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Dialectical Ideography

A Contribution to the Immanent Critique of Arithmetic

Postscripts

Epilogues

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Note: This essay is a partially self-exemplifying exposition of, and a record of an ongoing self-critique of, the ideas advanced herein. It is a 'meta-dynamical' and 'meta-evolving' conceptual object. Edition index [self-edit iteration number] and last revision date are stated on the title page; the most recent changes are coded as plum-colored text. The time sequence of changes in the form / content of this essay is predicted to be both an illustration and an instantiation of the meta-model of ontological meta-dynamics that this essay explores, as well as of the 'homeomorphic defect' of that meta-model. We expect that successive editions of this document will document an 'ideo-onto-dynamasis' rather than an 'ideo-onto-stasis'; a 'meta-evolving ideo-ontology'; a 'multi-meta-ontic idea-cumulum'; an expanding, and ever 'thickening', increasingly inter- and intra-connected, 'inter-acted' network of 'inter-implicatory', 'inter-determinate', 'inter-generative' ideas, elaborated on a mounting count of 'metafinite', 'meta-fractal' scales, all exemplifying a "non-standard", Contra-Boolean logic; the ontologically dynamical logic of the dialectical "law" of cognition signified by the 'ideo-ontological', 'pure-qualitative', Ω -algebraic inequation $x^2 \frac{1}{2} \neq x$.

This writing is an unpublished work, and one which is not sold or exchanged for remuneration or commercial gain of any kind, but is distributed «samizdat» to selected individuals and organizations, on a donation basis, free of charge. This work is a potential contribution to the collective creative property of the Terran human species: assimilate, disseminate, critique, and surpass at will. We, the authors, seek hereby to further neither our monetary riches, nor our public power, nor our personal fame. What we want, money cannot buy. We hope, with your help, to build a better us, and to help do our infinitesimal part in building a better universe ["Infinitesimal" differences can matter, as nonlinear dynamics demonstrates]. More monetary wealth will not buy that betterment. More political power cannot impose it. More fame would mainly distract from it. We hope that you have chosen, or will choose, to build a better you. We hold that this choice entails the profoundest consequences for one's life, as well as for the lives of others. We also hold that such choices belong to you alone. We wish to share, with you, the forthcoming conceptual riches. We will rejoice, and we will be compensated, if you teach us in turn, help us to correct our errors, and thus advance the common-wealth of all beyond this offering. We also request our readers' forgiveness in the areas of our many shortcomings, some of which, though determined to strive ceaselessly to overcome them, we will never, in a lifetime, overcome. We, the authors, are not publicly accessible, but will endeavor to continue private transmittals to you if you indicate publicly, however cryptically, and we recommend that it be cryptically, your desire that we should do so. We want not that our existences, let alone our egos, should be an impediment to that great reverberating propagation of new cognitions, and of emerging new forms of cognition, of which this essay is, at best, an incomplete, imperfect, transitory, and transitional manifestation. We therefore happily forego personal credit, and, by thus renouncing in advance the [remote] possibility of any notoriety resulting thereby, hope also to retain more lifetime for the continuation of this work. Dialectical ideography as set forth herein is interpreted variously as: (1) a calculus of 'quanto-qualitative change', encompassing an explicit, ideographical arithmetic for the dimensional unit[ie]s or metrical "monads" of classical "dimensional analysis", and, thereby, 'semantifying' the "meaningless" singularities [finite-time "infinite" values] of especially the "unsolvable" [in part, because of those very singularities] nonlinear integrodifferential equations and their solution-functions through their metrical 're-qualification' using those new, explicit 'metrical qualifiers' of this 'dimensional arithmetic', concretizing and operationalizing Plato's «arithmoi Monadikoi» and Diophantus' \mathbb{M} ; (2) an alternative, onto-logical contra-Boolean algebra; (3) an ideographic, 'onto-dynamical' "symbolic logic" for the state-space/control-parameter-space 'meta-dynamics' of 'meta-finite', conversion-singularity 'self-bifurcation'; (4) A mathematics for modeling the history of mathematical ideas as well as a [psycho-]historical algebra and arithmetic for modeling the 'meta-evolution' of the sciences generally; an ideography for the [psycho-]history of ideas; an ideography of the 'meta-dynamical' logic of conceptual self-innovation and self-development; a 'philosophical algebra' or trans-Leibnizian «characteristica universalis»; an arithmetic and algebra of innovative conception or of the creative conceptual process; (5) a rules-system for an ideographical language of qualitative self-escalation in self-transcending [meta-]systems; (6) a generic algorithm for the 'meta' operation regress; for a trans-Hegelian, autopoietic version of the 'aufheben' operation; and for a "dynamical", 'temporalized', diachronic, 'meta-evolutionary' version of the Russellian/Gödelian logical types hierarchy; (7) a model for a 'meta-fractal', non-Cantorian theory of totalities, of 'meta-finite' arithmetics, and of the "foundations" of mathematics; (8) an arithmetic, algebra, geometry, and analysis built on certain "non-standard natural numbers", i.e., on the 'Gödelian 'meta-natural' numbers', a space of 'evolute' "hypernumbers" of 2nd degree, 'made up out of' "standard", '1st degree' natural numbers, instantiating those "non-standard models of first order Peano arithmetic" whose possibility is implied by the first-order conjunction of Gödel's completeness and incompleteness theorems, as by the Löwenheim-Skolem theorem, constructing thereby an 'ontologically dynamical', 'de-Parmenideanized' actualization of Plato's 'arithmetic of dialectics', the «arithmoi eidetikoí». This essay, in addition to that of ideogramic, pictogramic, and phonogramic symbolization, draws also upon the power of neo-mythological, allegorical, and mythopoeic – that is, of psycho-historical – symbolization to aid in the conveyance of its most urgent messages. World-historically consequential universal labor, effective psycho-historical force, including individual 'psyche-ological', affective force, requires \mathbf{g}_{SPR} ; requires that its mythopoeic momenta, \mathbf{R} , be integrated, indeed, dialectically synthesized, with its Philosophical and Scientific momenta. Dialectical ideography is, we believe, a humble but potent seed. As with the several non-Euclidean geometries that arose from the failed attempts to prove the absoluteness of Euclid's geometry, these non-Parmenidean, contra-Boolean, and contra-Cantorian onto-logical and onto-dynamical arithmetics and their algebras of dialectics may bear fruit for humanity only if germinated through the intra- and inter-personal dialogue, and dialectic, of assimilation, critique, refutation, and supersession. We have avoided broadcast publication and indiscriminant distribution of this essay. We wish to base its circulation, and the selection of its recipients, upon our best judgement of its potential value to each candidate recipient. The taking to heart of the ideas "graphed", both ideographically and narratively, herein, can produce profound transformation in the very identity of the person so taking. Panic in response to perception of the early signs of such transformation in others may elicit, from some perceivers, a violent reaction. In particular, the intimations of the 'meta-human', Δh , implications of the 'cumulum' of human[oid] evolution is profoundly disturbing to some. We are therefore transmitting this document only to those whom we perceive, via their own published writings, to be already verging on similar or related conceptions as a result of their own protracted 'self-meta-evolution'. We have also decided not to disseminate the most "dangerous" of the results to date. We believe that you are eminently capable of 're'-discovering these results, if you have not yet discovered them already. Should you do so, we urge that you treat them, and their dissemination to others, with utmost care. The system, more accurately, the systems, of dialectical ideography glossed herein continue to evolve and 'meta-evolve' rapidly in our research. They burgeon beneath our feet. We expect to exercise a similar restraint and discretion in any future progress reports which we may send your way. We therefore lodge the Omni-Copyright statement above together with this countervailing caveat: we recommend that you disseminate this document, its ideas, and/or related ideas of your own discovery, with careful judgement. Give the friends of humanity a head start vis-à-vis their adversaries. Dialectics should inculcate humility. "Perfection" is not a final meta-state that can be finally manifested, but an open-ended, 'uncompleteable', asymptotic process, moving from greater to lesser imperfection. We realize that conceptual 'homeomorphic defect' is inescapable for cognizing beings such as ourselves. Even at best, we must always be partly wrong. Even at best, one cannot be finally, completely, and wholly right. One's mental constructs cannot ever be the truth, the whole truth, and nothing but the truth. But one may be right enough for one's time, for one's moment, for one's role, and for one's part; right enough to help one's contemporaries to live through, and beyond, one's time, and thus, potentially, to enjoy the privilege, the pain notwithstanding, of a vital [life-ful] and willing participation in the succeeding epoch of imperfection.

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No doubt because our interaction is non-commutative, to say the least, two 'Afterwords' have emerged. See if you can tell operator from operand, anima from animus, for each.

Karl Seldon

#

Sophya St. Germain

for

Foundation Encyclopedia Dialectica
Terminus, CA, 25 November 1999

Afterword [I]: Terran Humanity at the Brink

The calculus of retortion set forth above is also a calculus of accountability, and a calculus of retorsion: no action goes unrequited. Every action, every actor, is self-requiting. What we *have* visited upon others *will be* visited upon ourselves.

Is it well then, that *The Nonlinearity Barrier* still constrains us? *The Nonlinearity Breakthrough* can empower us to create a global renaissance of humanity. It can also enable us to create new and even more terrible weapons; to "create" planetwide destruction, with even greater human self-devastation than ever before.

To attain the former, and preclude the latter, it is not enough to have defeated one Hitler and one Stalin in our outer world during this century.

As the Hydra's headless necks sprouted two new maws for each one that Hercules hewed off, so this world can, and does, sprout new Hitlers and new Stalins *ad nauseam*.

Only to the extent that each living person fights World War II and the Cold War anew, *within*, and wins, can the world become safe for humanity, and humanity safe for the rest of the universe. Only to that extent can we confront, and overcome, the hideous and unnamed invisible face and invisible mind of the global "Invisible Hand". This monstrous power can, at will, and with the speed of the electron, throw down whole nations into decades of destitution and desperation, hurling emergent middle classes back into the dark ages of "Third World" deprecation and deprecation -- this monster that is the unconscious self-oppression of Terran humanity, mediated through its plutocratic econo-political elites.

Each human soul holds divine potential and satanic potential. Each harbors both the seed of divinity, and the seed of iblis. We have by now built a world which *potentially* empowers each individual, vis-à-vis our species, and its future, to unprecedented degree. The attainment, survival, and self-prosperization of humanity all depend upon the degree to which each of us slays -- or, if you prefer, tames -- the monster within each one of us. For each of you is, as am I, endowed with an inalienable blemish, and burden, of evil -- inalienable in that only you, through the expressed consent of your own self-effort, can enable it to be subdued and, at last, extinguished, before it extinguishes you, and, perhaps, with you, the rest of Terran human kind.

Insufficient inner, individual self-conquest on Earth to date has exacted an unended, refluent reverberation of outer conquest and subjugation. It has ended every social self-bifurcation with the abiding refrain: "meet the new boss, same as the old boss". Meet *self*. For our "bosses" can never be better than we deserve -- never better than a reflexion and a *refluxion* of ourselves, of the content and quality of our own past actions!

There is a quanto-qualitative requirement for this revival, and global renaissance, of Terran humanity. *Enough of us* must change ourselves. Enough of us must *change ourselves enough*, change ourselves enough in terms of the *quality* of our personal, human meta-evolution, hence action. Enough of us must change ourselves enough *in time*, before the window of opportunity for successful passage of its 'Planetary Selection Test' closes forever for planet Earth.

If -- and what an enormous and fragile if it is -- if enough of us relinquish our chronic denial, cease our incessant exo-projection of our own monstrous fault as if all of the fault were only in others, and see fit to confront and to accept what is required of us to conquer the Hitler and the Stalin inside each one of us, then we may yet acquit ourselves of our inhumanity. If we are so exonerated, in the years just ahead, by the jury of our own self-requital, then a new and vaster prospect opens before us and our posterity.

If we meet the mark, victorious in battle on the plain kurukshetra, then this terrene seed in which we inhere will become a planet in waiting, at the edge of the galaxy, at the brink of greater reality -- to turn the dust of a trillion worlds into blood and flesh, and to awaken its long-slumbering spirit.

Overcome we the iblis within ourselves, and the cosmos will be our canvas. We ourselves will be our works of art. We ourselves will be our work of art.

Kindred Spirits --

We have alluded throughout this essay to a cognitive transformation, an emergence of new faculties, which we have found concomitant with the exploration, codification, and application of the dialectical organon set forth herein, and with its praxis of conscious self-reflexion. Here, at last, is the locus to expand upon that theme.

We have been embarked upon a cognitive experiment, an experiment in cognitive meta-evolution, for all of these past many years. We have found our way to a mode of expanding self-re-production which is unprecedented, in our previous experience of both ourselves and others -- although, for all we know, and given the luminosity of your own work, it may be most prolifically preceded in yours.

We have watched ourselves enter into what, subjectively, *felt* like a state of deepening communion with many minds, across the ages, that had previously plumbed the depths of possible knowledge; a state of deepening communion with the inner nature of reality; a state of continuous revelation, of almost daily discovery, of breakthroughs minor and major nearly every morning, with nearly every breakfast, with nearly every dusk. The fruits of the Tantra, some might say.

It is as if, having renounced the "Midas Touch", we have found the 'Midas Mind', and every topic it touches turns to conceptual gold! It is as if a cornucopia has opened inside us -- a cornucopia that pours out to mind.

We have found it difficult to write down *Dialectical Ideography*, difficult to materialize its message, so fast has been the growth, within us, of this '*reflexivity paradigm*'. Burgeoning beneath our feet, it has oft made what we wrote yesterday obsolete for what we learned today. This essay, already too long, yet leaves out so much!

Sporadic at first, in the earlier years, this mental opening has mounted to a new norm of continual mental fertility, continual, intra-dual conceptual coition, continual conception, continual conceptual pregnancy, and continual giving-birth, that has lasted, *sustainedly*, not just for nine days, nor nine weeks, nor nine months, but for the last nine years. Yet, far from abating, it quickens still. We judge the results of this time of accelerated learning with respect to what we knew before. What we have learned may seem trivial next to your accomplishments. By our self-relative criterion, the gain has been enormous.

For those who have been long on Safari, hunting Hidden Treasure deep in the Wild Lands beneath the Sky Of Mind, new faculties emerge. The vast Dream Manufactory of "the sub-conscious mind" is pressed into the service of the conscious. The two-way commerce between them begins to flourish. The hidden looms that nightly weave our sleeping dreams harness too to the waking mind, playing back, or presenting *real-time-live*, the self-presenting waking dreams of the mathematical mind's eye.

Many mathematicians can attest to this.

But do our cognitive outcomes differ *usefully* from such as have been observed, however commonly, before?

Our approach has been to veer away from the well-paved Parmenidean path, and to pick up again that seldom trodden trail that Heraclitus heralded, however imperfectly, so many centuries ago. We did not expect to find perfection along this path -- or along any possible path. We did expect that it would prove to have been worth exploring. Indeed, at length, it has catapulted us into "*the empyrean vision*": ". . . Just Behind . . . Sky of Mind . . . Burning Bright . . . Sky Of Light".

Our cognitive opening has been enabled by an abandonment -- the abandonment of that pervasive Parmenidean paradigm of a timeless, static, and stagnant world, to which even Einstein, in his "greatest blunder", succumbed.

Our opening was enabled by the acceptance, terrifying to the form of self-identity with which we began, of an *Open Universe*, and an 'acceleratorily' ever-self-expanding one, ever self-opening in a qualitative, ontological, conceptual, and *meta-evolutionary* sense, as well as in the quantitative sense of spatio-temporal extent[ion].

Our self-experiment commenced with a realization and acceptance of *ever-expansion*, of *ever-imperfection*, and of *ever-perfection*. Ever-incompleteness. Ever-newness. Unfinishability.

The Sisyphean *Set Of All Sets* can serve as one mental mandala for this *Weltanschauung*.

This *self-motile* conceptual object, would-be centerpiece of Set Theory, as the set-theoretical definition of the set itself, can ever be at most '*The Set Of All Sets Except Itself-and-Subsets*'. Its essence -- its definition -- is ever at odds with its existence. Logically driven to attain itself, to comply with its definition, it must *swallow itself*, must *internalize all of its sub-sets, proper as well as improper*. But it thereby alters itself again, and so excepts itself once more. Its every attempt at *being itself excludes itself* all over again, at a higher level each time, thus ever regenerating this movement of self-inclusion/*all-subsets-incorporation* as unending, cumulative ever-expansion. Its *being is this continual motion*, this *constant change*. It is this unending movement of *self-/subsets-incorporation*.

That auto-dynamism, that uncompleteable self-engendered movement of inclusion, including that continual movement of *self-re-entry*, is the true, operatorial definition, the true verb-essence, the true "*intension*" of the Set Concept: set as operation, as 'eventivity'. It is also a *reductio ad absurdum* disproof of Standard Set Theory's implicit 'staticist', Parmenidean presumptions.

Our opening was also enabled by *an opening to diversity* -- to the diversity *within*, as well as to the diversity *without*. It was enabled by an acceptance of the qualitative in the sense of *qualitative heterogeneity*, of *alternativity* -- of a '*multi-chotomy*' to replace the vicious sectarianism of absolutist di-chotomy; the fanatics', the terrorists', and the inquisitors' di-chotomy of absolute right vs. absolute wrong, absolute true vs. absolute false, orthodoxy vs. heresy -- of "my way or the highway".

All over today's Earth, people are being brought to mutual slaughter over "qualitative differences", in the name of "the" "one" "right" -- culture, economy, morality, race, religion, etc.

All factions agree on "one", and only on "one" -- *their* "one", a different "one" for each.

But humanity is potentially, and to some degree already actually, far more than the 'sum' of its racial, religious, ethnic, genderal, and ideological parts. True strength, viability, true prosperity, richness of life -- all of these reside in 'cultural heterosis', in societal hybridization, in civilizational "hybrid vigor" -- via *interacting and thereby co-poeisic and co-generative, memetically-ontodynamical diversity*.

That too is a key message of U, the very *Calculus of Diversity*, of *Heterosis*, of *Hybrid Vigor*, at every level of a *continuing* cosmogenesis -- a *continuing Creation* in which all 'multi-globalized', multi-planetary humanities have a part, both as Created and as Creator. *Vives les différences!*

Alternativity abounds. As in the biosphere, so also in the '*ideosphere*'. As with the more realistic, *nonlinear* dynamical systems models generally, many 'species' "fit", and "are fit"; many attractors, many niches work, many *ontos*, many solutions obtain. Only *fictitious*, *linear*-dynamical [*pseudo*-dynamical] models exhibit a single, global, monolithic, static, equilibrium *fixed point*. The solution to the ontological 'Existence Equation' of this cosmos is an evolving one, and a 'multi-chotomous' one of ever-growing multiplicity; of '*ever-more-multi multi-chotomy*', akin to the late Gene Roddenberry's "IDIC" principle -- "Infinite Diversity in Infinite Combinations".

Speaking of diversity, we of the Foundation live with it "close to home".

There is our impersonal, sometimes dour, somewhat stiff, austere, Spartan, and Spock-like "Doctor Seldon", allotted, from the heart of his soul, and from an early age, to service as a secret guardian of humanity's future, and as a finder of the way forward.

There is that sprite, "Sophya", so often the playful, child-like ecstatic, somewhat lacking in seriousness -- even frivolous, in some eyes -- insufficiently burdened by the tragedies and the sorrows of the past, as by the urgency of the present hour and task.

Now that's "Diversity"!

Yet she is the love of his life, and he of hers. Deeply wedded to one another, they are a Team -- Intra-Dual. Each is the other's greatest foil. Ever at war. Ever in love. Ever at-on(e)-ing, and ever at one. And the incessant dueling *between* them is also 'intra-duel' -- modeling the dueling that is also within each one.

We have established, *for ourselves*, that this opening is *a* way of accelerated individual meta-evolution. It is not *the* way. It is not for everyone. It is probably for only a few. A question you may now be asking yourself, if you have not considered or even tested that question already is: "This way, of opening to an open, ever new universe; a Heraclitean/Hegelian/Chardinian, acceleratory universe of *cumulative but ever-unprecedented, ever-singular, "irreversible", 'irrepeatably' self-complexification* [and also of 'self-re-simplification', via the 're-meta-unitification' or 'meta-monadization' of predecessor relative-atoms / -units / -monads]; this way of reconstructing one's conceptual and perceptual structures by overcoming their Parmenidean, nounist grammatical conditioning -- might it be a way forward *for me*"? It is a way of no final answers, of temporary answers, opening to new questions, then new temporary answers. But *cumulatively* so. Each new transcendence, each new 'transfinite' meta-state -- which cumulatively contains all previously-manifested 'transfinite orders' -- is only relatively "*transfinite*", is *self-relatively metafinite*. Once attained, it is seen to be [meta]*finite*. And its own self-discrepancy, self-incompleteness, and *intra-duality* begin at once to generate *the next metafinity, the next transcendence, the next meta-evolutionary leap; gradually at first, then suddenly*. There is no orthodoxy here -- no sect, no bureaucracy, no party line, no dogma, no inquisition. There is respect for your right -- for the right of each one of us -- to create, to choose, to make our own way of life. There is Respect for *Individual Sovereignty*; affirmation of Diversity; appreciation for *The Ubiquity Of Alternativity*. We will never consent to so much as touch the Sauronic, Satanic Ring Of Power by which the global plutocracies rule, through their opinion management, their 'hired liars', their vast bribery machine ["lobbying"], their drug trafficking, their secret police, their death squads, their "population control" designer diseases, and all of their underworld enforcers. We reject violence as a method of social change. Those who use it succeed, *at most*, in seizing '*The Mandate of Satan*', and thus in incurring their own *total self-defeat -- becoming* the very evil that they had sought to overthrow. We possess but one power -- and we possess it only to the extent that we have first allowed it to possess us: the power of dialectical reason. That is the only power -- forever checked and countervailed -- that we ever want to possess. It is a power *over* no one, but an immanent power in and through each one who recognizes, in it, a moment of that one's own essence, and begins to act accordingly. Let reason lead us. Let dialectics lead, leaping like wildfire from one human mind to the next -- and from one human heart to the next -- until the whole world is alight with a Light and an enlightenment monopolizable and 'monolithizable' by no one. If humanity cannot win out in this way: through the blossoming growth of its productive, creative forces, and through what that growth fundamentally *is* -- through positive contagiousness; through a widening wake of knowledge-awakening; through a deepening diffusion of decency, an unstoppable pandemic of physical, moral, emotional, intellectual, and spiritual healing and well-being, mounting and merging into global regeneration; into worldwide renaissance -- then humanity cannot truly win. To do what you can do, *to contribute according to your potential, to the ever-burgeoning wealth of life, and to the successful graduation of Terran humanity in its looming Planetary Selection Test*, you need no more connexion with us than you have already. You need not even know of our existence at all. We think that those of you who do 'hear the call' will know this already, but will want to know that we know it too. And yet together, in whatever Respectful ways we can interact and act together, we may all be enabled to contribute more. Despite all of our incessant suffering, despite the horrors of the fate which the iblis-possessed, iblis-enslaved Enemies Of Life would visit upon us all, and despite the looming certainty of aging and eventual bodily death, no matter who wins that War, there is still something within us which is called to Revel in the Glory of Continual Revelation, and of its Continual Implementation. That is *The Song Itself, The Song Itself That Dialectic Sings, The Song Celestial, The Symphony Of The Spheres, The Music of the Ainur, The Rhapsody Of The Vortices*.

In this moment, we are moved to speak for it, to give it voice, NOW. If the image of the dialectic set forth herein helps you, in any way, for even a moment, to connect more deeply to the Living, Ever-Burgeoning Actuality of which that image is but a shadow on the wall of *The Cave Of Mind*, so much the better -- so much the better for you, for the other lives that your life touches, for all the vital tasks to which your Aliveness calls you, now.

The Foundation operates from two centers of operations for the continuation of the work presented here, and for the preparation of the Encyclopedia. One is located on the East Coast of the Terran North American continent, at Stars' End, New York. The other is located on the West Coast of that continent, at Terminous, California. The Foundation is *not* a U.S. tax exempt foundation. It accepts no *monetary* contributions. It attempts to accept creative, ideative contributions from all. We hope, even if in vain, to keep our theory abreast of our own actual life-praxis. Nor will we stop trying to keep up with yours!

Encyclopedia Dialectica [*E.D.*] --

- will give its *definitions* of each '[ev]entity' entry listed in its encyclopedic 'dictionary' in the form of a '[quanto-] qualitative', ideographical 'meta-model' of the 'meta-system meta-dynamics' -- of the reconstructed past, and the predicted future 'meta-evolution' -- of that[ev]entity, written in the languages of the 'dialectical ideographies'.
- will be, in effect, a "HoloDeck" [cf. Star Trek] for scenario-valued simulation, and for omni-esthesis representation, of dialectical-ideographic models of universe meta-evolution within all known 'levels' or "scales". These models, and the undergirding uninterpreted arithmetics of \underline{Q} , \underline{U} , and $\underline{\mu}$, form its core content, and the central-most drivers of all of its 3+-dimensional dynamic holographic data-visualization displays.
- aims to embody a *unified summary/archive* of the meristematic totality of extant Terran human knowledge to-date.
- is envisioned as an interactive, 'dialogic' presentation, mediated via a kind of 'dynamical book' software medium. Its 3+-dimensional virtual reality user interface will orchestrate multi-media holographic-dynamic peripheral displays. This user interface will unify animated, dynamic-pictographic, -textual, -numerical /-ideographic, and aural /voice/other-sound communications to and from its users.
- must be a meta-dynamical, meta-evolving, ever self-obsolescing, continually self-updating *eventity* which, like Hegel's "Owl of Minerva", ever-retrospectively assimilates and consolidates the latest branchings and leafings from the living meristem of The Tree of Human Knowing. It thereby records a 'snow-balling' of knowledge within itself [i.e., within the new terrain of itself, which it is continually laying down before itself], as also from the outer terrain laid down for it by external processes of knowledge formation.
- rather than an "En-Kyklios-Paideia", an "instruction in the circle of the arts and the sciences for the young", like the circle of Hegel's *Encyclopedia of the Philosophical Sciences*, Logic \rightarrow Nature \rightarrow Spirit \rightarrow Logic again, *E.D.* might better be termed an 'En-Speira-Paideia', an 'instruction in the *helical*, ever-opening, ever-rising, ever-cumulative *spiral* of knowledge, for the *ever-young*'. This '*Enspiropedia*' will thus constitute a 'Scripture Of Science'. By this we mean a compendium of empirically falsifiable, ever-self-superseding hypothesis, rather than of any "absolute" and "final", frozen truth. It will be a 'Book of Genesis' for the ongoing *Cosmogenesis* [cf. Chardin]; for the continuing self-creation of the cosmos, of which it is itself an "infra-finite" part, a [self-]changing, 'meta-evolving' part, like all of the rest. It will always be wrong, ever imperfect, forever afflicted with 'homeomorphic defect' -- always a process of perfection, moving from greater to lesser imperfection.
- using simulations of first-level, $n = 1$, meta-evolution models, $\{^1\underline{Q}_\tau\}$, $\{^1\underline{U}_\tau\}$, & $\{^1\underline{\mu}_\tau\}$, for the user-selected universe-of-discourse \underline{u} , portrayed per continually user-adjustable rate and mode, *E.D.* will enable users to "walk around inside" a multi-media model 'data-visualization-plus', equipped with 'meta-fractal' scaling zoom-in capabilities. Hypermedia links will enable users to jump to simulations of $\{^n\underline{Q}_\tau\}$, $\{^n\underline{U}_\tau\}$, & $\{^n\underline{\mu}_\tau\}$, wherein the $\{q_z\}$, $\{u_z\}$, & $\{\mu_z\}$ generic qualifiers are re-assigned to model deeper "scales" or "levels of sub-categorical detail",
 $n > 1$, of \underline{u} ; the intra-ontic order of \underline{u} 'meta-evolution' "within" each of \underline{u} 's first-level ontos.
- must implement capability to forecast the meaning-content of each $^n\underline{U}_{\tau+1}$, from its $^n\underline{U}_\tau$, namely, to predict:
[a.] *timings*; [b.] *changes in qualities*; added ontological, onto-dynamical attributes or sub-qualities defining each newly-emerged onto, and; [c.] *changes in quantities*, new quantifiers, or probability [frequency of encounter]/population [size, or percentage], $\{^n\rho_{\Delta\underline{u}_x}(\tau+1)\}$ & $\{^n\rho_{\Delta[\underline{u}_y; \underline{u}_x]}(\tau+1)\}$, $y < x = N_{\underline{u}}(^n\underline{U}_\tau)$, for all the new ontic qualities/qualifiers, $\Delta\underline{u}_x$ & $\Delta(^n\underline{u}_y)$; $\Delta(^n\underline{u}_x)$, in $\Delta(^n\underline{U}_\tau) \subset \Delta(^n\underline{U}_\tau) \subset ^n\underline{U}_{\tau+1}$, as well as for all of the old ontic qualities/qualifiers, $\{^n\underline{u}_x\}$ & $\{^n\underline{u}_y\}$, for all of their 'new [changed] quantities/quantifiers' in $\Delta(^n\underline{U}_\tau) \subset \Delta(^n\underline{U}_\tau) \subset ^n\underline{U}_{\tau+1}$, the new part of $^n\underline{U}_{\tau+1}$, updating to epoch-index $\tau+1$ the quantifications of old and new ontic qualifiers alike. These prediction *outputs* must compute principally and 'principally' from 3 inputs, each \underline{u} of $^n\underline{U}_\tau$: (1.) emergence *timings*; (2.) ontic [sub-]qualifiers, and; (3.) quantifiers, of all $\{^n\underline{u}_x\}$ & $\{^n\underline{u}_y\}$ in $^n\underline{U}_\tau$ and in all of its predecessors, which it, in turn, 'contains'; all of the *ancestors* in its evolute *cumulum*; all of the *preceding values* in its *time sequence = series*, all for the user's selected 'taxonomic level', n , within the user's selected universe-of-discourse \underline{u} .



The logo, reproduced above, represents a stylization of the ideogram $\underline{\underline{H}}$, [see *Glossary of Neograms*, below], wherein the double-underscore has become a double circle surrounding the \mathcal{H} as central symbol. The outer circle, in blue, denotes that which is carried forward, "conserved", in each conceptual self-bifurcation. The inner circle, in magenta, denotes that which is discarded in the course of each such 'meta-finite' transition. The ideogram $\underline{\underline{H}}$ forms a *still-life* symbol of the ever-self-re-moving *horizon* of the 'ultimate' mathematical Rules-System -- arithmetic, algebra, analysis, meta-analysis, and onward . . ., *ever-receding* as humanities and their *sequelae* advance, always just beyond those collective Subjects' then-present conceptual compass. This symbol thus signifies this Psycho-Historically, Psycho-Archaeologically derived and justified mental eventuality, an outcome of that empirical, retro-observational study of *The 'Meta-Evolution' Of Arithmetics* on planet Terra to-date, as of an analysis of the 'ideo-meta-dynamical' implications, for conceptual 'meta-evolution', of the Gödel Completeness and Incompleteness Theorems in conjunction, and of the Löwenheim-Skolem Theorem. We refer to this complex of implications, throughout this essay, as '*The Gödelian Ideo-Meta-Dynamic*' of dialectical '*Meta-Axiomatics*'.

This symbol has thus both a constant denotation and a variable denotation, at a conceptual locus where the concepts of 'variable' and 'constant' coalesce; a 'constantly variable' and 'variably constant' value. It is a 'change constant'; a 'constant of change', an implicand of **Heraclitean-Hegelian-Chardinian** = **—Parmenidean** foundational assumptions. The logo thus symbolizes the 'self-variable', 'meta-dynamical' character of the mathematical sciences, and of scientific knowledge -- of *empirically-disciplined hypothesis and theory* -- generally.

A humanity's conceptual horizon is ever '*self-variable*'; is 'internally', '*immanently self-variable*' as well as variable via the consequences of 'external' impacts upon it, including of its own "[pseudo-]external" impacts *back upon itself*. Empirically-disciplined and falsifiable, relative/revisable knowledge forms an immanently self-varying, self-growing 'ideo-ontological' conceptual '*cumulum*', one which is influenced in its developing content by the expansion of such a humanity's praxis, ecosphere, experience, and «*anti-physis*» in the course of that species' 'quanto-qualitatively' self-expanding self-reproduction. If that humanity truly lives, conceptual 'self-bifurcation' continues. 'Metafinite' self-transitionings of each "ultimate" axiomatization of its knowledge and «*organon*» continue. Conceptual 'self-singularity' -- the epochal, 'ideo-meta-dynamical' self-and-other-meta-development of its conceptual 'meta-system' -- goes ever on. The perimeter of its conceptual boundaries keeps advancing, widening; its conceptual horizon thus retreating; the circle of that horizon thus expanding. Such a humanity's "knowledge-base" keeps accumulating; in accumulating, keeps '[ideo-ontologically self-]bifurcating'; in so bifurcating, keeps self-revolutionizing.

The source of this symbol was the dream-vision of our founder, Karl Seldon, in his eleventh year. In that dream within a dream, he saw himself a nuclear scientist, cyclotron at his side, slumped asleep over his unrequited wee-hours' workings on the unity of Space/Matter and Time/Energy. In that scientist's ensuing dream, falling, spread-eagled-spinning through a green-and-violet vortex, thence repositied to a roomier, '8-D' realm, walking as if *through* rather than around as if translucent, curtain-like yet solid pillars, he met a wordless escort; a ~7-foot-tall, purple-enchitined, exoskeletal 'ant-man', who ushered him to the edge of a conical amphitheatre. Pointed-end-upward from bottom-center of that amphitheatre loomed a smaller, converse-conical stone altar. Carved topmost into that altar's stone?: the symbol \mathcal{H} . Below it?: a 90°-rotated, 'verticalized' equals sign. Below that?: a ramified symbolic expression replete beyond the mnemonic-capacity of his 11 years. Young Seldon instantly grasped that his escort's world was one in which mathematics and theology, science and philosophy, were as one. Thereafter, that *dream-within-a-dream* vision, and the quest for its \mathcal{H} equation, made \mathcal{H} the guiding, quickening symbol -- or '*sym-bolē*' -- for the '*bolē*', the life-trajectory, of Karl Seldon, counterpoint/counterpoise to its '*dia-bolē*', and the inspiration for his foundation of this Foundation.

The materials reproduced in excerpt below serve, for us, as a veritable symphony of hints and clues. Via this 'Psycho-Archaeological Excavation', we invite you to watch the wave of thought, and the broad river of consciousness of which this work is also but a minuscule part, wash over and through many minds across the ages [understanding that we do *not* mean by this to imply that the present work embodies some ultimate finality of that process, or to deny the burgeoning diversity of the ideational phenomena recorded below, embodying a vast heterogeneity of conceptual directions other than our own]. These extracts were selected for how they, in a catalytic way, favor, or disfavor, or even declare the impossibility of, the kind of 'Meta-Dynamical', *Dialectical «Characteristica Universalis»* that we seek. This *diachronically-ordered* sequence of extracts, reflecting the limitations of our own backgrounds, is, and will be, incomplete, omitting many thinkers whose thoughts belong in this train. We will add, in future editions, for as long as they continue, the selected psycho-artefacts of progenitors and 'co-genitors' -- newly discovered, or subsequently formed. Despite that, we shall inevitably fail to discover all that belongs here. [As elsewhere herein, *italic and sometimes bold and underscored black or blue-colored typeface* in a quote or extract signifies our emphasis, not the author's: throughout the quotations which follow, *such emphasis has been* added by F.E.D.].

a. From the period Before the Common / 'world market' Era [B.C.E.].

Ancient Babylonia, 8000 B.C.E., +: *A Theory of the Emergence of Written Language, via that of the «Arithmoi Monadikoi» --*

"Tokens are among the earliest, if not the first, clay artifacts in the Near East. . . .

The tokens constitute a rare source of information on the origin of mathematics. They show that counting evolved over an exceedingly long period and that abstract counting was preceded by a more archaic form of reckoning. The tokens' chronology suggests that the evolution from concrete to abstract counting proceeded as follows:

ca. 8000-3500 B.C.: Counters of multiple shapes indicate a system of concrete counting.

ca. 3500-3100 B.C.: Markings, impressed in one-to-one correspondence on [clay -- F.E.D.] envelopes [used to house/archive the clay tokens representing a trade -- F.E.D.] and tablets [flat clay "envelopes", which slowly supplanted the older envelopes that had to be broken open to "audit" the token-"records they contained" -- F.E.D.] and fusing together the notions of product and number, suggest that concrete counting still prevailed.

ca. 3100 B.C.: (Uruk IVa). Numerals were invented. They were the first symbols expressing numbers abstractly -- independently of the item counted.

ca. 3100-2500 B.C.: Archaic numerations for counting various categories of items still lingered, showing that the transition from concrete to abstract counting lasted over several centuries.

Tokens and clay tablets functioned as an extension of the human brain to collect, manipulate, store, and retrieve data. In turn, processing an increasing volume of data with more complex tokens brought people to think in greater abstraction. The token system dealt with data in concrete terms. First, each token represented a concrete entity: *one unit* of goods [of a specific variety, with *unit-quantifier* and *unit-qualifier* completely fused, in a 'primitive undifferentiated unity' -- F.E.D.]. Second, the tokens represented plurality as it is experienced perceptually. A set of three jars of oil was shown as it is in reality: one jar of oil plus one jar of oil plus one jar of oil. "Three" is an abstraction and there were no tokens to express such abstractions. . .

The tokens are unique in prehistoric assemblages in documenting the evolution of cognitive skills in pre-literate cultures. The steps from measures of grain to numerals, from tokens to writing, reflect the development of signs always further removed from real goods. Numerals represent the first mathematical symbols and the first signs expressing abstract concepts. In turn, these symbols mirror an increased capacity for abstract thinking.

The foremost function of tokens was counting goods. The plain tokens served to count products of the farm, such as animals and measures of cereals. Later, complex tokens kept track of industrial [better, hand- or hand-tool-based, not machine-based, but *handicraft* or *manufactured* -- F.E.D.] products famous in Mesopotamia such as textiles and garments; luxury goods such as perfume, metal, and jewelry; manufactured goods such as bread, oil, or trussed ducks. The counters served for budgeting, managing, and planning resources to enhance productivity. In turn, tokens can disclose to the archaeologist the resources of past communities ['economic archaeology' or 'social-reproductive archaeology' -- F.E.D.].

Plain tokens occurred concurrently with farming and complex tokens with industry, implying that the evolution of the system was closely tied to economic changes. Vice versa, the tokens can be clues for the domestication of plants and animals and for the development of workshops. . . .

Political power, which relied upon the control of real goods, depended upon counting and accounting. The more precise the accounting system, the more powerful institutions became.

The fact that tokens wielded power [or, less 'reifically' and "fetishistically" or 'subject/object-invertedly', *the fact that token wielders wielded socio-political power* -- F.E.D.] is illustrated by counters found in tombs of prestigious individuals. These artifacts suggest that tokens were status symbols and that counting was the privilege of the elite. This suggests that the token system was tied to the development of a redistributive economy. The plain tokens served to pool resources in early farming communities; complex tokens played an essential role in the collection of dues and tribute sustaining the first Mesopotamian city-states.

The two stages of the token system, plain and complex, correspond, therefore, to two phases in the evolution of social structures. Plain tokens imply a rank society, whereas complex tokens signal state formation in southern Mesopotamia. Furthermore, the geographic distribution of complex tokens in strategic administrative centers outlines the area controlled by the southern Mesopotamian bureaucracy and gives an insight into its organization ['tokens archaeo-geo-demography' -- F.E.D.]. . . .

The tokens shed light on the background of Mesopotamian writing. They reveal that *the first script derived from three-dimensional counters*. The path that led from *tokens* to *envelopes* and *tablets* can be reconstructed as follows:

8000-3000 B.C.: Plain tokens were characterized by mostly geometric shapes and a plain surface.

4400-3100 B.C. (Uruk XVII-IVa): Complex tokens had a larger repertoire of geometric shapes and also included more naturalistic forms. They bore a greater variety of linear and punched markings. Some specimens were perforated in order to be strung for safekeeping in archives.

3750 B.C. (Uruk X): Complex tokens formed series of counters of the same shape, with a variable number of lines or punctuations.

3500 B.C. (Uruk VI): The complex tokens reached a climax. At that time they had spread to sites of northern Mesopotamia, Susiana, and Syria, where the southern Mesopotamian bureaucracy was involved. This is indicated by artifacts typical of southern Mesopotamia such as monumental architecture with clay cone mosaics, cylinder seals, beveled-rim bowls, and nose-lugged jars.

3700-2600 B.C.: *Groups of tokens representing particular transactions were enclosed in [also clay -- F.E.D.] envelopes to be kept in archives.*

3500-2600 B.C.: (starting in Uruk VI-V): *Some [clay] envelopes bore on the outside the [wet-clay] impression of the tokens held inside. These envelopes bearing markings were the turning point between tokens and writing.*

3500-3100 B.C.: (starting in Uruk VI-V): *Tablets displaying impressed markings in the shape of tokens superseded the envelopes.*

3100-3000 B.C.: Pictographic script traced with a stylus on clay tablets marked the true takeoff of writing. *The tokens dwindled.*

The tokens give a new perspective on the evolution of communication in prehistory. *They point out that when writing began in Mesopotamia it was not a sudden, spontaneous invention, as previously thought, but the outgrowth of many thousands of year's worth of experience at manipulating signs. From tokens, the earliest script inherited fundamental aspects in form, content, and structure. . .*

The tokens give new insights into the nature of writing. They establish that in the Near East *writing emerged from a counting device* and that, in fact, *writing was the by-product of abstract counting*. When the concepts of *numbers* and that of *items counted* were abstracted [*'The Elision of the Qualifiers'* -- F.E.D.], the pictographs were no longer confined to indicating *numbers of units of goods in one-to-one correspondence*. With the invention of numerals, pictography was no longer restricted to accounting but could open to other fields of human endeavor. From then on, writing could become phonetic and develop into the versatile tool that it is today, able to store and to convey any possible idea. *The invention of abstract numerals was the beginning of mathematics, and it was also the beginning of writing.*

The tokens also raise new questions concerning the essence of writing. Was the first script of the Near East unique in deriving from a counting device? Or is literacy universally tied to numeracy. Is numeracy a prerequisite for literacy?

The tokens only begin answering Thomas Astle's question:

Whence did the wond'rous
mystic art arise
Of painting speech, and
speaking to the eye?
That we by tracing magic
lines are taught
How both to colour, and
embody thought?"

Denise Schmandt-Besserat, *Before Writing*, vol. I: *From Counting to Cuneiform*, University of Texas Press [Austin, 1992], pp. 6; 196-199.

[On the idea of the co-evolution of linguistic media, including mathematical notations, and cognitive powers-- F.E.D.]:

". . . I recognized that all written forms of language -- whether it be [non-mathematical] writing or mathematical notation -- have both a communications and an informatics dimension, as is also the case with computers. It was at this point that I formulated the two main theses of this book:

- all forms of verbal language have both a communications and an informatics dimension that facilitates human thought
- speech, writing, mathematics, science, and computing form an evolutionary chain of languages."

[On the idea of a quanto-qualitative ideography of [unit] qualifiers, quantifiers, and qualified quantities or quantified qualities as a [partial] return to origins, see also the 'psycho-historical' and 'psycho-archaeological' works of D. Schmandt-Besserat, cited above-- F.E.D.]:

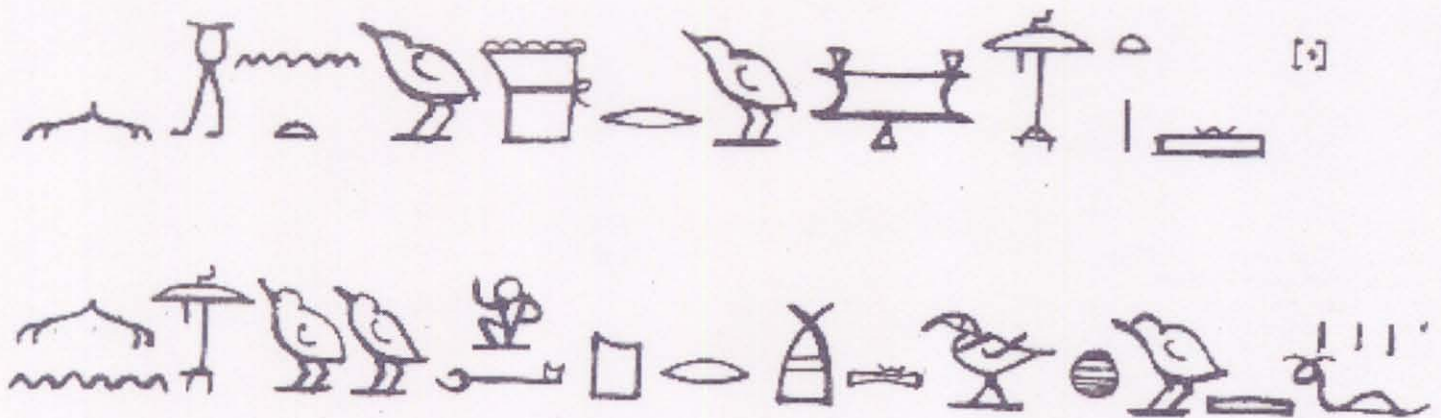
"The next step in the development of *notational technologies* were clay accounting tokens. . . Their use began in the prehistoric Middle east, circa 8000 B.C., at the very beginning of the agricultural age, which also had its roots in the Middle East. Each uniquely shaped token designated some measure of an agricultural commodity or product. The system which started with twenty-four kinds of tokens grew to 190 different types of tokens by 3300 B.C., just before the advent of writing and abstract numerals in Sumer. . . The token system may have evolved from pebbles. . . tokens differed from pebbles and other tallies in very significant ways. Tokens were not randomly collected objects put to the secondary use of counting. They were man-made artifacts prepared specifically for the purpose of accounting. Compared to a series of more or less identical counters such as grains or pebbles, tokens were molded into distinctive shapes that were easy to recognize and simple enough to be systematically reproduced. Compared to unspecified tallies, each token shape stood for a specific commodity. In other words, *tokens communicated both quantitative and qualitative information. . . the tokens were used in one-to-one correspondence: n ovoids were used to designate n jars of oil. . .* The token system underwent a linear sequence of development and enrichment from 8000 B.C. to 3100 B.C. which comprise the following steps or stages: 1. plain tokens; 2. complex tokens; 3. tokens in clay envelopes; 4. impressed logographs on clay envelopes containing [corresponding] tokens; and 5. impressed logographs on clay tablets. None of these changes represents a bifurcation, but they do represent either *an increase in the variety and number of tokens* or a morphological change of the tokens, that is, a change in their physical form. The meaning or function of a three dimensional token circa 8000 B.C., or an impressed logogram created by pressing a token into wet clay circa 3100 B.C., were the same. During the above five steps, neither writing nor abstract numerals emerged. The next development in *notational technology* was profound and followed shortly after the introduction of the two-dimensional clay tablet."

The impressed logograms on clay tablets not only underwent semantic and morphological variations but were subject to a series of Prigoginean bifurcations. . . The emergence of abstract numerals and writing can be described in terms of the following three separate *bifurcations*: Bifurcation 1: the split of the two-dimensional impressed logographic signs into two morphologically and semantically distinct sets of signs – the incised signs and the impressed signs, in which incised signs are used exclusively for verbal notation or writing and impressed signs used exclusively for accounting and numerical notation; Bifurcation 2: the split of the impressed logograms for the ban and the bariga into two semantic functions – the old one as “concrete numbers” for small and large measures of grain and the new one as the “abstract numbers” for 1 and 10, respectively; and Bifurcation 3: the split of the semantic function of the incised signs, in which they are used not only to code spoken words ideographically or pictographically, but also phonetically. . . *It would seem that the emergence of abstract numerals and ideographic writing were tied to each other and arose at the same point in history. . . The quantitative (from the Latin *quantus* – how great) and the qualitative (from the Latin *qualis* – of what kind) are two key categories of Western thinking that have, from the philosophical thoughts of the ancient Greeks to contemporary social science, been regarded as distinct and independent modes of analysis. The common origin of quantitative and qualitative notation from clay tokens argues against the notion that these two categories form a dichotomy. The fact that the two forms of notation emerged at the same point in history indicates the cognitive power that the interplay between the quantitative and the qualitative can release.”*

Robert K. Logan, *The Fifth Language*, (Toronto: Stoddart, 1995), pp. 4; 83-115.

Ptahhotep of Egypt, 2350 B.C.E. --

In Egyptian Hieroglyphic script:



In translation to contemporary English:

"One cannot attain the limit of craftsmanship, and there is no craftsman who acquires his total mastery."

Quoted in the Egyptian hieroglyphic form, and translated into English from the hieroglyphic form, in:

Webster's New World Dictionary Of American English, Third College Ed., Simon & Schuster (NY: 1988), p. 636.

Pythagoras of Samos, circa 570-500 B.C.E. --

"Greek philosopher. Probably born at Samos. He studied under Pherecydes and other Ionian philosophers. He is thought to have visited Egypt. *Saintly* in manner, *it was reported that he was once seen in two cities at exactly the same time*. He established a school in Croton, Italy, with many features of a religious sect, including secret initiation ceremonies, strict vows, *vegetarianism*, progress through ranks from novitiate to full member, *admitting women as well as men*, and requiring *communal sharing of goods*. . .

A. We shall attempt to state the insights of the founder, following this with the insights of the school. At least the following ideas would seem to have been the intellectual product of Pythagoras himself:

(1) *The end of life lies in gaining a relation to the divine*. Belief in God provides a principle of order for human life and for societies, providing a basis for constitutions, laws, and rights.

(2) The way to achieve relation to the divine is through philosophy. Seeking truth, one finds salvation. One decides which of three types of men he is to be: a lover of wisdom, a lover of success, a lover of pleasure; and the first is the superior type.

(3) The way of salvation was related to *a doctrine of the transmigration of the soul*, and *philosophy played a role in preparing man to escape from the cycles of reincarnation*.

(4) Pythagoras' *great discovery* was one concerning *the relation between mathematics and the physical world*, as exemplified by the relation between arithmetic ratios and the progression of tones produced by lengthening or shortening a vibrating string. This led either to the belief that *mathematics holds the key to the explanation of reality*, or that *number is the essence of reality*. Probably Pythagoras held both beliefs. . .

B. Beyond these principles there are others some of which may have been the work of Pythagoras and others the work of a number of his followers in the Pythagorean School.

(6) The *application of number to the universe* comes by means of *a list of opposites* which plays an important role in Pythagorean thought. The opposites are: limit (*peras*) and unlimited (*apeiron*); odd and even; *one and many*; right and left; male and female; *resting and moving*; straight and curved; light and darkness; good and bad; square and oblong.

(7) The list has cosmological-mathematical implications. The "unlimited" is space, *the "unlimited" limited once is the unit*. It stands also for *the point*, and hence *a possibility for identifying number and reality*. One is the *point*; two the *line*; three the *plane*; and four the *solid*. Hence, *by number we have constituted the world*. The sum of the critical numbers is *ten*. Ten is therefore the perfect number.

. . . This suggests some sort of association between unity, point, and pebble, *an atomism of number*. The *highly visual associations* used by the Pythagoreans derive, some say, from the *practice* of setting forth sums by laying out *pebbles* ["*calculi*" -- F.E.D.] on a smooth surface."

William L. Riese. *Dictionary Of Religion and Philosophy*, Eastern and Western Thought, Humanities Press, Inc. [Atlantic Highland, NJ: 1980], pp. 469-470.

Herakleitos of Ephesus, circa 540-475 B.C.E. --

"Greek philosopher. Born in the city of Ephesus. . . . A considerable number of written fragments from his thought remain. . .

(1) All things are in flux, changing in such a way that *it is not possible to step into the same river twice; for "other and yet other waters are flowing on"*.

(2) *Change takes place by means of opposites*. . . ."

William L. Riese. *Dictionary Of Religion and Philosophy*, Eastern and Western Thought, *ibid.*, p. 219.

Parmenides of Elea, circa 515-450 B.C.E. --

"Greek philosopher. Born in Elea. Influenced by the Pythagoreans. He is said to have prepared the laws for his native city of Elea. Prophet of *changelessness*, he founded the Eleatic philosophy. *It is sometimes said that he wrote in opposition to Heraclitus*; this is apparently not known to be the case; but *certainly the two philosophies present a marked contrast*. His stress on the unity of things makes him one of the earliest defenders of Monism. . .

- (1) Since *thought and being are identical*, one need only follow out the principle of consistency to gain the truth about reality.
- (2) *Sense experience*, on the other hand, is *full of contradiction* and *hence is mere appearance*.
- (3) Thought seeks the common, universal and invariant. What is common to all things is the being they have; and what is not-being is not existent [cf. the ancient Greek resistance to *the zero concept*, perhaps mistakenly confounded with the concept of *absolute 'Nothing[ness]'*, of global indeterminateness, or '*omni-in-determinateness*', of total *de*-manifestation, of *unqualified "abstract negation"* in Hegel's sense, or of *universal/general absence*, as distinct from mere *specific absence*, or '*determinate absence*', produced by "*determinate negation*" in Hegel's sense, in a context still of *general presence*, i.e., a context of mere '*mono-ontic*' or '*mono-metrical*' absence, as distinct from one of '*omni-ontic*' or '*omni-metrical*' absence -- F.E.D.]. Hence being is and not-being is not.
- (4) If being is and not-being is not, being cannot come from not-being. Hence *being is eternal*.
- (5) If anything changes, something which was not comes to be; since not-being is non-existent, *change is impossible*.
- (6) If anything moves, it must occupy a space where it was not; and since empty space is not-being and non-existent, *motion is impossible*.
- (7) If things are separate from each other, they are separated by empty space; hence, *things cannot be separate* from each other.
- (8) Hence, finally, *being is homogeneous throughout like the mass of a rounded sphere* "perfected on every side", *equally distant from the center at every point*. This picture of reality, so different from the world we see, was defended by the paradoxes of Parmenides' ablest student, Zeno of Elea."

William L. Riese. *Dictionary Of Religion and Philosophy, Eastern and Western Thought, ibid.*, pp. 412-413.

Zeno of Elea, circa 490-430 B.C.E. --

"Greek philosopher. A disciple of Parmenides and one of the most prominent Eleatic philosophers, he is known for his skillful defense of the Parmenidean doctrines. Regarded by Aristotle as *the inventor of dialectic*, Zeno supported Parmenides' view of *reality as changeless being* by developing *paradoxes* of space, time, motion, and change which he believed to be implicit in the commonsense conception of the world [cf. *failure of ancient Greeks to develop much dynamical mathematics, e.g., Calculus, versus their statical geometry, etc.* -- F.E.D.]. . .

Space is a contradictory notion and *reality is indivisible*, for the *opposite claim leads to absurdity*. Suppose that reality is divisible. It will be composed either of a finite or of an infinite number of parts. Reality could have a finite number of parts only if the magnitude of the parts disappeared in a finite number of divisions; but this would lose the finite space with which we began, since a finite number of parts without magnitude cannot produce a magnitude. If, on the other hand, reality has an infinite number of parts, the parts will have magnitude or else they will not. If they do not have magnitude, once again we have lost the space with which we began. If they do have magnitude, and we have an infinite number of them, we can construct a space as much larger than the initial space as we please. [cf. *the paradox associated with The Axiom of Choice in Zermelo-Fraenkel Axiomatic Set Theory* -- F.E.D.]. Hence, we must give up the idea of space, and of a divisible reality.

Time, likewise, is contradictory. . ."

William L. Riese. *Dictionary Of Religion and Philosophy, Eastern and Western Thought, ibid.*, p. 641.

Plato of Athens, circa 428-348 B.C.E. --

"The dialogues of the Socratic period provide that view of the world usually associated with Plato. *The period of transition and criticism, and the final synthesis, are little noted*; nor does the transition occur by an abrupt break, but rather by a pointing up of difficulties, and an introduction of new emphases. . . . The *Parmenides* can be taken as signaling the change. In this dialogue Socrates is unable to defend his Doctrine of Ideas. The problem of the utter difference between time and eternity sets the problem. As creatures of time it seems that we should have no capacity to know the universal forms, nor can we have, then, any connection with the universal God, or He with us. . . . (7) *Where the Republic and Phaedo stressed the unchanging nature of the soul, the emphasis in the Phaedrus is exactly reversed*. In this dialogue, the soul is the principle of *self-motion*, and we are told that *the soul is always in motion, and what is always in motion is immortal*. The difference now between spirit and matter is *not changelessness in contrast with change, but self-motion* ['auto-flexion' or 'self-re-flexion'; 'nonlinearity' -- F.E.D.], the essence of the soul, in contrast with *derived motion* ['allo-flexion', 'linearity' -- F.E.D.]. The emphasis on *self-motion* is continued even in the *Laws*, Plato's final dialogue."

William L. Riese. *Dictionary Of Religion and Philosophy*, Eastern and Western Thought, *ibid.*, pp. 442-443.

"...disputation and debate may be taken as a *paradigmatic model* for the general process of *reasoning in the pursuit of truth*, thus making the transition from rational *controversy* to rational *inquiry*. There is nothing new about this approach. Already the *Socrates* of Plato's *Theaetetus* conceived of *inquiring thought as a discussion or dialogue that one carries on with oneself*. Charles Saunders Peirce stands prominent among those many subsequent philosophers who held that discursive thought is always *dialogical*. But *Hegel*, of course, was the prime exponent of the conception that all *genuine* knowledge must be developed *dialectically*. ...These conclusions point in particular towards *that aspect* of the *dialectic* which lay at the *forefront* of *Plato's concern*. He insisted upon two fundamental ideas: (1) that a starting point for *rational* argumentation cannot be merely assumed or *postulated*, but *must itself be justified*, and (2) that the *modus operandi* of such a *justification can be dialectical*. Plato accordingly mooted the prospect of *rising above a reliance on unreasoned first principles*. He introduced a special device he called "*dialectic*" to *overcome this dependence upon unquestioned axioms*. It is worthwhile to see how he puts *[this]* in his own terms: There remain geometry and those other allied studies which, as we have said, do in some measure apprehend reality; but we observe that they cannot yield anything clearer than a dream-like vision of the real so long as they leave the assumptions they employ *unquestioned* and can *give no account of them*. *If your premiss is something you do not really know and your conclusion and the intermediate steps are a tissue of things you do not really know, your reasoning may be consistent with itself, but how can it ever amount to knowledge?*...So...the *method of dialectic* is the *only one* which takes this course, *doing away with assumptions*. ...*Dialectic* will stand as the *coping-stone* of the *whole structure*; there is *no other study that deserves to be put above it*. *Plato's writings do not detail in explicit terms the exact nature of this crucial enterprise of dialectic*. Presumably we are to gain our insight into its nature not so much by way of *explanation* as by way of *example* -- the example of Plato's own *practice* in the *dialogues*."

Nicholas Rescher, *Dialectics: A Controversy-Oriented Approach to the Theory of Knowledge*, SUNY Press [Albany, New York: 1977], pp. 46-48.

Translator's note [by Eva Brann]: "The Greek word *arithmos* (*αριθμος*) is rendered in the German text as *Anzahl*: "a number of things", to distinguish it from our modern *Zahl*: "number". Since English approximations to *Anzahl* are either obsolescent (e.g., "tale") or awkward (e.g., "counting-number", "numbered assemblage"), *Anzahl*, like *Zahl*, has been rendered simply as "number", although it is a chief object of this study to show that [the ancient -- F.E.D.] Greek "*arithmos*" and modern "*number*" do not mean the same thing, that they differ in their intentionality, for the former intends things, i.e., a number of them, while the latter intends a concept, i.e., that of quantity (cf. Pp. 206-208). *Intentionality* and *conceptualization* are both used to translate the German word *Begrifflichkeit*. The following Greek terms occur so frequently that they have been incorporated into the text in transliterated form:

aisthesis (*αισθησις*) -- sense perception
aistheton (*αισθητον*) -- object of sense
analogia (*αναλογια*) -- proportion
aoristos [*dyas*] (*αοριστος δυας*) -- indeterminate, infinite [*dyad*]
apeiron (*απειρον*) -- sb.: the limitless, infinite
apodeixis (*αποδειξις*) -- demonstration, [strict] proof
aporia (*απορια*) -- quandary
arché, pl. *archai* (*αρχη, αρχαι*) -- [governing] source, beginning

chorismos (*χωρισμος*) -- separation

dianoia (*διανοια*) -- [the faculty and activity of] *thinking*
dynamis (*δυναμις*) -- *power*

eidetikos (*ειδητικός*) -- adjective from *eidos*
eidos, pl. *eide* (*ειδος, ειδη*) -- literally: "*looks*"; *kind*, *form*, *species*, "*idea*"; sometimes: "*figure*"
episteme (*επιστημη*) -- Lat.: *scientia*, knowledge, science

genos, pl. *gene* (*γενος, γενη*) -- genus, family, class; often: *the higher eide*

hen (*εν*) -- one
hyle (*υλη*) -- *material*

idea (*ιδεα*) = *eidos*

koinon (κοινον) -- sb.: *common thing*
koinonia ton eidon (κοινωνια των ειδων) -- *community* of the *eide*
kata to plethos (κατα το πλεθος) -- according to multitude
kath auto (καθ αυτο) -- *by itself*
katholou pragmataia (καθολου πραγματεια) -- general treatment or study
kat eidos (κατ ειδος) -- according to kind
kinesis (κινησις) = *change*

logistiké (λογιστικη) -- logistical art
logos, pl. *logoi* (λογος, λογουι) -- *reasonable speech*; also: *ratio*

mathema (μαθημα) -- *learning matter*
mathematika (μαθηματικα) -- sb.: *mathematical objects*
mathesis (μαθησις) -- *study, discipline*
methexis (μεθεξις) -- *participation*
monas (μονας) -- *monad, unit*
morphé (μορφη) -- *shape*

noeton (νοητον) -- *that which is for thought, object of thought*
noesis (νοησις) -- *activity of thought, intellection*
nous (νους) -- [direct and perfect *activity* of] *thought, intellect*

on; me *on* (ον, μη ον) -- being; *non*-being

pathos, pl. *pathé* (παθος, παθη,) -- characteristic, property
poson (ποσον) -- Lat. *quantum*, sb.: the object of the question "how many", that which has quantity; to avoid awkwardness, rendered as "quantity".
pros allo (προς αλλο) -- in relation to another
pros ti (προς τι) -- sb.: [that which is] in relation to something

stasis (στασις) -- *rest*

tauton (ταυτον) -- the same
taxis (ταξις) -- order
téchne (τεχνη) -- art, skill
thateron (θατερον) -- the other

Jacob Klein, *Greek Mathematical Thought and the Origin of Algebra*, [New York: Dover, 1992], pp. vii - ix.

... The strange *koinonia* among *on*, *kinesis*, and *stasis* is none other than that between "being" and "non-being".

Thus the relation of *stasis* to *kinesis* forms the nucleus of all subsequent discussion. But from now on the conversation grows broader. . . ; the point of departure is that "gift of the gods to human beings". . . , namely the "astounding". . . assertion that each thing is "one" and "many" ". . . at once". Next it is asked how the "many" are conjoined to form the "unity" of any being. This question is nothing but the generalization of the original problem of the "two" aspects, i.e., of the meaning of "at one and the same time". . . or of "both". . . It is raised at first in the most general terms, namely in reference to any being, and then transformed into the narrower ontological *methexis* problem, into the question of the *koinonia ton eidon* (or, of the *gene*). Thereupon three possibilities, and no more, arise. . . : (1) There is no *koinonia* at all. (2) All the *eide* are mutually related. (3) There is *partial* *koinonia*, in the sense that some *eide* can "mix" with each other but others not.

Since the two first possibilities are not in fact realizable, the third alone, of necessity, remains. . . But the very formulation of this possibility indicates the *arithmos structure of the gene*; for what is it but the division of the whole realm of the *eide* into single groups or assemblages such that each *eidos*, which represents a unique eidetic "unit". . . , i.e., a "*monas*". . . , can be "thrown together" *with the other ideas of the same assemblage*, but *not* with the ideas of other assemblages?

The *eide*, then, form assemblages of monads, i.e., *arithmoi* of a peculiar kind. The *units* of which the assemblages consist are *not mathematical monads*, for *these are, as we have seen, completely similar and can therefore all be "thrown together"* (Aristotle, *Metaphysics* M 7, 1081 a 5 f.: "capable of being thrown together and indifferent" . . .). While the numbers with which the arithmetician deals, the *arithmoi mathematikoi* or *monadikoi*, *are capable of being counted up, i.e., added*, so that, for instance, *eight monads and ten monads make precisely eighteen monads together, the assemblages of eide, the*

"arithmoi eidetikoi", cannot enter into any "community" with one another [non-amalgamative -- F.E.D.]. Their monads are all of different kind [$\frac{1}{2}$ -- F.E.D.]

and can be brought "together" *only "partially"*, namely only insofar as they happen to belong to one and the same assemblage, whereas insofar as they are "entirely bounded off" from one another . . . they are incapable of being thrown together, *in-comparable* [incapable of being counted as repetitions of the same unity; incomparable quantitatively -- $\frac{1}{2}$ -- F.E.D.]. The notion of an "*arithmetical*" *structure of the realm of ideas* now permits a solution of the ontological *methexis* problem (cf. *Parmenides* 133 A).

The monads which constitute an "*eidetic number*", i.e., an *assemblage of ideas*, are nothing but *a conjunction of eide which belong together*. They belong together because they belong to one and the same *eidos of a higher order*, namely a "class" or *genos*. But all will together be able to "partake" in this *genos* (as for instance, "human being", "horse", "dog", etc., partake in "animal") *without "partitioning" it among the (finitely) many eide and without losing their indivisible unity only if the genos itself exhibits the mode of being of an arithmos*. Only the *arithmos* structure with its special *koinon* character is able to guarantee the essential traits of the community of *eide* demanded by *dialectic*; the indivisibility of the single "monads" which form the *arithmos* assemblage, the limitedness of this assemblage of monads as expressed in the joining of many monads into one assemblage, i.e., into one idea, and the *untouchable integrity of this higher idea as well*.

What the single *eide* have "in common" is theirs only in their community and is not something which is to be found "beside" and "outside". . . them. . . The unity and determinacy of the *arithmos* assemblage is here rooted in the content of the idea. . . that content which the *logos* reaches in its characteristic activity of uncovering foundations "analytically". A special kind of number of a particular nature is not needed in this realm, as it was among the *dianoetic* numbers. . . to provide a foundation for this unity. In fact, it is impossible that any kinds of number corresponding to those of the *dianoetic* realm should exist here, since each *eidetic* number is, by virtue of its *eidetic* character, unique in kind [$\frac{1}{2}$ -- F.E.D.], just as each of its "monads" has not

only unity but also uniqueness. For each idea is characterized by being always the same and simply singular [∴ additively idempotent -- F.E.D.] in contrast to the unlimitedly many homogeneous monads of the realm of mathematical number, which can be rearranged as often as desired into definite numbers. . . The "pure" mathematical monads are, to be sure, differentiated from the single objects of sense by being outside of change and time [early Platonic theory, later rejected by Plato in favor of the «*auto-kinesis*» theory [see extract above by Reese] -- F.E.D.], but they are not different in this sense -- that they occur in multitudes and are of the same kind (Aristotle, *Metaphysics* B 6, 1002 b 15 f.: [Mathematical objects] differ not at all in being many and of the same kind. . .), whereas each *eidos* is, by contrast, unreproducible [Ξ idempotent addition, 'hyper-amalgamativity', or 'non-addability': $\hat{A}_k + \hat{A}_k = \hat{A}_k$ -- F.E.D.] and truly one (*Metaphysics* A 6, 987 b 15 ff.: "Mathematical objects differ from objects of sense in being everlasting and unchanged, from the *eide*, on the other hand, in being many and alike, while an *eidos* is each by itself one only" . . .). In consequence, as Aristotle reports (e.g., *Metaphysics* A 6, 987 b 14 ff. and N 3, 1090 b 35 f.), there are three kinds of *arithmoi*: (1) the *arithmos eidetikos* -- idea-number, (2) the *arithmos aisthetos* -- sensible number, (3) and "between". . . these, the *arithmos mathematikos* or *monadikos* -- mathematical and monadic number, which shares with the first its "purity" and "changelessness" and with the second its manyness and reproducibility. Here the "aesthetic" number represents nothing but the things themselves which happen to be present for *aisthesis* in this number. The mathematical numbers form an independent domain of objects of study which the *dianoia* reaches by noting that its own activity finds its exemplary fulfillment in "reckoning [i.e., account-giving] and counting". . . The *eidetic number*, finally, indicates the mode of being of the *noeton* as such -- it defines the *eidos* ontologically as a being which has multiple relations to other *eide* in accordance with their particular nature [i.e., with their content -- F.E.D.] and which is nevertheless in itself altogether indivisible [i.e., «*a-tom-istic*», 'un-cuttable' -- F.E.D.].

The Platonic theory of the *arithmoi eidetiko* is known to us in these terms only from the Aristotelian polemic against it (cf., above all, *Metaphysics* M 6-9). It is questionable whether Plato sketched out more than the general framework of the theory. In his lecture *On The Good*. . . he seems to have limited the realm of *eidetic* numbers to ten (cf. *Metaphysics* A 8, 1073 a 20; M 8, 1084 a 12 ff., 25 ff., and *Physics* Γ 6, 206 b 32 f., and elsewhere). In this he, as well as his successor *Speusippus*, remained true to the Pythagorean tradition; indeed, the *eidetic numbers* might, in their foundational function, be most easily compared to the Pythagorean "roots". . . of the realm of mathematical number. Now in understanding the *arithmoi* in the only way in which they can be understood in their very own, mathematical, domain, Aristotle exhibits the many contradictions which must arise from the transfer of the universal character of the countable as such to the *eide*, each of which has a special nature. For Plato, however, it is precisely this unmathematical use of the *arithmos* structure which is essential. For the *arithmoi eidetiko* are intended to make intelligible not only the inner articulation of the realm of ideas but every possible articulation, every possible division and conjunction -- in short, all counting [and not just all cardinality, but all distinctions of kind/quality, and all [including genetic] ordinality or ordering -- F.E.D.]. While the arithmetician and the logistician "suppose" certain *eide* to "underlie" the unlimitedly many monads of his domain in order to have "hypothetical" grounds on which they may be comprehended into single monadic assemblages (cf. Pp. 72 and 78), only the dialectician is able to give the true grounds for the existence of such *eide* of numbers and of each single number of pure units.

Only because there are *eide* which belong together, whose community in each case forms a "kinship" which must, due to the "arithmetical" tie among its "members", be designated the six or the ten, can there be arbitrarily many numbers, such as hexads or decads, in the realm of pure units as well as in the realm of sensibles; furthermore, only because of this can numbers exhibit such definite, unifying kinds (cf. Pp. 55 f.) as the "even-times-even" or the "triangular". Only the *arithmoi eidetiko* make something of the nature of number possible in this world. They provide the foundation for all counting and reckoning, first in virtue of their particular nature which is responsible for the differences of *genus* and *species* in things so that they may be comprehended under a definite number, and, beyond this, by being responsible for the infinite variety of things, which comes about through a "distorted" imitation of ontological *methexis* (cf. Pp. 82, 80 and 98-9; cf. *Philebus* 16 C-E; *Timaeus* 43). This foundational function guarantees their separate, independent and "absolute" being in relation not only to the *aistheta* but also to the "pure" mathematical numbers (see P. 71). What the Pythagoreans undertook with respect to the world of sense in which they believed that all beings were comprised (*Metaphysics* A 8, 990 a 3-5), Plato now undertakes to do with respect to the world opened up by the *logos*, the world of *noeta* which has true being. For him, as for them, the "numerical" being of the *noeta* means their ordered being, their *taxis*. The mathematical monads by themselves form a homogeneous field, but the "sequence" of monadic numbers (cf. P. 52) is grounded in the original order of the *eidetic* numbers. Every *eidetic number* is either "superior" or "inferior" in this order with respect to its "neighbor", so that the subsumption of all of these numbers under one idea common to all, namely "number in general", is quite impossible. This *taxis* of *eidetic numbers* is "logically" expressed in the relation of "being superior" or "inferior" in the order of *eide*; the relation of family "descent" [i.e., of 'ideo-genealogy' -- F.E.D.] between the higher and lower ideas corresponds to the "genetic order of the *eidetic numbers*". The "higher" the *genos*, i.e., the less articulated the *eidetic* number, the more original and "comprehensive" it is. In this order the "first" *eidetic* number is the *eidetic* "two"; it represents the *genos* of "being" as such, which comprehends the two *eide* "rest" and "change". That this last tenet, at least, is genuinely Platonic can be seen, if not "with complete clarity" yet clearly enough, from the *Sophist*.

The third possibility, that of a "partial" *koinonia ton eidon* (cf. P. 89) is here investigated only for the "greatest genera". . . namely, the following "five": being, rest, change, the same, the other. . . This "count", the only one possible within the *dianoetic-dialogic method*, does as little justice to the true ontological state of affairs as the assertion that the sequence "sophist", "statesman", "philosopher" are three *gene* of equal importance to which, therefore, three dialogues should correspond (217 A-B; cf. 254 B and *Statesman* 257 A-B). The purpose of the inquiry concerning the *koinonia* among the "five" greatest *gene* is, as is expressly stated, to grasp "being" as well as "non-being" in a manner suited to the present mode of examination, though for this very reason not completely adequate: "So that if we are not able to grasp being and non-being with complete clarity, we may yet at least miss as little of the account as the present manner of inquiry allows". . . It is shown (254 D-257 A) that there is indeed a "community" among all the *gene*, although this kind of "community" is by no means the same in all cases. The immediate basis of the discussion is the incompatibility of *stasis* and *kinesis*, and this fact must not be overlooked. Their "mixing" had already been mentioned (cf. P. 87) in the strongest terms as "impossible by the greatest necessity". . . This is again stated in 254 D 7-9: "Stranger: And we do say that the two of them are not to be mixed with one another. Theaetetus: We stress it." . .

"But both 'are', and from this follows the 'triad' of *stasis*, *kinesis*, and *on* (D 12), although it has already been shown that the *on* is not to be understood as a "third thing beside" or "outside these". . . since this would lead to the "most impossible thing of all" (250 B-D, see P. 88). In respect to *on*, *kinesis* and *stasis*, the *logos* fails! It fails because it must count "three" when in truth there are only "two", namely *stasis* and *kinesis*, which are "each one" and "both two" (. . cf. also *Theaetetus* 185 B, *Timaeus* 38 B). The *logos* cannot conclude the count with "two" because it says that *stasis* and *kinesis* "are" not only "together" but also "singly", while in the case of "two mathematical monads" it understands each of these by itself as only one and precisely not as "two" (cf. P. 81).

On, *kinesis*, and *stasis*, in spite of their "arithmetical" *koinonia*, cannot be "counted" at all -- this defines the "failure" of the *logos*. The dianoetic understanding is clear only about this much, that each of the "three" things presupposed by it, insofar as each is "itself" exactly that which it is, and is grasped in its "self-sameness", is an "other" than the "two" others: "The each of them is other than the two, but the same with itself". (. . . 254 D 14 f.). This crucial quandary of the *dianoia* is compounded by the introduction of yet a further "pair": "self-sameness" and "otherness". . . , but with these also emerges the solution which must suffice within the dianoetic realm (cf. 257 A 9-11): It consists in this, that "the other", *thateron*, analogously to the vowels among the letters (253 A 4-6), ranges "through all" (. . . 253 C 1) *gene* whatsoever (255 E 3 f.), and is, accordingly, both a *connecting* "bond". . . and the agent "responsible for their *division*" (. . . 255 D 1; cf. D 6 f.), that is, "the other" is the "ultimate source" of all articulation whatsoever. *Thateron* is in itself two-fold, for "the 'other' is always in relation to an 'other' [i.e., to a second, *another other* relating to the *first other* in *mutual otherness* -- F.E.D.]". (. . . 255 D 1; cf. D 6 f.).

This means that the possibility of "otherness" is dependent on the "self-sameness" of the participants in the relation of "otherness"; therefore *tauton*, too, pervades all the *gene*. This *koinonia* between *tauton* and *thateron*, which is nothing but another expression for the internal "twofoldness" of *thateron* itself. . . , permits the *dianoia* to understand the "duplicity" of "being", namely, that it means not only ever self-identical "rest", but conjointly also "change", and that this alone makes possible the "imaging" of being in "re-cognition", that is, of "knowing" and "being known" (. . . 248 B ff.), and, beyond this, all image making (cf. *Cratylus* 439 E-340 A). "Conjointly" does not mean here, as for instance in the dialectic of *Nicholas of Cusa* or *Hegel*, a "coincidence of opposites".

Just as the *dianoia* finds in the realm of the "more and less". . . the "opposition", the "obstacle". . . which first "awakens" it (see Pp. 75 ff.), so it must finally, at the end of its "dialectic" activity, come to see that the "conjunction" of opposites is in truth the "co-existence" of elements other in kind [$\frac{1}{2}$; \oplus --

F.E.D.]. "Otherness" makes possible an "arithmetic community" among eidetic monads which are not capable of "being mixed" although they "belong together"; the paradigm for such a *koinonia*, which is no longer accessible to the *logos*, or, therefore, to counting, is the eidetic "two" which consists of *stasis* and of *kinesis*; *kinesis* is that which, in confrontation with *stasis*, is the "other" without which even *stasis* itself cannot "be", since precisely only "both together" amount to "being". This means that "being" itself is accompanied of necessity by a "not"; just as *stasis* is not *kinesis*, so *kinesis* is not *stasis*. "Otherness" turns out to be the ontological aspect of "non-being", which can never be separated from "being": "Of necessity, then, non-being is [immediately] involved in change and occurs throughout all the *gene* [of being]; for throughout these the nature of 'the other' works on each being to make it other and [thus] a non-being [namely not thus and such a being but another], and accordingly we rightly speak of all things whatsoever as 'non-beings', and conversely, because things partake of being, as 'being' and 'beings'". (. . . 256 C-D). The shadow of "non-being" necessarily attends all the "being" of that which is -- just as the sophistic refutation, the *elenchos* [refutation by argument -- F.E.D.], belongs to the essential business of the philosopher (cf. 230 A-231 B), just as the mutability of the cosmos perpetually returning into itself is the most immediate image of the true cosmos in its immutability. Everywhere "not being" is only "being other", "not something the contrary of, but only other than, being" (. . . 257 B; cf. 258 B), for the contrary of being would amount to the unthinkable as well as unspeakable "nothing"; what we everywhere have is an "opposition" of one being to an other being before us, "a confrontation of being with being" [connoting also, perhaps, a confrontation of each being with[in] its own being, with[in] itself -- a 'self-duality', 'intra-duality', 'in[ternal]-tension', 'self-opposition', 'self-antithesis', or 'ontological-existential [vs. propositional] self-contradiction', or 'indiv[isible]-duality', that gives rise to 'self-change', 'self-development', 'temporary self-destabilization' of linear equilibria/temporary 'self-re-stabilization', in short, to "autokinesis" -- F.E.D.](. . . 257 E 6). This is the reason for the possibility of a "mistake" or "interchange" of the "one" and the "other" or of "being" and "non-being", a possibility on which all "contradiction" (cf. 232 B), all "illusion", all "error", and every "lie" depend (260 B-264 B, also 266 D-E). This "duplicity" of being is the ontological foundation and justification of the method of "division" [cf., below, *Boole's presentation of "dichotomy" in relation to his "fundamental law of thought" or "law of duality" -- F.E.D.*]. . . The "divisions" at the beginning and at the end of the *Sophist* (as also in the *Statesman*) are intended to hint repeatedly at this "duplicity" (cf. also *Statesman* 287 C; 306 C ff.; also *Charmides* 159 B ff.). They are intended at the same time to train the dialectician in "dividing according to genera" (. . . cf. *Sophist* 253 D; *Statesman* 285 C-D), an activity in which he must be expert if he is to reach the primal "genetic" order of eidetic numbers. . . In particular, the *Sophist* is intended to show that all articulated arrangements of the *gene*, in other words, the "arithmetic community of ideas", can be understood only by means of "the other". Its very "nature". . . consists in "being broken up into parts", which makes it -- certainly not accidentally -- akin to the discerning *episteme*: "The nature of the other seems to me to be all broken up just like knowledge". (. . . 257 C 7 f.; 258 D-E; cf. *Parmenides* 142 E, 144B and E; *Theaetetus* 146 C ff., *Meno* 79 A, C.). It is always, as it were, only a "part". . . of itself, namely "other of another". . . a "counter-part". In the context of the Platonic search for foundations (cf. P. 71) this arché of all doubleness must be recognized as the "two-fold in general", the *aoristos dyas*. . . This *dyas* is *aoristos* because it does not itself represent "two" beings of some particular kind such as are mutually delimited and univocally determined. . . Rather, in endowing the being of such a thing with "imageability", it "doubles" every thing, and so first allows it to come into "being" at all -- it is "two-making" (. . . Aristotle, *Metaphysics* M 8, 1083 b 35 f.; M 7, 1082 a 15). Thus by a continual "duplication" of the *eide* -- which the *logos* grasps in the "division" of the *gene* -- it makes the "genetic" order of the eidetic numbers possible. But it can do all this only because the "head" of the *taxis* of eidetic numbers, at once concluding and introducing them, is the *One Itself* in its "absolute" priority (cf. P. 77). Since it is beyond all articulation, beyond the "two", and thus "beyond being itself" (. . . *Republic* 509 B), it is not, like the mathematical unit, *one among many*. . . , but rather the original, perfect, all-comprehensive Whole (cf. *Sophist* 244 D-245 D; also *Parmenides* 137 C, 142 D).

As the "Whole" it is that which needs no "other" at all, that which is altogether "finished". In this sense it is "the perfect itself", namely, the model of every possible "relative" wholeness which is "delimited" in respect to an "other": it is the "Idea of the Good". . .

"The doctrine of the *gene* as eidetic numbers must, finally, also furnish the foundation of an eidetic logistic. For instance, sound-mindedness. . . and justice. . . in Book IV of the *Republic* (cf. I 337 A-C; Aristotle, *Nicomachean Ethics*, E 6, 7), or the *taxis* of the elemental materials in the *Timaeus*, can be understood only by means of analogia, proportion (cf. *Theaetetus* 186 A-B). So also the relation of the ontological to the dianoetic *methexis* problem (cf. P. 80), as well as the relation of the original to copy in general, becomes comprehensible only in "logistic" terms. What is usually overlooked in discussions of the *methexis* question is the secondary, the *imaging*, character of the whole *methexis* relation, i.e., the relation of one *eidos* to a series of *aistheta*. Only when these relations are reduced to relations of "community" within the realm of the *eide*, can we see the *methexis* problem in its original form. But one of the possible solutions to this higher problem is precisely the conception of the arithmos eidetikos.

This solution at once gives the final answer to the problem of the "one and many": The arithmos eidetikos exhibits in itself the possibility of an immediate unification of the many. But this solution is bought, as we have seen, at the price of the transgression of the limits which are set for the *logos*, for from this point of view the ordinary mode of predication, such as: "the horse is an animal", "the dog is an animal", etc., is no longer understandable. Above all, the "natural" meaning intended when a multiplicity of things is called an "arithmos" is now lost. Here, therefore, are the points of departure for further development and correction of the Platonic doctrine by *Speusippus* and *Xenocrates*. But only Aristotle's critique exposes the root of this, as of all the other related difficulties, namely the postulate of the "separation". . . of all noetic formations, and in particular the *chorismos* of the *arithmoi monadikoi*, the numbers of "pure" monads."

Jacob Klein, Greek Mathematical Thought and the Origin of Algebra, *ibid.*, pp. 88-99.

"Plato seems to have realized *the gulf between arithmetic and geometry*, and it has been *conjectured* that he may have tried to bridge it by *his concept of number* and by the *establishment of number upon a firm axiomatic basis* similar to that which was built up in the nineteenth century independently of geometry; but we cannot be sure, because *these thoughts do not occur in his exoteric writings* and were not advanced by his successors. If Plato made *an attempt to arithmetize mathematics* in this sense, *he was the last of the ancients to do so*, and the problem remained for modern analysis to solve. *The thought of Aristotle we shall find diametrically opposed to any such conceptions. It has been suggested that Plato's thought was so opposed by the polemic of Aristotle that it was not even mentioned by Euclid.* Certain it is *that in Euclid there is no indication of such a view of the relation of arithmetic to geometry*; but *the evidence is insufficient to warrant the assertion that*, in this connection, *it was the authority of Aristotle which held back for two thousand years a transformation which the Academy sought to complete.*"

Carl B. Boyer, *The History of the Calculus and its Conceptual Development*, Dover [NY: 1949], p. 27.

Aristotle of Athens and of Macedonia, circa 384-322 B.C.E. --

"...The absence of a potentiality in a thing is called a privation. To the complete fulfillment of a thing on the other hand, Aristotle applies the terms "entelechy" and "energeia", the former referring to the state of completion and the latter to the power of that state. *One of Aristotle's chief contributions to philosophy was the concept of potentiality.* It has many applications in his philosophy. For example, his most recurrent interpretation of infinity is that of "potential infinity". Potential infinity is the statement of a capacity; it applies to that which can be infinitely divided, augmented, or diminished. But the infinitely divisible is not actually divided into an infinite series. According to Aristotle one of the mistakes of Zeno in his elaboration of the paradoxes of time and motion is that he did not distinguish between actual and potential infinities. *Aristotle's definition of continuity also requires the idea of potential infinity [versus Cantor's -- F.E.D.].* The continuous is that *subdivision of the contiguous whose limits are one and the same, and contained in each other.* This implies that the parts of the continuous are infinitely divisible; or as Aristotle says "divisible into divisibles that are infinitely divisible" (Physics 231b). *The potentiality-actuality contrast allowed him to develop a dynamic reality, which he yet constrained within rather narrow limits.* He is able to express dynamically the sense in which an acorn is potentially an oak, but he is not able to express the sense in which an entire species might have developed from a potentiality. Indeed, in his view, *species are fixed and immutable.* Potentiality is also limited by the fact that of the three basic types of change -- "alteration" or *change of quality*, "growth and diminution" or *change of quantity*, and "locomotion" or *change of place* -- Aristotle believes that the last is primary in the sensible world."

William L. Riese. *Dictionary Of Religion and Philosophy, Eastern and Western Thought, ibid.,* p. 30.

Euclid of Alexandria, circa 300 B.C.E. --

"The cream of the mathematical work created by the men of the classical period has fortunately come down to us in the writings of two men, Euclid and Apollonius. Chronologically, both belong to *the second great period of Greek history, the Hellenistic or Alexandrian.* It is quite certain *that Euclid lived in Alexandria about 300 B.C.* and trained students there, though his *own education* was probably acquired in *Plato's Academy.* This information, incidentally, is about all we have on Euclid's personal life and even this comes from a one-paragraph passage in Proclus' *Commentary.* Apollonius died in 190 B.C., so his life too falls within the Alexandrian period. *It is customary, however, to identify Euclid's work with the classical period, because his books are accounts of what was developed in that age.* Euclid's work is actually an organization of the separate discoveries of the classical Greeks; this is clear from a comparison of its contents with what is known of the earlier work. The *Elements* in particular is *as much a mathematical history of the age just brought to a close as it is the logical development of a subject.*... Since the *Elements* is *the first substantial source of mathematical knowledge* and one that was *used by all succeeding generations, it influenced the course of mathematics as no other book has.* The very *concept of mathematics, the notion of proof, and the logical ordering of theorems* were learned by studying it, and its contents determined the course of subsequent thinking. Hence we should note the characteristics that influenced so strongly the future of mathematics. Though, as mentioned before, the *form of presentation* of the *individual propositions* is not original with Euclid, the *form of presentation* of the *entire work* -- the *statement of all of the axioms at the outset, the explicit statement of all definitions, and the orderly chain of theorems* -- is his own. Moreover, *the theorems are arranged to go from the simple to more and more complex ones.*... Despite some omissions and errors of proof that we shall point out shortly, Euclid's *choice of axioms* is remarkable. *From a small set he was able to prove hundreds of theorems, many of them deep ones.* Moreover, his *choice* was *sophisticated.* His handling of the *parallel axiom* is especially clever. Euclid undoubtedly knew that any such axiom states *explicitly or implicitly* what must happen in the infinite reaches of space and that any pronouncement about what must be true of infinite space is *physically dubious* because *man's experiences are limited.* Nevertheless, *he also realized that some such axiom is indispensable.* He therefore chose a version that states conditions under which two lines will meet at a *finitely distant* point. Moreover, *he proved all the theorems he could before calling upon this axiom.*"

Morris Kline, *Mathematical Thought from Ancient to Modern Times*, vol. I., Oxford U. Press [NY: 1972], pp. 56, 86-87.

Apollonius of Alexandria, circa 262-190 B.C.E. --

"The other great Greek who belongs to the classical period, in the two senses of summarizing and adding to the kind of mathematics the classical period produced, is Apollonius (c. 262-190 B.C.). Apollonius was born in Perga, a city in the northwestern part of Asia Minor, which was under the rule of Pergamum during his lifetime. He came to Alexandria in his youth and learned mathematics from Euclid's successors. As far as we know, he remained in Alexandria and became an associate of the great mathematicians who worked there. His chief work was on the conic sections but he also wrote on other subjects. His mathematical powers were so extraordinary that he became known in his time and thereafter as "the Great Geometer". His reputation as an astronomer was almost as great. The conic sections, as we know, were studied long before Apollonius' time. In particular, Aristaeus the Elder and Euclid had written works on them. Also Archimedes' work, which we shall study later, contains some results on the subject. Apollonius, however, stripped the knowledge of all irrelevancies and fashioned it systematically. Besides being comprehensive, his *Conic Sections* contains highly original material and is ingenious, extremely adroit, and excellently organized. As an achievement it is so monumental that it practically closed the subject, at least from the purely geometrical standpoint. It may be truly regarded as the culmination of classical Greek geometry... The strict deductive mathematics of Euclid and Apollonius has given rise to the impression that mathematicians create by reasoning deductively. Our review of the three hundred years of activity preceding Euclid should show that conjectures preceded proofs and that analysis preceded synthesis. In fact, the Greeks did not think much of propositions obtained by simple deduction. Results that sprang readily from a theorem the Greeks called corollaries or porisms. Such results, obtained without additional labor, were regarded by Proclus as windfalls or bonuses."

Morris Kline, *Mathematical Thought from Ancient to Modern Times*, vol. I., Oxford U. Press [NY: 1972], pp. 89-90, 99.

Archimedes of Alexandria and of Syracuse, circa 287-212 B.C.E. --

Archimedes, in a work long lost until recent times entitled *The Method*, wrote as follows to his correspondent Eratosthenes about a procedure which Archimedes had employed as a method of discovery, but not ever yet as a method of presentation, of exposition, or of demonstration/rigorous proof, a method which involved the use of metaphors of a partly dynamical, partly statical mechanical-geometry -- a partial 'geometry of mechanics' or 'mechanics of geometry' -- and a heuristic, as opposed to a logically rigorous, use of infinitesimals, a method perhaps only brought to axiomatic "completion" and fruition ~1900 years later, in Newton's «*Philosophia Naturalis Principia Mathematica*», Newton's extension of Euclid's and Archimedes' Geometry into Mechanics/Mechanical Dynamics, and, with respect to a logically rigorous, axiomatically-grounded use of infinitesimal numbers or 'infinitesimal quantifiers', perhaps only brought to fruition ~2200 years later, with the formulation of *Non-Standard Analysis* by Abraham Robinson:

"The proofs then of these theorems I have written in this book and now send to you. Seeing moreover in you, as I say, an earnest student, a man of considerable eminence in philosophy, and an admirer [of mathematical inquiry], I thought it fit to write out for you and explain in detail in the same book the peculiarity of a certain method, by which it will be possible for you to get a start to enable you to investigate some of the problems in mathematics by means of mechanics. This procedure is, I am persuaded, no less useful even for the proof of the theorems themselves; for certain things became clear to me by a mechanical method, although they had to be demonstrated by geometry afterwards, because their investigation by said method did not furnish an actual demonstration. But it is of course easier, when we have previously acquired, by the method, some knowledge of the questions, to supply the proof than it is to find it without any previous knowledge. This is a reason why in the case of the theorems the proof of which Eudoxus was the first to discover, namely that the cone is a third part of the cylinder, and the pyramid of the prism, having the same base and equal height, we should give no small share of the credit to Democritus who was the first to make the assertion with regard to the said figure* though he did not prove it. I am myself in the position of having first made the discovery of the theorem now to be published [by the method indicated], and I deem it necessary to expound the method partly because I have already spoken of it† and I do not want to be thought to have uttered vain words, but equally because I am persuaded that it will be of no little service to mathematics; for I apprehend that some, either of my contemporaries or of my successors [cf. Newton's use of dynamical-geometrical/mechanical metaphors and analogies, as well as of logically 'irrigorous', 'heuristic' infinitesimal quantifiers, in his concept of 'fluents' or of 'flowing quantities', and "Method of Fluxions"; his development of what we now call the differential and integral calculus ~1,900 years later -- F.E.D.], will, by means of the method when once established, be able to discover other theorems in addition, which have not yet occurred to me."

Quoted in: Thomas Heath, *The Works of Archimedes*, Dover [NY: 2002], pp. 13-14.

"*The death of Archimedes portended what was to happen to the entire Greek world.* In 216 B.C.[E. -- F.E.D.] Syracuse allied itself with Carthage in the second Punic war between that city and Rome. The Romans attacked Syracuse in 212 B.C.[E. -- F.E.D.]. While drawing mathematical figures in the sand, Archimedes was challenged by one of the soldiers who had just taken the city. Story has it that Archimedes was so lost in thought that he did not hear the challenge of the Roman soldier. The soldier thereupon killed him, despite the order of the Roman commander, Marcellus, that Archimedes be unharmed."

Morris Kline, *Mathematical Thought from Ancient to Modern Times*, vol. I., Oxford U. Press [NY: 1972], p. 106.

The materials reproduced in excerpt below serve, for us, as a veritable symphony of hints and clues. Via this 'Psycho-Archaeological Excavation', we invite you to watch the wave of thought, and the broad river of consciousness of which this work is also but a miniscule part, wash over and through many minds across the ages [understanding that we do *not* mean by this to imply that the present work embodies some ultimate finality of that process, or to deny the burgeoning diversity of the ideational phenomena recorded below, embodying a vast heterogeneity of conceptual directions other than our own]. These extracts were selected for how they, in a catalytic way, favor, or *disfavor*, or even *declare the impossibility of*, the kind of 'Meta-Dynamical', *Dialectical «Characteristica Universalis»* that we seek. This *diachronically-ordered* sequence of extracts, reflecting the limitations of our own backgrounds, is, and will be, incomplete, omitting many thinkers whose thoughts belong in this train. We will add, in future editions, for as long as they continue, the selected psycho-artefacts of progenitors and 'co-genitors' -- newly discovered, or subsequently formed. Despite that, we shall inevitably fail to discover all that belongs here. [As elsewhere herein, *italic and sometimes bold and underscored black or blue-colored typeface* in a quote or extract signifies our emphasis, not the author's: throughout the quotations which follow, *such emphasis has been* added by F.E.D.].

b. From the period of Late Antiquity [early Common Era, 1 to 640 C.E.].

Heron of Alexandria, circa 1st Century C.E. --

Regarding the cultural/psycho-historical context of Hellenistic Ancient Alexandria:

"The period following the Peloponnesian War was one of political disunity among the Greek states, rendering them easy prey for the now strong kingdom of Macedonia which lay to the north. King Philip of Macedonia was gradually extending his power southward and Demosthenes thundered his unheeded warnings. The Greeks rallied too late for a successful defense and, with the Athenian defeat at Chaeronea in 338 B.C.[E. -- F.E.D.], Greece became a part of the Macedonian empire. Two years after the fall of the Greek states, ambitious Alexander the Great succeeded his father Philip and set out upon his unparalleled career of conquest which added vast portions of the civilized world to the growing Macedonian domains. Behind him, wherever he led his victorious army, he created, at well-chosen places, a string of new cities. It was in this way, when Alexander entered Egypt, that the city of Alexandria was founded in 332 B.C.[E. -- F.E.D.]. ...From its inception, Alexandria showed every sign of fulfilling a remarkable future. *In an incredibly short time, largely due to its very fortunate location at a natural intersection of some important trade routes, it grew in wealth, and became the most magnificent and cosmopolitan center of the world.* ...

After Alexander the Great died in 323 B.C.[E. -- F.E.D.], his empire was partitioned among some of his military leaders, resulting in the eventual emergence of three empires, under separate rule, but nevertheless united by the bonds of the Hellenistic civilization that had followed Alexander's conquests. Egypt fell to the lot of Ptolemy. ...He selected Alexandria as his capital and, *to attract learned men to his city, immediately began the erection of the famed University of Alexandria. This was the first institution of its kind. ...Report has it that it was highly endowed and that its attractive and elaborate plan contained lecture rooms, laboratories, gardens, museums, library facilities, and living quarters. The core of the institution was the great library, which for a long time was the largest repository of learned works to be found anywhere in the world, boasting, within forty years of its founding, over 600,000 papyrus rolls. It was about 300 B.C.[E. -- F.E.D.] that the university opened its doors and Alexandria became, and remained for close to a thousand years, the intellectual metropolis of the Greek race* [and not of the Greek "race" alone, but of the Occidental Afro/Euro/Near-Asian hemisphere of humanity entire! -- F.E.D.]."

Howard Eves, *An Introduction to the History of Mathematics*, 3rd Ed., Holt, Rinehart & Winston [NY: 1969], pp. 112-113.

"*No other city has been the center of mathematical activity for so long a period as was Alexandria from the days of Euclid (ca. 300 B.C.[E. -- F.E.D.]) to the time of Hypatia (†A.D. 415 [C.E. -- F.E.D.]).* It was a very cosmopolitan center, and the mathematics that resulted from Alexandrian scholarship was not all of the same type. ..."

C. Boyer, U. Merzbach, *A History of Mathematics*, 2nd. Ed., John Wiley & Sons, Inc. [NY: 1991], p. 178.

"About 290 B.C.[E. -- F.E.D.] Ptolemy Soter built a center in which scholars could study and teach. This building, dedicated to the *Muses*, became known as the *Museum*, and it housed poets, philosophers, philologists, astronomers, geographers, physicians, historians, artists, and most of the famous mathematicians of the Alexandrian Greek civilization. Adjacent to the Museum, Ptolemy built a library, not only for the preservation of important documents but for the use of the general public. This famous library was said at one time to contain 750,000 volumes, including the personal library of Aristotle and his successor Theophrastus. Books, incidentally, were more readily available in Alexandria than in classical Greece because Egyptian papyrus was at hand. In fact, Alexandria became the center of the book-copying trade of the ancient world. *The Ptolemies also pursued Alexander's plan of encouraging a mixture of peoples, so that Greeks, Persians, Jews, Ethiopians, Arabs, Romans, Indians, and Negroes came unhindered to Alexandria and mingled freely in the city. Aristocrat, citizen, and slave jostled each other and, in fact, the class distinctions of the older Greek civilization broke down. The civilization in Egypt was influenced further by the knowledge brought in by traders and by the special expeditions organized by the scholars to learn more about other parts of the world. Consequently, intellectual horizons were broadened. The long sea voyages of the Alexandrians called for far better knowledge of geography, methods of telling time, and navigational techniques, while commercial competition generated interest in materials, in the efficiency of production, and in improvement of skills. Arts that had been despised in the classical period were taken up with new zest and training schools were established. Pure science continued to be pursued but also applied science.*"

Morris Kline, *Mathematical Thought from Ancient to Modern Times*, vol. I., Oxford U. Press [NY: 1972], pp. 102-103.

"The mechanical devices created by the Alexandrians were astonishing even by modern standards. Pumps to bring up water from wells and cisterns, pulleys, wedges, tackles, systems of gears, and a mileage measuring device no different from what may be found in the modern automobile were used commonly. *Steam power was employed to drive a vehicle along the city streets in the annual religious parade.* Water or air heated by fire in secret vessels of temple altars was used to make statues move. ... Water power operated a musical organ and made figures on a fountain move automatically while compressed air was used to operate a gun. New mechanical instruments, including an improved sundial, were invented to refine astronomical measurements."

Ibid., p. 103.

Regarding the achievements of Heron: "Proclus refers to Heron as *mechanicus*, which might mean a *mechanical engineer* today, and discusses him in connection with *the inventor Ctesibius, his teacher.* Heron was also a good surveyor. ...*The striking fact about Heron's work is his commingling of rigorous mathematics and the approximate procedures and formulas of the Egyptians. On the one hand, he wrote a commentary on Euclid, used the exact results of Archimedes (indeed he refers to him often), and in original works proved a number of new theorems of Euclidean geometry. On the other hand, he was concerned with applied geometry and mechanics and gave all sorts of approximate results without apology. He used Egyptian formulas freely and much of his geometry was also Egyptian in character.* ...His applied works include *Mechanics, The Construction of Catapults, Measurements, The Design of Guns, Pneumatica* (the theory and use of air pressure), and *On The Art of Construction of Automata.* He gives designs for water clocks, measuring instruments, *automatic machines*, weight lifting machines, and war engines."

Ibid., pp. 116-117.

Regarding the [self-]bifurcation of late-ancient Geometry into Geometry plus Arithmetic/Algebra -- the emergence of Arithmetic/Algebra as [an] independent subject(s)-of-study: "We have been reviewing the methods of doing arithmetic employed by the Greeks in both periods but more especially in the Alexandrian *period when the geometry and trigonometry became quantitative.* But the major development with which this chapter is concerned is the rise of arithmetic and algebra as subjects independent of geometry. The arithmetical work of Archimedes, Apollonius, and Ptolemy was a step in this direction, but they *used arithmetic to calculate geometric quantities.* One might infer that the numbers were meaningful to them because they represented geometric magnitudes and the logic of the operations was guaranteed by *geometrical algebra.* But there is no question that *Heron, Nichomachus* (c. A.D. 100), who was probably an Arab from Gerasa in Judea, and *Diophantus* (c. A.D. 250), *a Greek of Alexandria, did treat arithmetical and algebraic problems in and for themselves and did not depend on geometry either for motivation or to bolster the logic.*"

Ibid., pp. 135.

Heron, Arithmetic, and Algebra: "More significant than Heron's arithmetical work of finding square and cube roots is the fact that he formulated and solved algebraic problems by purely arithmetic procedures. . . . There are many such problems in his work. Of course *this is precisely the old Egyptian and Babylonian style of presentation, and there is no doubt that Heron took much material from the ancient* [already "ancient", even from the perspective of Heron's time, and so 'doubly-ancient' from the perspective of ours -- F.E.D.] *Egyptian and Babylonian texts. There, we may remember, algebra was independent of geometry and, as for Heron, an extension of arithmetic.* In his *Geometrica*, Heron speaks of adding an area, a circumference, and a diameter. *In using such words he means, of course, that he wants to add their numerical values. Likewise, when he says that he multiplies a square by a square, he means that he is finding the product of the numerical values.* Heron also translated much of Greek geometrical algebra into arithmetic and algebraic processes. *This work of Heron (as well as his use of approximate Egyptian area and volume formulas) is sometimes evaluated as the beginning of the decline of Greek geometry. It is more fitting to regard it as a Hellenized improvement on Babylonian and Egyptian mathematics. When Heron adds areas and line segments, he is not misapplying classical Greek geometry but merely continuing the practice of the Babylonians, for whom area and length were just words for certain arithmetic unknowns.*"

Ibid., pp. 136.

Nichomachus of Gerasa, circa Second Century C.E. --

"An Hellenic philosopher born in Arabia. A neo-Pythagorean with an admixture of Platonic and Philonic ideas, *Nichomachus held ideas to be numbers, providing the model for creation.* The numbers pre-exist in the mind of God in his view. The one, or unity, is itself the principle of reason and the divine, while the two or dyad is the principle of matter. His treatise on arithmetic was translated by Boethius and Apuleius, was the subject of numerous commentaries, and was used as a text well into the Renaissance."

William L. Riese. *Dictionary Of Religion and Philosophy, Eastern and Western Thought*, Humanities Press, Inc. [Atlantic Highland, NJ]: 1980], p. 390.

"More remarkable from the standpoint of the reemergence of an independent arithmetic is the work of Nichomachus, who wrote the *Introductio Arithmetica* in two books. It was the first sizable book in which arithmetic (in the sense of the theory of numbers) was treated entirely independently of geometry. *Historically its importance for arithmetic is comparable to Euclid's Elements for geometry.* Not only was this book itself studied, referred to, and copied by dozens of later writers, but *it is known to be typical of many books by other authors of the same period* and so *reflects the interests of the times.* Numbers stood for quantities of objects and were no longer visualized as line segments as in Euclid. *Nichomachus uses words throughout*, whereas Euclid used a letter, such as A, or two letters, such as BC -- referring, in the second case, to a line segment -- to speak about numbers. Hence Nichomachus' phrasing is clumsier. He treats only whole numbers and ratios of whole numbers."

"Nichomachus was a Pythagorean; and though the Pythagorean tradition was not dead, he reanimated it. Of the four subjects stressed by Plato -- arithmetic, geometry, music, and astronomy -- Nichomachus says arithmetic is the mother of the others. This he maintains is "not solely because we said that it existed before all the others in the mind of the creating God like some universal and exemplary plan, relying upon which as a design and archetypal example the creator of the universe sets in order his material creations and makes them attain to their proper ends; but also because it is naturally prior in birth..." Arithmetic, he continues, is essential to all the other sciences because they could not exist without it. However, if the other sciences were abolished, arithmetic would still exist... The Introductio had value because it was a systematic, orderly, clear, and comprehensive presentation of the arithmetic of [positive -- F.E.D.] integers and ratios of [positive -- F.E.D.] integers freed of geometry. It was not original as far as ideas were concerned, but was a very useful compilation. It also incorporated speculative, aesthetic, mystical, and moral properties of numbers, but no practical applications. The Introductio was the standard text in arithmetic for a thousand years. At Alexandria, from the time of Nichomachus, arithmetic rather than geometry became the favorite study."

Morris Kline, Mathematical Thought from Ancient to Modern Times, vol. I., Oxford U. Press [NY: 1972], pp. 136-138.

Diophantus of Alexandria, circa Third Century C.E. --

"The highest point of Alexandrian Greek algebra is reached with Diophantus... His work towers above that of his contemporaries; unfortunately, it came too late to be highly influential in his time because a destructive tide was already engulfing the civilization."

Morris Kline, Mathematical Thought from Ancient to Modern Times, vol. I., Oxford U. Press [NY: 1972], pp. 138-139.

Diophantus himself, introducing his famous proto-ideographical algebraic work, the «Arithmetica», wrote:

"All numbers are made up of some multitude [«arithmos» -- F.E.D.] of units [«monads», i.e., a "natural" number is an «arithmos monadikos», an assemblage, or "population", or "ensemble", or "'set'" of two or more monads/units, all of the same, generic kind -- F.E.D.]. Among them are --

squares, which are formed when any number is multiplied by itself; the number itself is called the side of the square;

cubes, which are formed when squares are multiplied by their sides;

square-squares, which are formed when squares are multiplied by themselves;

square-cubes, which are formed when squares are multiplied by the cubes formed from the same side;

cube-cubes, which are formed when cubes are multiplied by themselves;

and it is from the addition, subtraction, or multiplication of these numbers, or from the ratio which they bear to one another or to their own sides, that most arithmetical problems are formed; you will be able to solve them if you follow the method shown below.

Now each of these numbers, which have been given abbreviated names, is recognized as an element in an arithmetical science; the square [of the unknown quantity] is called dynamis [«ΔΥΝΑΜΙΣ» -- F.E.D.] and its sign is Δ with the index Y, that is, Δ^Y ; the cube is called kubos [«ΚΥΒΟΣ» -- F.E.D.] and has for its sign K with the index Y, that is, K^Y ; the square multiplied by itself is called dynamo-dynamis and its sign is two deltas with the index Y, that is, $\Delta^Y\Delta$; the square multiplied by the cube formed from the same root is called dynamo-kubos and its sign is ΔK with the index Y, that is, ΔK^Y ; the cube multiplied by itself is called kubo-kubos and its sign is two kappas with the index Y, $K^Y K$.

The [unknown -- F.E.D.] number which has none of these characteristics, but merely has in it an undetermined multitude of units, is called arithmos [«αριθμος» -- F.E.D.] and its sign is ς . There is also another sign denoting the invariable element in determinate numbers, the unit [i.e., the "MONAD" or «ΜΟΝΑΣ» -- F.E.D.], and its sign is M with the index O, that is, $\overset{O}{M}$."

Translation from Thomas, Selections Illustrating the History of Greek Mathematics, quoted in: Victor Katz, A History of Mathematics: An Introduction, [2nd Ed.], Addison-Wesley [NY: 1998], p. 174, and:

"... It is well that one who is beginning this study should have acquired practice in the addition, subtraction, and multiplication of the various species [types of terms]. He should know how to add positive and negative terms with different coefficients to other terms, themselves either positive or likewise partly positive and partly negative, and how to subtract from a combination of positive and negative terms other terms either positive or likewise partly positive and partly negative..."

If a problem leads to an equation in which certain terms are equal to terms of the same species but with different coefficients, it will be necessary to subtract like from like on both sides, until one term is found equal to one term. If by chance there are on either side, or on both sides, any negative terms, it will be necessary to add the negative terms on both sides, until the terms on both sides are positive, and then again to subtract like from like, until one term only is left on each side. This should be the object aimed at in framing the hypotheses of propositions, that is to say, to reduce the equations, if possible, until one term is left equal to one term, but I will show you later how, in the case also where two terms are left equal to one term, such a problem is solved."

Translation from Heath, Diophantus of Alexandria: A Study in the History of Greek Algebra, quoted in: Victor Katz, A History of Mathematics: An Introduction, *ibid.*, [NY: 1998], p. 175.

"Diophantus wrote several books that are lost in their entirety... His great work is the *Arithmetica* which, *Diophantus* says, comprises thirteen books. We have six [surviving in Greek; four more were discovered, in 1973, in the Mashad Shrine Library, in Arabic translation, possibly translations of Hypatia's commentaries on books four through seven, rather than of Diophantus' originals -- F.E.D.]... *One of Diophantus' major steps is the introduction of symbolism* [i.e., of *proto-ideography* -- F.E.D.] in algebra... The appearance of such symbolism is of course remarkable but *the use of powers higher than three is even more extraordinary* [given the apparent, sensible dimensionality of our external-spatial world: three dimensions, and no more; *cubic* capacity and nothing greater -- F.E.D.]. *The classical Greeks could not and would not consider a product of more than three factors because such a product had no [then-known -- F.E.D.] geometrical significance. On a purely arithmetical basis, however, such products do have a meaning; and this is precisely the basis Diophantus adopts.* Addition is indicated in Diophantus' notation -- F.E.D.] by putting terms alongside one another [i.e., by that 'juxtapositioning' of variables' ideograms which typically denotes *multiplication* -- not addition -- in modern algebras -- F.E.D.]. Thus [using 'overscored' Greek letters to denote "Natural" numbers, via alphabetic-ordinal correspondence, e.g., $\alpha \leftrightarrow 1, \beta \leftrightarrow 2, \gamma \leftrightarrow 3, \delta \leftrightarrow 4, \epsilon \leftrightarrow 5, \dots$, and $\iota \leftrightarrow 10, \therefore \bar{\iota}\bar{\beta} \leftrightarrow 12$, etc. -- F.E.D.]

$$\Delta^y \bar{\gamma} \bar{\mu} \bar{\iota} \bar{\beta} \text{ means } x^2 \cdot 3 + 12.$$

The $\bar{\mu}$ is a symbol for unity and indicates that a pure number not involving the unknown follows. Again

$$\Delta^y \alpha \zeta \bar{\beta} \bar{\mu} \bar{\gamma} \text{ means } x^2 + x \cdot 2 + 3.$$

For subtraction he uses the symbol \wedge . Thus for $x^6 - 5x^4 + x^2 - 3x - 2$ he writes

$$K^y K \alpha \Delta^y \alpha \wedge \Delta^y \Delta \bar{\epsilon} \zeta \bar{\gamma} \bar{\mu} \bar{\beta},$$

putting all of the negative terms [i.e., all of the "subtracted" terms -- F.E.D.] after the positive ones [i.e., after all of the "added" terms -- F.E.D.]. There are no symbols for addition, multiplication, or division as operations. The symbol ι^σ is used (at least in the extant versions of the *Arithmetica*) to denote equality. The coefficients of the algebraic expressions are specific numbers; there are no symbols for general coefficients. Because he does use some symbolism, Diophantus' algebra has been called *syncopated*, whereas that of the Egyptians, the Babylonians, Heron, and Nichomachus is called *rhctorical*. Diophantus writes out his solutions in a continuous text, as we write prose. His execution of the operations is *entirely arithmetical*; that is, there is *no appeal to geometry* to illustrate or substantiate his assertions... The first book of the *Arithmetica* consists mainly of problems that lead to determinate equations of the first degree in one or more unknowns. The remaining five books treat mainly indeterminate equations of the second degree. But this segregation is not strictly adhered to... The most striking feature of Diophantus' algebra is his solution of indeterminate equations. Such equations had been considered before, as for example in the Pythagorean work on solutions of $x^2 + y^2 = z^2$, in the Archimedean cattle problem, which leads to seven equations in eight unknowns (plus two supplementary conditions), and in other odd writings. Diophantus, however, pursues indeterminate equations extensively and is the founder of the branch of algebra now called, in fact, *Diophantine analysis*... (in *modern Diophantine analysis* one seeks *integral solutions only*)... Diophantus shows great skill in reducing equations of various types to forms he can handle. We do not know how he arrived at his methods. Since he makes no appeal to geometry, it is not likely that he translated Euclid's methods for solving quadratics. Moreover, *indeterminate problems* are not in Euclid and as a class are new with Diophantus. Because we lack information on the continuity of thought in the later Alexandrian period, we cannot find traces of Diophantus' work in his predecessors. As far as we can tell, his work in pure algebra is remarkably different from past work... He *accepts only positive rational roots* [i.e., *solutions* to his proto-equational problems -- F.E.D.] and ignores all others. Even when a quadratic equation has two positive roots he gives only one, the larger one. When an equation, as it is being solved, clearly leads to two *negative* or *imaginary roots*, he *rejects the equation* and *says it is not solvable*. In the case of *irrational roots*, he retraces his steps and shows how by altering the equation he can get a new one that has rational roots. Here Diophantus differs from Heron and Archimedes. Heron was a surveyor and the geometrical quantities he sought could be irrational. Hence he accepted them, though of course he approximated them to obtain a useful value. Archimedes also sought exact answers, and when they were irrational he obtained inequalities to bound the irrational. Diophantus is a pure algebraist; and since algebra in his time did not recognize irrational, negative, and complex numbers, he rejected equations with such solutions. It is, however, worthy of note that fractions for Diophantus are numbers, rather than just the ratio of two whole numbers. He has no *general methods*. Each of the 189 problems in the *Arithmetica* is solved by a different method.

There are more than 50 different types of problems but no attempt is made to classify them by type. His methods are closer to the *Babylonian* ones than to those of his Greek predecessors, and there are *indications of Babylonian influences*. In fact, he does solve some problems just as the Babylonians did. But it has not been established that there was any direct connection between Diophantus' work and Babylonian algebra. *His advance in algebra over the Babylonians* consists in *the use of symbolism* and *the solution of indeterminate equations*. In determinate equations he went no further than they did, but his *Arithmetica* assimilated *logistics*, which Plato, among others, had banned from mathematics. Diophantus' variety of methods for the separate problems dazzles rather than delights. He was a shrewd and clever virtuoso but apparently not deep enough to see the essence of his methods and thereby attain generality. (it is still true [i.e., to this day -- F.E.D.] that *Diophantine analysis* is a maze of separate problems).

Unlike a speculative thinker who seeks general ideas, Diophantus sought only correct answers. There are a few results which might be called general, such as that no prime number of the form $4n + 3$ can be the sum of two squares. *Euler did credit Diophantus with illustrating general methods that he could not display as such because he did not have literal coefficients. And there are others who credit Diophantus with recognizing that his material belonged to an abstract and basic science.* But this view is not shared by all. *His work, as a whole, however, is a monument in algebra.*"

Morris Kline, *Mathematical Thought from Ancient to Modern Times*, *ibid.*, pp. 139-140.

Hypatia of Alexandria, circa 370 - 415 C.E. --

"The first notable *female astronomer and mathematician*, she *taught at Alexandria and became head of the Neoplatonist school there*. She was the *daughter of Theon*, a writer and commentator on mathematics [and the redactor of Euclid's *Elements* -- F.E.D.], with whom she collaborated, and was herself the *author of commentaries on mathematics and astronomy*, though none of these survives. She was renowned for her beauty, eloquence and learning, and *drew pupils from all parts of the Greek world, Christian as well as pagan*. Cyril, Archbishop of Alexandria, resented her influence. ..."

Melanie Parry, Ed., *Chambers Biographical Dictionary*, Larousse plc [NY: 1997], p.947.

Regarding the assassination of Hypatia by the mafia of Roman-Imperial [pseudo-]Christian power-idolators:

"The fate of Hypatia, an Alexandrian mathematician of note and the daughter of Theon of Alexandria, symbolizes the end of the era. *Because she refused to abandon the Greek religion, Christian fanatics seized her in the streets of Alexandria and tore her to pieces.*"

Ibid., Morris Kline, *Mathematical Thought from Ancient to Modern Times*, vol. I., p. 181.

Regarding the [pseudo-]Christian Dark Ages which that assassination epitomized and portended:

"From the standpoint of the history of mathematics, the rise of Christianity had unfortunate consequences. Though the Christian leaders adopted many Greek and Oriental myths and customs with the intent of making Christianity more acceptable to converts, they opposed pagan learning and ridiculed mathematics, astronomy, and physical science; Christians were forbidden to contaminate themselves with Greek learning. Despite cruel persecution by the Romans, Christianity spread and became so powerful that the emperor Constantine (272-337 [C.E. -- F.E.D.]) was obliged to consign it a privileged position in the Roman Empire. The Christians were now able to effect even greater destruction of Greek culture. *The emperor Theodosius proscribed the pagan religions and, in 392 [C.E. -- F.E.D.] ordered that the Greek temples be destroyed. Pagans were attacked and murdered throughout the empire. Greek books were burned by the thousands.* In the year that Theodosius banned the pagan religions, the Christians destroyed the temple of Serapis [in Alexandria -- F.E.D.], which still housed the only extensive collection of Greek works. *It is estimated that 300,000 manuscripts were destroyed [Thus book-burnings did not begin with the Nazis, but with the [Roman-Imperial pseudo-]"Christians" -- F.E.D.]. Many other works written on parchment were expunged by the Christians so that they could use the parchment for their own writings.*"

Ibid., Morris Kline, *Mathematical Thought from Ancient to Modern Times*, vol. I., p. 180-181.

"In 529 [C.E. -- F.E.D.], the Eastern Roman emperor Justinian closed all the Greek schools of philosophy, including Plato's Academy... The final blow to Alexandria was the conquest of Egypt by the uprising Moslems in A.D. 640 [C.E. -- F.E.D.]. *The remaining books were destroyed* on the ground given by Omar, the Arab conqueror: "Either the books contain what is in the Koran, in which case we do not have to read them, or they contain the opposite of what is in the Koran, in which case we must not read them." And so for six months the baths of Alexandria were heated by burning rolls of parchment [Thus the [pseudo-]Islamists also preceded the Nazis in book-burning -- F.E.D.]. After the capture of Alexandria by the Mohammedans, the majority of the scholars migrated to Constantinople, which had become the capital of the Eastern Roman Empire. Though *no activity along the lines of Greek thought could flourish in the unfriendly Christian atmosphere of Byzantium*, this flux of scholars and their works to comparative safety *increased the treasury of knowledge that was to reach Europe eight hundred years later. It is perhaps pointless to contemplate what might have been. But one cannot help observe that the Alexandrian Greek civilization ended its active scientific life on the threshold of the modern age. It had the unusual combination of theoretical and practical interests that proved so fertile a thousand years later. Until the last few centuries of its existence, it enjoyed freedom of thought, which is also essential to a flourishing culture. And it tackled and made major advances in several fields that were to become all-important in the Renaissance: quantitative plane and solid geometry; trigonometry; algebra; calculus; and astronomy.* It has often been said that man proposes and God disposes. It is more accurate to say of the Greeks that God proposed them and man disposed of them. The Greek mathematicians were wiped out. *But the fruits of their work did reach Europe.* ..."

Ibid., Morris Kline, *Mathematical Thought from Ancient to Modern Times*, vol. I., p. 181.