k}$-- the coordinates defining the physical-spatial position of $\mathbf{s t a r}_{\mathrm{k}}$ at instant t , plus 3 physical-spatial-directional components for momentum, defining the $\mathbf{3}$ additional perpendicular-directional "'mass-times-velocity"' components for star ${ }_{k}-$ generically ${ }_{k} p_{x}(t),{ }_{k} p_{y}(t), \&_{k} p_{z}(t)$.
- The state-variables for each individual star as a "'system"' - or for each star as a '"sub-system"', if the multi-star stellar-system level is modeled as the 'system' level - are those of the "stellar main sequence" Hertzsprung-Russell Diagram, or Temperature-Luminosity Diagram, employed here as a 2-dimensional, $\mathbf{T}_{\mathrm{k}}(\mathrm{t}) \perp \mathrm{L}_{\mathrm{k}}(\mathrm{t})$, state-space for each star, denoted generically as $\mathbf{s t a r} \mathrm{k}_{\mathrm{k}}$.


## The Dialectic of the Dialectical Arithmetics: Specimen of the Higher Arithmetics -

Applied to the 'Self-Meta-Evolution' of a Hypothetical, Collision-Singularity-Destined Triple-Star System

## Definitions of Ideographic Symbols Used-

t $=$ The Real-number time-variable; $\underline{\mathbf{e}}_{8}$ denotes the ortho-normal unit-length vector in the 8 th linearly-independent direction;
$\mathbf{m}_{\mathbf{k}}=$ mass of star $\mathbf{k}_{\mathbf{k}}$ control-parameters for stellar-super-system;
$\underline{\mathbf{r}}_{k}(t)=$ position in physical-space of mass-center of star ${ }_{k}$ as of time $t$, as 3 -vector ${ }_{k} r_{k}(t) \underline{e}_{x}+{ }_{k} r_{\gamma}(t) \underline{e}_{y}+{ }_{k} r_{x}(t) \underline{e}_{z} ;$ state-variables for the stellar-super-system';
$\underline{\underline{r}}_{k}(t)=\underline{v}_{k}(\mathbf{t})=$ change-rate of position with time-change, or velocity, of star $r_{k}$, as of time $\mathbf{t}$, as 3 -vector ${ }_{k} v_{x}(t) \underline{e}_{x}+{ }_{k} v_{y}(t) \underline{e}_{y}+{ }_{k} v_{z}(t) \underline{g}_{z}$ $\mathbf{m}_{k} \underline{r}_{k}(\mathbf{t})=\mathbf{p}_{k}(\mathbf{t})=$ momentum, of star ${ }_{k}$, as of time $\mathbf{t}$, as $\mathbf{3}$-vector ${ }_{k} \mathbf{p}_{x}(t) \underline{e}_{x}+{ }_{k} \mathbf{p}_{\gamma}(t) \underline{e}_{y}+{ }_{k} \mathbf{p}_{z}(t) \underline{e}_{z} ;$ state-variables for the stellar-super-system;
$\mathbf{r}_{\mathbf{j k}}(\mathbf{t})=$ radial distance between mass-center, or mass-point, of star $_{\mathrm{j}}$, and mass-point of star $\mathbf{k}_{\mathrm{k}}$, as of time $\mathbf{t}$, such that $\mathbf{r}_{\mathbf{j k}}(\mathrm{t})=\mathbf{0}$;
$\oplus \quad=$ Addition operation, generalized to encompass [quanto-]gualifier meta-numbers as well as pure quantifier numbers;
$\Theta=$ Subtraction operation, generalized to encompass [quanto-lqualifier meta-numbers as well as pure quantifier numbers;
$\cap=$ "'Dieiectorial diacritical mark" indicating that a 'meta-numeral so marked denotes a unit-gualifier meta-number [of unit "length" or unit "modulus"]

- = "'Diophantine"', 'omicron headdress'; dialectorial diacritical mark"' indicating that a 'meta-numeral' so marked denotes a quantiflable qualifier meta-number,
$\checkmark=$ "'Dialectorial diacritical mark"' indicating that a 'meta-numeral so marked denotes a 'strongly contra-Boolean' qualifier meta-number,
$\widehat{B}_{k}=$ 'meta-numeral denoting an unquantifiable ontological qualifier 'meta-number' for a super' ${ }^{\prime}$-system:
$\widehat{\hat{\alpha}}_{\mathrm{o}}=$ 'meta-numerar denoting an quantifiable ontological qualifior 'meta-number' for the kth super${ }^{\circ}$-system, i.e., for the kth [a super'-system-subsumed] system;
$\stackrel{\text { - }}{\stackrel{y}{r}}=$ 'meta-numeral denoting an quantifiable ontological qualifier 'meta-number' for the kth state-variable, or for the kth a control-parameter,
$\stackrel{\mu}{\stackrel{\mu}{\circ}}$
$\underline{T}_{k}(\mathbf{t})=$ Surface $^{\mathbf{T}}$ emperature of star ${ }_{k}$ as of time $\mathbf{t}$; state-variable for each star as a system subsumed within multi-star stellar-super-system as a whole;
$\underline{\mathbf{L}}_{\mathbf{k}}(\mathbf{t})=\underline{\text { Luminosity }}$ of star $_{k}$ as of time $\mathbf{t}$; state-variable for each star as a system subsumed within multi-star stellar-super-system as a whole;
t. $\quad=$ The moment of irruption of the Newlonian gravitational-force collision singularity for star ${ }_{1}$ and star ${ }_{2}$


## The Dialectic of the Dialectical Arithmetics: Specimen of the Higher Arithmetics -

Applied to the 'Self-Meta-Evolution' of a Hypothetical, Collision-Singularity-Destined Triple-Star System Dimensional Analysis Arithmetic used for 'Metrical Qualifler meta-numbers' and for 'Metrically \& Ontologically Co-Qualified Quantifiers' -

$$
\begin{aligned}
& \stackrel{\stackrel{0}{H}}{\stackrel{\circ}{\dot{u}}} \\
& =\text { 'meta-numeral denoting quantiflable metrical qualifler 'meta-number' for the 't(monad)-of measure', or "'unit-of-measure"; of Time, } \underline{\mathbf{T}}^{11} \text { : } \\
& =\text { 'meta-numeral' denoting quantifiable metrical qualifier 'meta-number' for the 'रmonad"-of measure', or "'unit-of-measure'", of Mass, M } \\
& \underset{\underset{\sim}{\underset{\sim}{u}} \underset{\sim}{\circ}}{\underset{\sim}{\sim}} \\
& =\text { 'meta-numeral denoting quantifiable metrical qualifier 'meta-number' for the '《<monad"-of measure', or "unit-of-measure"', of L्ength, } \underline{\underline{L}}^{\text {in }} \text {; }
\end{aligned}
$$

$x$
$=$ 'quanto-qual/fier', or 'qualo-quantifier', representing the triple-product of a quantifier, a 'metrical gualifier', \& an 'ontological qualifier':
$\stackrel{O}{0}_{n}$
$=$ Physical-spatial position-vector value for star ${ }_{k}$ as of lime $t$. fully-qualified, metrically [in $\underline{L}$ units], \& ontologically [as state-variable type 1]:
$\stackrel{\mathrm{O}}{\mathrm{m}}_{\mathrm{k}} \quad=$ mass value for star ${\underset{k}{k}}$, fully-qualified, metrically [in $\underline{\mathbf{M}}$ units], \& ontologically [as super-system state-variable/control-parameter type 3$]$;
$\circ$
$T_{k}$
${ }^{\circ}$
$=$ Iemperature value for star ${ }_{k}$ at t , fully-qualified, metrically [in $\underline{\theta}$ units], \& ontologically [as ssystem-level state-variable type 1];
$L_{k}$
$=$ Luminosity value for star ${ }_{k}$ at t , fully-qualified, metrically [in $\underline{\underline{L}}^{2} \underline{M}^{\underline{T}} \underline{\underline{T}}^{3}$ units], \& ontologically [as system-level state-variable frpe 2].

The Dialectic of the Dialectical Arithmetics: Specimen from the Higher Arithmetics Applied to the 'Self-Meta-Evolution' of a Hypothetical, Collision-Singularity-Destined Triple-Star System

3-Body [Meta-Super-]System "'Meta-Model"', via a syntactic unit / <monad) / 'meta-numeral' of the ${ }_{\mathbf{R}} \mathbf{q}_{\text {RAMON }} \mathbf{m}_{\mathbf{R}} \mathbf{q}_{\text {BAMU }} \leftrightarrow \mathbf{q}_{31}$ 'Dialector' Arithmetic

(b)

-- with "'auxiliary quantifier singularity-factor"' $\mathbf{f =} \mathrm{r}_{\mathbf{1 2}}(\mathrm{t})$, modifying the 'system qualifiers' of $\mathrm{star}_{\mathbf{1}}$ \& star $_{\mathbf{2}}$, but not those of star $\mathbf{r}_{\mathbf{3}}$. More compactly -
b
 8 wherein $\boldsymbol{\pi}$ denotes the highest positive integer expressible within the word-size of the computer in use for this modeling application.

The Dialectic of the Dialectical Arithmetics: Specimen from the Higher Arithmetics -
Applied to the 'Self-Meta-Evolution' of a Hypothetical, Collision-Singularity-Destined Triple-Star System

for $\mathrm{t}=\mathrm{t}$., the 'state/control meta-space' " "meta-state"' of this $3 \rightarrow 2$-star [meta-supert-]system in " meta-system transition"' is represented by -


-- wherein $\mu_{0}$ asserts the absent, no-longer-extant, presently unmanifest character of star $\mathbf{1}_{1}$ as of its collision with $\boldsymbol{s t a r}_{\mathbf{2}}$ at $\mathrm{t}=\mathrm{t}$, and where $\mu_{0}$ likewise, asserts the absent, no-longer-extant, presently unmanifest character of star $\mathbf{z}_{\mathbf{2}}$, as of its collision and coalescence with $\mathbf{s t a r} \mathbf{1}_{\mathbf{1}}$ at $\mathbf{t}=\mathrm{t}$, therein forming / irrupting star ${ }_{4}$.

# The Dialectic of the Dialectical Arithmetics: Specimen from the Higher Arithmetics - 

## Applied to the 'Self-Meta-Evolution' of a Hypothetical, Collision-Singularity-Destined Triple-Star System

3-Body [Meta-Super-]System '"Meta-Model'", via a syntactic unit / <monad" / 'meta-numeral' of the $\mathbf{R}^{\mathrm{q}_{\mathrm{BAM}}} \mathbf{m}_{\mathbf{R}} \mathrm{q}_{\text {BAMu }} \leftrightarrow \mathrm{q}_{31}$ 'Dialector' Arithmetic

for $\mathbf{t}=\mathbf{t}_{*}+\Delta \mathbf{t}$, the 'state/control meta-space' "'meta-state" of this $\mathbf{2}$-star [meta-super ${ }^{1}$-]system is represented by --


Commentary on the 'Meta-Monadic', 'Units within ... Unit', '«Monads»> within . . <<Monad»' Syntax of the Numerals / Units of the Higher Systems of Dialectical Arithmetic. The 'symbols-formation' above instantiates the "'syntactics'" of the higher systems of dialectical arithmetic, that arise, in the 'meta-systematic dialectic' of the presentation of
the progression of the systems of dialectical arithmetic and of dialectical algebra, as generated by the Seldon-function [ $\left.\frac{\mathbf{N}}{\underline{2}}\right]^{\mathbf{t}}$, from the system of dialectical algebra/arithmetic of the ${ }_{\boldsymbol{N}} \mathrm{q}_{\text {AMU }} \leftrightarrow \underline{q}_{15}$ 'meta-numbers', onwards. These "'syntactics'" involve a "'non-reduclive"' operation of 'gualitative division'. By 'qualitative division' here, we mean the division of a [quanto-]qualitative unit fy], or "(monad"), into its constituent, heterogeneous, [quanto-]qualitative sub-units, or sub-«monads", so that each 'super"-system is represented as a single unit, i.e., as a singular unity, a "'hol", but also, simultaneously, concurrently as a "'composite"', as a "'composition"', or c(arithmos), of multiple, heterogeneous sub-units, sub-unifies, or sub-ho/s, each of which may, in turn, be also, simultaneously, represented as a "composition, or "tarithmos", of multiple, helerogeneous sub-sub-units, and so on, in a ""finite regress"", to some finite exlent, sufficient for the descriptive/ modeling purpose at hand. This is accomplished by a syntax involving multiple
"'fraction bars", which are also, in this (quanto-]qualifier 'meta-number' context, "'meta-fractal bars'". Here, in the 'symbols-formation' above, a super'-system-qualifier 'meta-numeral' "'"ultimate numerator"', consisting of a single beta 'super'-system qualifier', represents the super'-system as a unity, with a " first denominator'", below the first bar, consisting of an inhomogeneous, non-amalgamative sum of metrically and ontologically-qualified state-variable and control-parameter quantifiers, specifying the 'meta-state' of that super'-system as a whole as a function of time, followed by a " second denominator"', below the second bar, consisting of meta-number non-amalgamative [quanto-lqualifiers, the two, differently-subscripted alphas, non-amalgamatively summed, representing that 'super'-system also as a multiplicity, via the two distinct, qualitatively different, 'system-qualifiers', or super-system qualifiers, denoting the two 'system-constituents' of that super'-system, followed by a "third denominator'" for each super${ }^{\circ}$-system qualifier, such that each of these denominators consists of an inhomogeneous, nonamalgamative sum of metrically and ontologically-qualified state-variable and control-parameter quantifiers, specifying the 'meta-state' of its specific super - -system as a function of time. That is, below the first bar that is yet another bar, beneath which is an inhomogeneous, non-amalgamative sum of two distinct, alpha, or super${ }^{0}$ - system, i.e., system, qualifiers, each representing a different systemcomponent of that overall super'-system. Alternatively, we might say that alpha-sub-three and alpha-sub-four each denote a distinct sub-system of the system, denoted, as a unitfy], by beta. Each of these sub-unities, alpha-sub-three and alpha-sub-four, is, in its turn, represented also as a multiplicity- that of an inhomogeneous, non-amalgamative sum of metrically-qualified and ontologically-qualified state-variable [and, in general, also of controt-parameter] quantifiers. All of these 'non-reducing', non-amalgamative sums are made possible by the recognition and application of the relationship of mathematical, arithmetical, but non-quantitative inequality, i.e., of gualitative inequality, that holds between the various terms of these sums, and between the various levels of 'qualitative numerators' and 'qualitative denominators', representing the successive 'meta-fractal', 'meta- « monadic,' '"'scales'" of these 'meta-super-system-atic' '"hols'", or 'unitliels'. These "'syntactic"' principles thus construct structures of 'meta-fractal fractions', of '[quanto-]qualitative fractions', based upon operations of '"non-reductive'"' 'qualitative division', as well as of "'non-reductive"', 'qualitative addition', involving 'division(s) of the qualifiers', "'qualitatively-divided unitfliejs'", and multi-leveled, scaled, explicit '"divisions of unity'", creating these [finitely-]continued [quanto-]qualitative fractions, with multiple levels of 'Iquanto-]qualitative numerators' and of their 'quanto-]qualitative denominators'.

