$$
\begin{gathered}
\left\{\underline{x}^{2} * \underline{x}\right\} \\
\text { Dialectical } \\
\text { Ideography }
\end{gathered}
$$

## A Contribution to the Immanent Critique of Arithmetic

## Part I. c.

## Prolegomena: Briefings on $\underline{\underline{\text { Xf }}}$

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Note: This essay is a partially selfexemplifying exposition of, and a record of an ongoing self-critique of, the ideas advanced herein. It is a 'meta-dynamical' and 'meta-evolving' conceptual object. Edition index [self-edit iteration number] and last revision date are stated on the title page; the most recent changes are coded as magenta-colored text. The time sequence of changes in the form / content of this essay is predicted to be both an illustration and an instantiation of the meta-model of ideo-ontological ideo-meta-dynamics that this essay explores, as well as of the 'homeomorphic defect' of that meta-model. We expect that successive editions of this document will document an 'ideo-onto-dynamasis' rather than an 'ideo-onto-stasis'; a 'meta-evolving ideo-ontology'; a 'multi-meta-ontic idea-cumulum'; an expanding, and ever 'thickening', increasingly inter- and intraconnected, 'inter-acted' network of 'inter-implicatory', 'inter-determinate', 'inter-generative' ideas, elaborated on a mounting count of 'metafinite', 'meta-fractal' scales, all exemplifying a "non-standard", Contra-Boolean logic; the ideo-ontologically dynamical logic of the dialectical "law" of cognition signified by the 'ideo-ontological', "pure-qualitative", $\underline{\underline{Q}}$-algebraic $\underline{\text { inequation }} \underline{x}^{2} \frac{3}{2} \underline{X}$.
This writing is an unpublished work, and one which is not sold or exchanged for remuneration or commercial gain of any kind, but is distributed «samizdat» to selected individuals and organizations, on a donation basis, free of charge. This work is a potential contribution to the collective creative property of the Terran human species: assimilate, disseminate, critique, and surpass at will. We, the authors, seek hereby to further neither our monetary riches, nor our public power, nor our personal fame. What we want, money cannot buy. We hope, with your help, to build a better us, and to help do our "infinitesimal" part in building a better universe ["Infinitesimal" differences can matter, as nonlinear dynamics demonstrates]. More monetary wealth will not buy that betterment. More political power cannot impose it. More fame would mainly distract from it. We hope that you have chosen, or will choose, to build a better you. We hold that this choice entails the profoundest consequences for one's life, as well as for the lives of others. We also hold that such choices belong to you alone. We wish to share, with you, the forthcoming conceptual riches. We will rejoice, and we will be compensated, if you teach us in turn, help us to correct our errors, and thus advance the common-wealth of all beyond this offering. We also request our readers' forgiveness in the areas of our many shortcomings, some of which, though determined to strive ceaselessly to overcome them, we will never, in a lifetime, overcome. Others' voices need to be raised -- perhaps your own voice -- to check and balance our biases. We, the authors, are not publicly accessible, but will endeavor to continue private transmittals to you if you indicate publicly, however cryptically, and we recommend that it be cryptically, your desire that we should do so. We want not that our existences, let alone our egos, should be an impediment to that great reverberating propagation of new cognitions, and of emerging new forms of cognition, of which this essay is, at best, an incomplete, imperfect, transitory, and transitional manifestation. We therefore happily forego personal credit, and, by thus renouncing in advance the [remote] possibility of any notoriety resulting thereby, hope also to retain more lifetime for the continuation of this work. Dialectical ideography as set forth herein is interpreted variously as: (1) a calculus of 'quanto-qualitative change', encompassing an explicit, ideographical arithmetic for the dimensional unit [ie]s or metrical "monads" of classical "dimensional analysis", and, thereby, 'semantifying' the "meaningless" singularities [finite-time "infinite" values] of especially the "unsolvable" [in part, because of those very singularities] nonlinear integrodifferential equations and their solution-functions through their metrical as well as ontological 're-qualification' using those new, explicit 'metrical qualifiers' of this 'dimensional arithmetic', as well as 'quantifiable' kinds of ontological qualifies, concretizing and operationalizing aspects of Plato's «Arithmoi Monadikoi» and Diophantus' $\mathbf{H}$, last extant circa 250 C.E.; (2) an alternative, onto-logical contra-Boolean algebra; (3) an ideographic, 'onto-dynamical' "symbolic logic" for the state-space/control-parameter-space 'meta-space' 'meta-dynamics' of 'meta-finite', conversion-singularity 'self-bifurcation'; (4) A mathematics for modeling the history of mathematical ideas as well as a [psycho-]historical algebra and arithmetic for modeling the 'meta-evolution' of the sciences generally; an ideography for the [psycho-]history of ideas; an ideography of the 'meta-dynamical' logic of conceptual self-innovation and self-development; a 'philosophical algebra' or trans-Leibnizian «characteristica universalis»; an arithmetic and algebra of innovative conception or of the creative conceptual process; (5) a rules-system for an ideographical language of ontological self-escalation in self-transcending [meta-]systems; (6) a generic algorithm for the 'meta' operation regress; for a trans-Hegelian, autopoiesic version of the 'aufheben' operation; and for a "dynamical", 'temporalized', diachronic, 'meta-evolutionary' version of the Russellian/Gödelian logical types hierarchy; (7) a model for a 'meta-fractal', non-Cantorian theory of totalities, of 'metu-finite' arithmetics, and of the "foundations" of mathematics; (8) an arithmetic, algebra, geometry, and analysis built on certain "non-standard natural mumbers", i.e., on the 'Gödelian 'meta-natural' numbers', a space of 'evolute' "meta-numbers" 'of 2 nd degree', 'made up out of "standard", '1st degree' natural numbers, instantiating those "non-standard models of first order Peano arithmetic" whose possibility is implied by the first-order joint applicability of Gödel's completeness theorem and first incompleteness theorem, as also by the Löwenheim-Skolem theorem, constructing therehy an 'ontologically dynamical', 'de-Parmenideanized' actualization of Plato's "arithmetic of dialectics", the «Arithmoi Eidetikoi». This essay, in addition to that of ideogramic, pictogramic, and phonogramic symbolization, draws also upon the power of neo-mythological, allegorical, and mythopoeic symbolism -- that is, of psycho-historical symbolism -- to aid in the conveyance of its most urgent messages. World-historically consequential universal labor, the evocation of effective psycho-historical force, including individual 'psyche-ological', affective force, requires $\underline{q}_{s P R}$ requires that its mythopoeic momenta, denoted $\underline{R}$, be integrated, indeed, dialectically synthesized, with its Philosophical and Scientific momenta. Dialectical ideography is, we believe, a humble but potent seed. As with the Riemannian, and the other non-Euclidean geometries that arose from the failed attempts to prove the absoluteness of Euclid's geometry, these non-Parmenidean, contra-Boolean, and contra-Cantorian onto-logical and onto-dynamical arithmetics and their algehras of dialectics may bear fruit for humanity only if germinated through the intra- and inter-personal dialogue, and dialectic, of assimilation, critique, refutation, and supersession. We have avoided broadcast publication and indiscriminant distribution of this essay. We wish to base its circulation, and the selection of its recipients, upon our best judgment of its potential value to each candidate recipient. The taking to heart of the ideas "graphed", ideographically, 'pictographically', and narratively, herein, can produce profound transformation in the very identity of the person so taking. Panic in response to perception of the early signs of such transformation in others may elicit, from some perceivers, a violent reaction. In particular, the intimations of the 'meta-human', $\Delta \mathrm{h}$, implications of the 'cumulum' of human[oid] evolution is profoundly disturbing to some. We are therefore directly transmitting this document only to those whom we perceive, via their own published writings, to be already verging on similar or related conceptions as a result of their own protracted 'self-metaevolution'. We have also decided not to disseminate the most "dangerous" of the results to date. We believe that you are eminently capable of 're'discovering these results, if you have not yet discovered them already. Should you do so, we urge that you treat them, and their dissemination to others, with utmost care. The system, more accurately, the systems, of dialectical ideography glossed herein continue to evolve and 'meta-etolve' rapidly in our research. They burgeon beneath our feet. We expect to exercise a similar restraint and discretion in any future progress reports which we may send your way. We therefore lodge the Omni-Copyright statement above together with this countervailing caveat we recommend, should you choose to disseminate this document, its ideas, and/or related ideas of your own discovery, that you do so with careful judgment as to the recipients you select. Give the friends of humanity a head start vis-à-vis their adversaries. Dialectics should inculcate humility. "Perfection" is not a final meta-state that can be finally manifested, but an open-ended, 'uncompleteable', asymptotic process, moving from greater to lesser imperfection. We realize that conceptual 'homeomorphic defect' is inescapable for cognizing beings such as ourselves. Even at best, we must always be partly wrong. Even at best, one cannot be finally, completely, and wholly right. One's mental constructs cannot ever be the truth, the whole truth, and nothing but the truth. But one may be right enough for one's time, for one's moment, for one's role, and for one's part; right enough to help one's contemporaries to live through, and beyond, one's time, and thus, potentially, to enjoy the privilege, the pain notwithstanding, of a vital ['life-ful'] and willing participation in the succeeding epoch of imperfection.

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| :---: | :---: |
| $\left[\mathrm{q}_{\mathrm{QN}}\right] \leftrightarrow \mathrm{q}_{3}:$ | Briefing on U Quanto-Qualitative Calauli Modeling Meta-Dynamics of पniverse-of-Discourse Multi-Population Meta-Distributions. |
| $\left[\mathrm{g}_{\text {Mu }}\right] \leftrightarrow \mathrm{g}_{7}$ : | Briefing on a , Quanto-Qualitative Calculus of Ontologically and Metrically Ounlified Quantifiers as State-Variables and Control-Parameters for |

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## Potential Conceptual/Terminological Unfamiliarities.

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Plato's term «Dialekticós».
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$\mu_{0}$ as ultimate recession / vanishing-point /'meta-infra-finite' remnant of $\underline{\underline{U}}$ within ${ }_{\alpha} \underline{\underline{\boldsymbol{u}}}$ ?

## $[\underline{\mathbf{N}}] \leftrightarrow \underline{q}_{2}: \quad$ Briefing on $\underline{\underline{\mathbf{Q}}}$

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Rule 2. [The Rule of Ontological Parsimony].
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Qtatical Algebra and Statical Geometry.
Q Meta-Dynamical Algebra.
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Q Arithmetic and [Meta-]Systematic Dialectics.
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Partial Syntheses -- Partial Solutions: Steppingstones and Half-Way Houses on the Way to Full Synthesis. Progressive Partial Syntheses as also Progressive Partial Antitheses.
$\left[\underline{q}_{\underline{M N}}\right] \leftrightarrow \underline{q}_{5}$ ' 'Peanic', Quantifiable, 'Contra-Boolean' Arithmetic Mapping Multiples of the Metrical Units arising from the 'Onto-Dynamasis' of a Single Metric/Unit «Archén.
$\left[\underline{q}_{M Q}\right] \leftrightarrow \underline{q}_{6}:$ 'Peanic', $\underline{U_{n}}$-Quantifiable, 'Boolean' Units Mapping 'Diophantine Monads' for an Indefinite Multiplicity of Qualitatively/Ontologically Distinct Monadic Species.

Synthesis -- Self-Re-Unity ['Meta-Finite', 'Meta-Fractal', 'Self-Subsumptive' Self-Re-Unificution]: $\underline{q}_{\text {MON }}$ or $\underline{q}_{\text {MU }}$ as the ${ }_{\alpha} \underline{\underline{\boldsymbol{u}}}$ Ideography. Some Aspects of 'Quanto-Qualitative Computation' and 'Quanto-Qualitative Modeling' in ${ }_{a} \underline{\underline{\mu}}$.
Interpretation of $\mathbf{n}_{\mathbf{u}} \underline{\underline{\mu}}_{\tau}$ 'Cumula' as Models of 'Meta-System' 'Meta-Dynamics' -- of Sequences of Systems separated by 'Meta-Fractal', 'Meta-Finite', Self-Bifurcation 'Conversion Singularities', Modeled via Unified State-Space/Control-Space 'Meta-Spaces', describing such 'Meta-Evolution' via Ontological/Dimensional/Axial/Metrical ['Quanto-Qualitative'] Net Self-Expansion of the 'Meta-Space'.

The Problem of Zero Division.
The 'Paradox of Singularity' in Dynamical 'Total-Differential' and 'Partial-Differential' Systems.
The Paradox of Infinite Error: "Instantaneous" Transition from Micro-Finite Residuals to Infinity Residuals. The Metrico-Ontological 'Fixity' of Dynamical Systems Models -- Corollary of 'The Parmenidean Postulate'? Singularity and 'The Nonlinearity Barrier'.

The Two Components of "Unsolvability".
Differential Equation Higher Degree / Degree > 1 Nonlinearity ties to Proneness to 'Moveable-Pole' Singularity, due to 'denominatorized'/ "neggative degrec" dymamical finite differences -- time-varying finctions minus control parameters.
The 'Conversion Meta-Dynamic' and Zero Division.
Physical Meanings of [Meta-]Dynamical 'Conversion-Singularity' as Guide to its Mathematical Representation. The «a posterioris, "synthetic", inductive, empirical Principle of 'Metafinity'. Singularity and Onto-Dynamasis. Pure-Quantitative Zeros versus Pure-Qualitative Zeros versus Quanto-Qualitative Zeros.
The 'Method of [Ontological-Metrical] Re-Qualification' and 'Singularity Semantification'.
Computation of Singularities via 'The Method of Re-Qualification' - Some Illustrations. The Rocket Equation: «Gedankens* Experiment. The Newtonian Mamy-Body Problem. The Ultraviolet Catastrophe. The 'Meta-Dynamics' of Stellar 'Meta-Evolution': Conversion Singularity Nucleosynthesis Inter-Epochal Transitions.
Transition to ${ }_{\beta} \underline{\underline{\mu}}$ and Beyond: Some Descriptive Limitations/Inadequacies of the ${ }_{\alpha} \underline{\underline{\mu}}$ Ideography.

[^0]I- xix Distributed «Santizdat» by Foundation Encyclopedia Dialectica
$\left[\underline{q}_{\underline{M u}}\right] \leftrightarrow \underline{q}_{7}:$ Briefing on ${ }_{\alpha} \underline{\underline{\boldsymbol{u}}}$,
Quanto-Qualitative Calculus of Ontologically and Metrically Qualified Quantifiers for Unified, Numerical/Dimensional Analysis and Singularity 'Semantification' ['Re-Qualified Metafinite Analysis'] [continued \& concluded].

Transition to the Next Section: The Precocity of these Briefings and the Work of Part II.
The Scale of Systematic-Dialectical 'Ideo-Meta-Evolution' from $\underline{\mathbf{N}}$ to $\underline{\mathbf{W}}$ to $\underline{\underline{Z}}$ to $\underline{\mathbf{Q}}$ to $\underline{\mathbf{R}}$ and Beyond.
The Scale of 'Ideo-Meta-Evolution' from $\underline{\underline{N}}_{\underline{\underline{A}}}^{\tau}$ to $_{\underline{w}} \underline{\underline{A}}_{\tau}$ to $\underline{\underline{z}}^{\boldsymbol{A}_{\tau}}$ to $\underline{\underline{Q}}_{\underline{\boldsymbol{A}_{\tau}}}$ to $\underline{\underline{R}}^{\underline{A}_{\tau}}, \ldots$ within each $\underline{\underline{A}}_{\tau}$.
The Psycho-Historical Ideo-Dialectic of $\mathbf{N}$.

This sub－section is designed to familiarize the reader with certain notational practices，as well as with some conceptual background，that might otherwise be unfamiliar，and which might thereby constitute a barrier to the reader＇s appropriation of the three Briefings which follow．

Potential Notational－Conceptual Unfamiliarities．The notation used in these Briefings may be unfamiliar mainly because it involves a qualitatively expanded arithmetic，algebra，and geometry；including a formulation of，and a mode of calculation with，＇pure qualifiers＇as well as with＇pure quantifiers＇，and also with＇quanto－qualifiers＇or＇qualo－quantifiers＇．While we endeavor to make the incremental notation，as far as we are able，consistent with－－indeed，a coherent extension or generalization of－－standard mathematical notation，the meanings of new signs are seldom self－evident from their form alone，without some conceptual introduction－－some explication of the idea behind the＇idea－gram＇．We find it best to begin such explication with the＇neo－ideograms＇which inaugurate the incremental，qualifier side of mathematics：
－The ideogram or ideographic symbol $\frac{⿱ 亠 二 又}{र}$ denotes the relation of non－quantitative，or qualitative，inequality．
There is an immanent need，even within today＇s standard mathematics，to make explicit its implicit involvement with relations of inequality，$\neq$ ，that are not equivalent to quantitative inequality，？．The relations＇greater than＇ and＇less than＇do not account for certain meanings of inequality that are already long－extant in mathematical usage．Consider everyday＂dimensional analysis＂，where we have 5 ［in．］$\neq 3$［in．］，in that $5[\mathrm{in} .]^{1}>3$［in．］$]^{1}$ ，but also $5[\mathrm{in} .]^{1} \neq 5[\mathrm{in} .]^{2}=5[\mathrm{sq} . \mathrm{in} .]^{1}$ ，and clearly $5[\mathrm{in} .]^{1} \neq 5[\mathrm{sq} . \mathrm{in} .]^{1}$ ，but also $5[\mathrm{in} .]^{1} \& 5[\mathrm{sq} . \mathrm{in} .]^{1}$ ，even though，by some idealizations，there are an infinite number of 1－dimensional line－segments［linear inches］in a 2－dimensional square＇plane－segment＇［square inch］，so that［in．］${ }^{2} \ggg$ in．$]^{1}$ ．We say that both line－segments \＆ ＇plane－segments＇are finite，but dimensionally，qualitatively different，and that a＇plane－segment＇is＇meta－finite＇ relative to a line－segment．Thus，we write：［in．$]^{2} \frac{\frac{7}{k}}{k}[\mathrm{in} .]^{1}$ ．Consider also the mathematical domain of set theory，where distinct ＂logical individuals＂$a \sim b, c, d ; b=c, d ; c=d$, imply $\{a, b\} \sim\{c, d\}, y e t,\{a, b\} \ngtr\{c, d\}, \&\{a, b\} \notin\{c, d\}$ ，despite the fact that these 2 sets
 ！＜ 1 either．Thus：！专 1 ．Consider also the calculus of＂vectors＂－of＂quantities＂which have both＂magnitule＂［quuntitatioc］，and＂direction＂［टepartly qualitative，though measurable in terms of quantitative angular＂degrees＂？］－－for either its＂scalar product rule＂［or＂dot product＂rule］，or its＂vector product rule＂［or＂cross product＂rule］，e．g．，of $\hat{\underline{e}}_{1} \cdot \hat{\underline{e}}_{1}=\cdot \dot{\hat{e}}_{1}^{2}=1$ ，and of $\underline{\hat{e}}_{1} \cdot \underline{\hat{e}}_{2}=0$ ，or $\underline{\hat{e}}_{1} \times \underline{\hat{e}}_{1}=\times \underline{\hat{e}}_{1}^{2}=0$ ，and $\underline{\hat{e}}_{1} \times \underline{\hat{e}}_{2}=\underline{\hat{e}}_{3}$ ，where either $\underline{\hat{e}}_{1}^{2}=\underline{\hat{e}}_{1}^{1}$ ，and
 ＂matrices＂or＂＇number－arrays＂＂is pervaded by the $\frac{\frac{y}{k}}{\}}$ relation to an even greater degree than is the＂space＂of vectors．

However，the need for the＇$\frac{1}{k}$＇symbol in Dialectical Ideography is even greater than in the standard mathematics of today．The＇dialectical mathematics＇of Dialectical Ideography begins with the formulation of new systems of non－standard arithmetic，systems whose＇meta－numerals＇model＇metrical qualifiers＇and／or ＇ontological qualifiers＇．Dialectical Ideography begins with＇meta－numbers＇which represent qualities，not［or not only］quantities．Therefore，the typical relation between any pair of distinct＇dialectical numbers＇is that denoted by＇娄＇，not that denoted by＇$\geqslant$＇．Standard mathematics is pervaded by a＂trichotomy principle＂which holds that the relation between any pair of numbers is，$\frac{2}{<}$ ，i．e．，that the possibilities of such relationship are exhausted by the three relations＇disjuncted＇in that sign，and contained in the three－element set $\{=,>,<\}$ ． Dialectical Ideography explicitly extends that＂trichotomy principle＂to a＇tetra－chotomy principle＇，and that set to［at least］the four－element set $\left\{=,>,<, \frac{y}{\}}\right\}$ ，for which there are two distinct and opposing＇ideo－ontological＇ ＇species＇or＂＇kinds＂＇of the＇［sub－］genus＇of inequality：$\neq$ as \}, versus $\neq$ as $\frac{\frac{7}{k}}{}$ ．
－ $\boldsymbol{\Delta}, \underline{\boldsymbol{\Delta}}, \& \underline{\underline{\Delta}}$ denote＂［meta－1finite differencing＂operations；＇incrementors＇，of，respectively，＇pure quantity＇，（ $\boldsymbol{\Delta}$ ），
＇pure quality＇， $\mathbb{I} \underline{\Delta} \mathbb{\rrbracket}$ ，and mixed／combined／unified＇quanto－quality＇，（ $\underline{\underline{\Delta}}$ ）．
－1，－C cumulum［＂total＂］＂order＂signs for historical dialectic \＆［meta－］systematic dialectic resp．［homologous to＜in pure－quantitative＇consecua＇， ＇contigua＇，and continua］．E．g．，$\hat{\mathbf{g}}_{1} \leftrightarrow \underline{\mathbf{x}}+\boldsymbol{y} \leftrightarrow \hat{\mathbf{G}}_{2}$ ，if $\underline{\mathbf{x}} \& \underline{\mathbf{y}}$ refer to＇intensions＇of objective－extensive or＇physio－ontos＇［＇«physis» ontos＇， or＂＇physical＂＇＇ontos＇］，$\underline{X} \rightarrow \mathbf{Y}$ if to subjective－intensive／intensional categories or＇ideo－ontos＇．

Thus, $\boldsymbol{\Delta}(\mathbf{x})=\Delta \mathbf{x}$ denotes some standard-increment part of that «arithmos» of "'quantifier"' units -- that abstract, pure, unqualified 'quantifier' value -- for which $\mathbf{x}$ stands. The expression $\underline{\boldsymbol{I}} \underline{\mathbf{x}} \boldsymbol{\rrbracket}=\underline{\mathbf{x}}$, on the contrary, denotes some 'meta-fractally' higher likeness of that "arithmos" of 'qualifier' units -- the abstract, pure, unquantified 'qualifier' value -- for which $\underline{\mathbf{x}}$ stands. The expression $\underline{\underline{\Delta}}[\underline{\mathbf{x}}]=\underline{\underline{\mathbf{x}}}$ denotes a differencing operation involving the 'quanto-qualifier' ['quantified qualifier'] or 'qualo-quantifier' ['qualified quantifier'] for which $\underline{\underline{\boldsymbol{X}}}$ stands.

[^1]- [. . .】 for 'pure ontological qualifier' contents [contents here denoted generically by ellipsis dots '...']. For convenience, in contexts devoid of the 'co-occurrence' of ontological and metrical 'qualifiers', where confusion thus should not arise, we may use [. . .] to enclose pure 'ontological qualifiers';
- [ ...] for 'pure metrical qualifier' contents;
- <..>
for unquantified [or unit-quantified] and explicitly 'mixed' qualifier/qualifier cases -- for 'metrically-qualified ontological qualifiers' or 'ontologically-qualified metrical qualifiers';
- ( . . ) for 'full-multiplicity' [i.e., not confined to the unit interval] 'pure quantifier' contents;
- (...) for 'mixed', 'quanto-qualifier', or 'qualo-quantifier' products-contents, i.e., for 'qualified quantities' or 'quantified qualities' as expressed via 'qualified quantifier' or 'quantified qualifier' symbols.

Symbols of 'meta-finite', "'reoolutionary"' transition, or of dialectical-logical 'followership', are used abundantly throughout -

- $\rightarrow$ denotes 'self-causal implication' or 'temporal becoming' in historical dialectic; movements of external event[itie]s,
- $\rightarrow$ denotes the self-causation of cutegorial, ideational progressions in [meta-]system-atic dialectic; movements of mind;
- -1 denotes the postulated movement from any given 'meta-numeral' value to its self-«aufheben» self-negation via [self-lsquaring, in the context of a 'minimally-interpreted' syntax of a dialectical arithmetic, as yet applied neither for the construction of an historical dialectical model, nor for that of a [meta-]systematic dialectical model;

The relationship between a symbol which denotes an [ontological] "intension[ality]" or "meaning" for a universe of discourse that is to be modeled, and a 'dialectical meta-mmeral', "assigned" to that symbol as its "'model" or "interpretation"' within Dialectical Ideograplny, is expressed via a special sign herein --

- $\leftrightarrow$ denotes the relationship or the operation of 'interpretation' in the above sense. Thus, an expression such as 'humanity $\leftrightarrow \hat{\mathbf{g}}_{256}$ ', asserts that the 'meta-numeral' $\hat{\mathbf{g}}_{256}$, interpreted as an 'ontological qualifier', is assigned to the 'onto' ['ontological category'] symbol or 'eventity' symbol 'humanity', as a model or imperfect -- or with homeomorphic defect -- representation of the 'eventity' thus designated, in a dialectical-ideographic model of the universe of discourse being specified, in part, by that expression.
Dialectical-ideographic models also involve kinds of 'containment' that generalize, and differ qualitatively from, those familiar from standard set theory, viz., $\in$ [is an element of , and $\subset$ [is a subset of] --
- ᄃ, コ denote relationships of 'sub-system' or 'sub-component' explicit containment at any level, which may hold between intension-symbols, even when, for their extensions, the $\in$ and $\subset$ relations do not hold.
- ㄷ, $\exists$ \| denote relationships of implicit containment, as embryonic versus as extant or manifest possibility.

We use 3 ideograms for 3 aspects of each present's vision of the 'Ultimate Arithmetic', always receding, in response to our advance, into ever-higher 'meta-fractal' layers of 'ideo-ontology' as we approach it, per 'The Gödelian Ideo-Meta-Dynamic', or 'The Gödelian Dialectic':
) H (, ) $\mathrm{X}($, )( denote the current horizons of (1) the ultimate space or "set" of 'standard meta-numbers' [starting with $\mathbf{i}=+\sqrt{-1] ; ~(2) ~ t h e ~ u l t i m a t e ~ s p a c e ~ o r ~ " s e t " ~ o f ~ ' n o n-s t a n d a r d / d i a l e c t i c a l ~ m e t a-n u m b e r s '[s t a r t i n g ~}$ with $\hat{\mathbf{G}}_{1}$, and (3) the ultimate rules-system for the arithmetics, + , of both the former and latter.

We follow a practice of recent mathematics in denoting, by univocal symbols, whole "spaces" or [potentially-infinite] ""sets"' of numbers of various "'species'", thus forming, as a conceptual object, and directing readers' attentions to, the common essence, "'intension"', or totality of a given, entire such "'species'" as a subject-matter of discourse.

For example, ' $\mathbf{N}$ ' standardly denotes the so-called "Natural" numbers "set" or "'space"', namely, $\{1,2,3, \ldots\}$, and $\mathbf{R}$, the "irrational" plus "ratio-nal" -- the so-called "Real" -- numbers. We go further here, denoting, by the underscoring of such 'number-space' or 'number-set' symbols, the total 'rules-system', including various axiomatizations, of the arithmetic of that 'number-space', viz. $\mathbf{N} \& \underline{\mathbf{R}}$.

Since we use $\underline{\mathbf{Q}}$ to denote the 'number-space' of the 'meta-numbers' interpretable as Ontological $\mathbf{Q u a l i f i e r s , ~ t o ~}$ help distinguish it from © which traditionally denotes the "Quotient" or "ratio-nal" numbers, we use $\underline{\underline{\mathbf{Q}} \text { to }}$ denote the rules-system of 'pure-qualifier' arithmetic. Single-underscored number-space symbols indicate '"second-order"' and "'higher-order"' specifications or axiomatizations of such arithmetics.


#### Abstract

Paradoxically, double-underscored number-space symbols here denote "'first-order"' specifications or axiomatizations of such arithmetics. We foment this notational paradox to subliminally remind the reader, and ourselves, that the "'first-order" rules-systems encompass more arithmetic content, including so-called "non-standard" versions of those arithmetics, than do the higher-order specifications [as implied by the conjunction of the Gödel [semantic] completeness and [syntactic] incompleteness theorems for "'first-order"' systems, a conjunction which does not pertain for "'higher-order"' systems, for which the completeness theorem does not hold, but only the incompleteness theorems]. Specifically, the "'first-order"" rules-systems encompass 'The Standard/Non-Standard Models intra-duality'. The 'meta-systematic dialectic' generated by/expressive of that 'intra-duality' is the central focus of the Briefings which follow. Also, we use these rules-system symbols not as passive, nounic labels or designators, but as dialectical operators; as 'self-«aufheben»' self-sublators, e.g.:


## 

A brief catalogue of our symbols for some of the various "standard" arithmetics encountered in these Briefings is given below [for fuller detail, see Postscripts, Glossaries, Glossary of Ideograms] --
$\mathbf{N} \pm\{1,2,3, \ldots\}$, the one-dimensional space or 'consecuum' of the so-called "Natural" Numbers;
$W \equiv\{0,1,2,3, \ldots\}$, the one-dimensional space or 'consecuum' of the so-called 'Whole" Numbers;
$\mathbf{Z} \equiv\{\ldots,-1,-2,-3,0,+1,+2,+3, \ldots\}$, the $1-\mathrm{D}$ space or 'consecuum' of the "'Integral"' Numbers, or "Integers";
Q $=\{\ldots,-4 / 3, \ldots,-1 / 2, \ldots, 0, \ldots,+1 / 2, \ldots,+4 / 3, \ldots\}$ ' contiguum' of the "Quotient" Numbers, or "Ratio[-nal]" Numbers;
$\boldsymbol{R} \equiv\{\ldots,-\pi, \ldots,-e, \ldots,-\sqrt{2}, \ldots, 0, \ldots,+\sqrt{2}, \ldots,+e, \ldots,+\pi, \ldots\}$, so-called "continuum" of the so-called "Real" Numbers;

$\mathbf{H} \equiv\left\{\mathbf{R r}+\boldsymbol{R r} \underline{\mathrm{i}}_{1}+\boldsymbol{\operatorname { R r }} \underline{\mathrm{i}}_{2}+\boldsymbol{\operatorname { R r }} \underline{\mathrm{r}}_{3}\right\}$, the 4-D [four-dimensional] space or "contimuum" of the $\mathbf{H}$ amilton Quaternions;

$\mathbf{V} \equiv\left\{\boldsymbol{R}_{\underline{\mathbf{e}_{1}}}+\boldsymbol{R} \underline{\mathbf{e}}_{2}+\boldsymbol{R} \underline{\mathbf{e}}_{3}+\ldots\right\}$, the n -dimensional space or "continuum" of the Heaviside/Gibbs/'quasi-Grassmannian $\underline{\mathbf{v}}$ ectors.
We use 'pre-subscripts', such as the $\mathbf{u}$ in ${ }_{\mathbf{u}}{ }_{\underline{Q}}{ }^{2}{ }^{2}$, which specifies the $\underline{\text { universe }}$ of discourse which the $\underline{\mathbf{Q}}$ have been applied to model in the case at hand, as well as the usual '[post-]subscripts', such as the $\tau$ in the same compound-ideogram symbol. We also employ 'pre-superscripts', such as the $\mathbf{n}$ in ${ }_{u}{ }_{u} \underline{Q}_{\tau}{ }^{2}$, which specifies the ordinal-numbered taxonomic level within $\mathbf{u}$, which the $\underline{\underline{Q}}$ have been applied to model, in addition to the usual '[post-]superscripts', such as the power or exponent 2 in the same symbol-complex [for more on these conventions of notation, please see Part I. a., sub-section entitled Higher Level [Sub-]Universe-Models].

We delimit major hypotheses -- typically textual, and denoted generically, here, by ellipsis dots, '...' -- as follows: $\quad$ [though the majority of the muterial, so enclosed or not, remains conjectural], vs. [proven] theorems, derived deductively from explicit premises, via $\quad$.... . Single quote-marks enclose 'self-quotes' of our own coinages. Double quote-marks enclose exact quotes of others. Triple quote-marks enclose approximate, paraphrased quotes of others. Double 'angle marks', «...》, enclose non-English words, whether transliterated or rendered in their native alphabets].

We also employ－－in contexts where the $\mathbf{3}$ major＇application－domains＇of the $\underline{\underline{\mathbf{Q}}}$ dialectical arithmetic require rigorous distinction－－distinct but mutually similar notations for the＇generic dialectical self－movement＇，or ＇generic onto－dynamical «autokinesis»＇．That is，we employ－－
（1）for the generic，minimally－interpreted $\underline{\underline{Q}}$ arithmetic：

（2）for that arithmetic interpreted for historical dialectic，or＂＇dialectic of［both pre－human and human］nature＂＇：

（3）for that arithmetic interpreted for meta－system－atic dialectical，categorial－progression，idea－systems－progression idea－exposition：


The above expressions each assert，among other principles，one which holds that each $\hat{\underline{\underline{x}}}$ is $\hat{i t s}$ own＇auf－its own self－－aufhebens＂operator，its own ＇meta－evolutionary＇，＇meta－dynamical＇，＇meta－finite＇，＇meta－monadic＇dialectical self－negation／self－transformation operation．

Herein we may，for typographical convenience，drop the ${ }^{\wedge} \boldsymbol{N}$＇symbol－element，or＇ideographical diacritical mark＇，［which signifies the unit－status［e．g．，modulus equal to unity，etc．］of those＇dialectical meta－numbers＇ which bear it］，where the presence of other，contextual cues so allows．Also for typographical convenience，we
 ＂pure＂，unquantifiable＇ontological qualifiers＇，the $\hat{\mathbf{G}}_{n}$ ，the $\hat{\underline{u}}_{n}$＇quantifiable ontological qualifiers＇of $\underline{\underline{U}}$ arithmetic，and the $\stackrel{\hat{\mu}}{\mu}_{\underline{\hat{u}}}$＇metrical qualifiers＇of the ${ }_{a} \boldsymbol{\mu}$［＇alpha－mu＇］arithmetic，where context permits this without confusion．

We may also use the standard＇+ ＇sign in place of the＇⿴囗十丌＇sign of the minimally－interpreted $\underline{\underline{\mathbf{Q}}}$ arithmetic，the＇ $\mathbf{\omega}^{\prime}$ sign of the $\underline{\underline{\mathbf{Q}}}$ arithmetic interpreted for historical exo－dialectic，and the＇$\omega^{\prime}$ sign of that arithmetic interpreted for categories－progression／systems－progression＇［ meta－］system－atic ideo－dialectic＇，in contexts where the generalization of＇+ ＇to encompass such operations of superposition，aggregation，or＂addition＂＇，including their ＇non－amalgamative＇，as well as their idempotent，or＇super－amalgamative＇aspects，is clear．

Along with the above 3 variants of the signs for the analogue of ordinary－arithmetic＇s operation of addition in the＇pure qualifier＇arithmetic of $\underline{\underline{\mathbf{Q}}}$ ，and beyond，we have also a corresponding 3 variants of the signs for the nearest analogues of ordinary arithmetic＇s operations of multiplication，subtraction，and division：
－区，囚，＂＇multiplication＇＂／proliferation／＇multiplicitation＇of ontological，metrical，etc．，qualities／of ontological，metrical，etc．，＇qualifier dialectors＇；
－$\square, \ominus, \ominus$＂＇subtraction＂＇of ontological，metrical，etc．，qualities／of ontological，metrical，etc．，＇qualifier dialectors＇；
－$\rightarrow \uparrow, \ominus$＂＇division＇＂of ontological，metrical，etc．，qualities／of ontological，metrical，etc．，＇qualifier dialectors＇．
The latter two analogous operations－－the $\underline{\underline{Q}}$－and－beyond analogues of subtraction and division－－first become fully definable only in $\mathbf{w} \mathbf{Q}$ and $\mathbf{z} \mathbf{Q}$ ，respectively，but are not comprehensively meaningful in $\mathbf{N} \mathbf{Q}$ ， where the $\mathbf{3}$ progressive variants of the $\underline{\underline{\mathbf{Q}}}$ arithmetic just referenced are defined as follows：
$\cdot{ }_{\mathbf{N}} \underline{Q} \equiv\left\{\hat{\mathrm{~g}}_{1}, \hat{\mathrm{~g}}_{2}, \hat{\mathrm{~g}}_{3}, \ldots\right\} ;$

$z^{Q}=\left\{\ldots, \hat{\mathbf{g}}_{-3}, \hat{\underline{G}}_{-2}, \hat{\underline{G}}_{-1}, \hat{q}_{0}, \hat{\underline{G}}_{+1}, \hat{\underline{G}}_{+2}, \hat{\underline{G}}_{+3}, \ldots\right\}$ ．

- Plato's term «Dianoia» ["The [human faculty of] Understanding"; cf. Hegel's «Verstand»], names, in our usage herein, those human factor(s), faction(s), and institutions upholding exclusively the mode and "'instrumentality of thought"' [or «organon»] of reductionist, atomistic-analytical, mechanical/formal logic. This logic initially holds, at the beginning of its history, that the axioms, primitives, base definitions, and rules of inference of mathematics, and of formal logic itself, are unique and fixed in perpetuity; are immutable, timeless, changeless, eternal truths.

But it also holds that these ideas cannot be accounted for or justified formally; that they must arise/be discovered outside, external to, the axiomatic systems resulting from the applications of formal logic; that there is no formal-logical means for arriving at these ideas; that formal deduction/rules of inference provide no justification, no method for arriving at, no formal account of, these starting points -- the points from which all else is deduced. It offers only the essentially authoritarian, dogmatic, and doctrinaire principle that may be voiced as follows: "I am an expert; I have tried and experienced the consequences of every possible relevant alternative; I therefore know that this starting point is optimal -- the best of all possible starting points -- so first just accept these "STANDARD" postulates. All else that I have promised -- the best possible results -- follow from them. Lust trust me [on pain of ridicule, ostracism, . . . death, etc.]".

- Plato's term «Dialekticós» [or «Dialektiké», cf. Hegel's «Dialektik», and «Vernunft», for "[Dialectical] Reason"], in our usage herein, names an «organon» that includes: (1) mental instrumentalities for the immanent critique of existing axiomatizations; (2) for finding -- and optimizing/justifying, relative to known alternatives -- axioms, definitions, primitives, \& rules of inference, for emergent axiomatizations, and; (3) for accounting, "'psycho-historically'", for the order of 'meta-evolution' of systems of axioms, definitions, primitives, \& rules of inference across human-historical time. This includes comprehending and modeling, reconstructively, and even predictively, the empirically/'psycho-archaeologically' observable patterns of that 'meta-evolution'. Even before the discovery of the Non-Euclidean Geometries; even before Gödel's formalization of the logical phenomenon of "undecidability" or "axiomatic independence", the Platonic dialectical tradition held that, in a given universe of discourse, there is usually no single, unique, optimal system of axioms, definitions, primitives, and rules of inference; that trade-offs and options exist, even abound -- that 'alternativity abounds' -- for framing qualitatively, 'ideo-ontologically' different, differentially useful, differentially advantaged-and-dis-advantaged axiomatic systems. Moreover, the «organon» of 'dialectics' is a thought-tool whose users are, by their use of it, brought to notice, through their own 'psycho-historical'/'psycho-archaeological' observation of human history, that even mainline, "standard" logical/mathematical/scientific axiomatic systems 'meta-evolve', i.e., change qualitatively, expanding and progressing in the 'ideo-ontology' they imply, along with the general progress of humanity. They may thus discern patterns, 'ideo-meta-dynamics', meta-"laws" of this 'ideo-ontological expansion'; of this mapping of what we call 'The Gödelian Ideo-Meta-Dynamic' onto human history.
The heart of the dialectical tradition thus centers upon the collective, pluralistic, democratic, social principle of ever-present, living, open, collective self-questioning; the Socratic/Platonic principle of continual social self-interrogation [no longer limited, as within the horizons of the ancient Greek epoch of human society, to democratic deliberation restricted to the slave-holding/eligible citizen-elite, exduding the rest of societyl, as opposed to the monolithic, dictatorial, tendentially totalitarian, dogmatic, doctrinaire, and consequently typically sub-optimal impositions of received authority.

This «Dialectics» is the standpoint of this essay, and the name of the movement to which it belongs. As a result, we do not confine our concerns to the discussion of, e.g., N, W, Z, Q, R, \& C as if fixed and final/pre-completed Number-Spaces, nor to that of their rules-systems, denoted $\mathbf{N}, \boldsymbol{W}, \mathbf{Q}, \mathbf{R}$ \& $\mathbf{C}$ respectively, as if they were fixed, eternal, immutable systems-of-truths, nor to that of their usual pedagogical interconnexion or [meta-]system-atic order-of-exposition, e.g., $\underline{\mathbf{N}} \rightarrow \underline{\mathbf{W}} \rightarrow \underline{\mathbf{Z}} \rightarrow \underline{\mathbf{Q}} \rightarrow \underline{\mathbf{R}} \rightarrow \underline{\mathbf{C}}$, nor to that of their historical progression and order-of-discovery, which is, per our Hypothesis, for occidental humanity at least: $\underline{\mathbf{N}} \rightarrow \underline{\underline{\mathbf{Q}}}_{+} \rightarrow \underline{\mathbf{R}}_{+} \rightarrow \underline{\mathbf{R}}_{\mathrm{n} \text { so }} \rightarrow \underline{\mathbf{R}}_{-} \rightarrow \underline{\mathbf{R}} \rightarrow \underline{\mathbf{C}} \rightarrow \underline{\mathbf{H}} \rightarrow \underline{\mathbf{0}} \rightarrow \ldots \rightarrow \underline{\mathbf{V}} \rightarrow \ldots$, wherein $\underline{\mathbf{R}}_{+}$denotes $\mathbf{R}$ exclusive of 0 and "negatives numbers", and wherein $\underline{\mathbf{R}}_{-1}{ }_{0}$ denotes the same, but inclusive of positive infinitesimals.
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We are also concerned here，as in the Briefings which follow，with their＇ideo－meta－dynamics＇or＇＂ideo－meta－system transitions＇＂in the direction of＇contra－Boolean＇，dialectical＇quantifier／qualifier＇arithmetics，such as：

## $[\underline{\underline{\mathbf{W}}}] \ni[\underline{\underline{\mathbf{W}}} \oplus \underline{\underline{\mathbf{Q}}}] \rightarrow[\underline{\underline{\mathbf{W}}} \oplus \underline{\underline{\mathbf{Q}}} \oplus \underline{\underline{\mathrm{U}}} \oplus \underline{\underline{\mathbf{M}}}] \rightarrow\left[\underline{\underline{\mathbf{W}}} \oplus \underline{\underline{\mathrm{Q}}} \oplus \underline{\underline{\mathrm{U}}} \oplus \underline{\underline{\mathbf{M}}} \oplus \hat{\mathrm{G}}_{\underline{M N}} \oplus \underline{\hat{\mathrm{G}}_{\underline{M O}}} \oplus \hat{\mathrm{G}}_{\underline{M O N}} \oplus \underline{\underline{M}}\right]$.

I．e．，we also address the＇ideo－ontodynamasis＇of the arithmetical＇meta－system＇，or＇ideo－meta－fractal＇ sequence of systems of arithmetic，in that direction of conception which encompasses dialectical ideography as developed herein and to－date，and as outlined in these Briefings．

We conduct，in these Briefings，an exposition of this＇ideo－metasystem＇of dialectical ideography by means of a ＇metasystematic－dialectical＇＂categorial progression＂method of presentation and of dialectical derivation of each successive system of arithmetic as＇idea－system＇，＇rules－system＇，or＇category＇／＂＂species＇＂of ideography．We do so using the $\underline{\underline{\mathbf{Q}}}$ arithmetic itself－－which is also one of the systems of dialectical arithmetic arising in this progression of＇Peanic＇［or］arithmetical systems－－to model the whole progression：for the dialectical derivation of each system in the sequence，itself included．Denotations of our symbols for the first seven of these＇ideo－ontological＇ categories－of－arithmetics in that＇metasystematic－dialectical＇order，starting from not from $\underline{\underline{\mathbf{N}}}$ as «arché»，are：

 space of＇quanto－qualitative＇or＇qualo－quantitative meta－numbers＇which are doubly qualifed，of quantifiers qualificd both ontologically and metrically，able to model the meta－system succession of a given＇meta－finite＇，＇meta－fractal multi－ontic cumulum＇in a＇guanto－qualitative＇\＆＇multi－metrical＇way，and in terms of any spectrum of metrics；used to characterize the state－variables and control－parameters of the＇meta－esolving＇，＇meta－dynamical＇state－ space／control－space＇unified meta－space＇of each successive system of the diachronic＇meta－system＇，encompassing＇metafinite self－bijurration singularities＇，for evolving and＇meta－evolving＇unitiereses of discourse in general．

In the above－listed systems－progression，$\underline{\underline{W}}$ is taken as conceptual «arché»［ever－present origin；ever－ controlling source；initial condition］of number－in－general，while $\underline{\underline{w}} \underline{\underline{Q}}$ denotes the＂arché» of the dialectical ideographies，of the explicitly dialectical arithmetics presented herein．In the notations for the＇meta－numerals＇of these dialectical arithmetics，we employ the underscore ideogram element，or＂＇ideographical diacritical mark＂＇，＇－＇， to signify those＇meta－numbers＇or＇meta－scalar＇operators whose＇unit－qualifier＇self－multiplication is＇strongly contra－Boolean＇，i．e．，whose＇self－product－tion＇is＇ontologically＇or qualitatively＇meta－potent＇，whether＇evolute＇or ＇convolute＇．Such＇ontology self－multiplications＇have the form $\underline{\mathbf{x}} \cdot \underline{\mathbf{x}}=\underline{\mathbf{x}} \times \underline{\mathbf{x}}=\underline{\mathbf{x}}[\underline{\mathbf{x}}]=\underline{\mathbf{x}}=\underline{\mathbf{x}}^{2}=\underline{r} \underline{\mathbf{x}} \mathbf{⿴ 囗 十 \underline { \mathbf { x } }} \underline{\text { ，such }}$
 ontological qualifiers，to signify qualifier＂unities＂：＂units＂，＂monads＂，＂forms of unity＂，＂Non－Standard＂forms of 1，i．e．，transcendental operators of＇meta－modulus＇＂unity＂［of＂length＂＂one unit＂；of＂unit length＂］，in short，to designate＇dialector unit－qualifiers＇in the tradition of the traditional＂unit vectors＂，herein denoted $\underline{\hat{\mathbf{x}}}, \underline{\mathbf{y}}, \underline{\mathbf{z}}$ ， or $\hat{\mathbf{E}}_{1}, \hat{\mathbf{e}}_{2}, \hat{\mathbf{e}}_{3}, \ldots$ ．We sometimes，when contextual cues allow，omit the＂hat＂component for typographical convenience．We similarly employ the ＇Á＇＇headdress＇ideogram－element to signify quantifiable zunit qualifiers［reprising Diophantus＇＂syncopated＂symbolization of the abstract，generic homogeneous units，or «monads＂，of the Platonic＋aarilhmoi monadikois，via ${ }^{\circ}$ in the proto－algebraic notation of his circa 250 C．E．work，the ＂Arithmeticas．］

- The philosophical term "Ontology" is employed herein in a non-reductionist mode, perhaps a mode rarely encountered in explicit form. For example, if we suppose that molecules are the immediate 'meta-imits' or 'meta-«monads»' of atoms -- 'meta-atoms', each made up out of a heterogeneous multiplicity of atoms - constructed via densified/intensified mutual interactions of atoms after localized atomic populations within the 'meta-evolving' cosmos attain sufficient multiplicity and density, we nevertheless conceive the ontological category of molecules as a "tertium quid"; as holding an independent ontological status vis-à-vis that of atoms if we find that the 'behavioral qualities', the 'types of action', the "emergent properties" evident in the phenomenology of molecules 'qualitatively exceed' and add to those manifest in atomic matters which were, or which remain, without any organization at the molecular level. We then, in that case, do not say that molecules are "just" "collections" of atoms. We do not "reduce" molecules to atoms simply because molecules "contain" atoms, or constitute a 'self-subsumption' of atoms. We see the self-movement of the 'meta-evolving' cosmos, from a 'meta-state' containing atoms but no molecules to one containing both -- to one containing 'atoms $\oplus$ molecules' -- as one in which the cosmos adds new ontology to itself; as an ontology expansion; as a 'meta-dynamical process' and also as an 'onto-dynamical' self-movement, an 'onto-dynamasis'; as an 'onto-auto-kinesis' and as an 'auto-onto-kinesis'.

Overview of Briefings. Let us characterize these first $\mathbf{7}$ arithmetics in more detail, as a preview of what is to come:


- $\underline{\underline{Q}}_{\underline{Q}}^{\underline{Q} \text { denotes a system of arithmetic whose units/ «monads» are 'Peanic' [compliant with the first-order Peano Postulates] }}$ but 'contra-Boolean' in self-multiplication, and interpretable as "pure", i.e., 'unquantifiable ontological unit-㗐ualifiers' [in polar contrariety to $\mathbf{N}$ interpreted as a system of arithmetic of "pure", i.e., of "abstract" or 'unqualified' quantifiers'], i.e., with each $\hat{\mathbf{g}}_{n}$ 'meta-number' as a non-additive/'non-addable' [additively idempotent] 'ontological qualifier'.
 also self-multiplicatively 'contra-Boolean', but which are still interpretable as 'ontological qualifiers', and which thus combine 'ontological qualification' with '"quantification"', but which still lack the capacity to express 'metrical qualifiers', or any other, higher species of qualifiers, ideographically and algorithmically.
- $\underline{\underline{\mathrm{M}}}=\underline{\underline{\mathrm{M}}}$ denotes a system of arithmetic which models the ontology and 'onto-dynamics' of a[ny] unit or «monad» which is interpretable as a metrical qualifier or dimensional unit [y], and which unit[y] is 'Qualo-Peanic', and 'contra-Boolean' in self-multiplication, but, again, non-additive [additively idempotent], and restricted, in its modeling capability, to a capability to model but one single "arché" species of metric/metrical unit/dimension/dimensional unit at a time.
- $\mathbf{M}_{\text {MN }}$ denotes a system of arithmetic whose units / qualifiers are 'quantifiable' as well 'Qualo-Peanic' and self-multiplicatively 'contra-Boolean', interpretable as [metrical qualifiers], and which thus combine 'metrical qualification' with its opposite, "quantification"', but only for one single «arché» species of metric/metrical unit/dimension/dimensional unit at a time.
- $\underline{\mathbf{A}}_{\mathrm{MQ}}$ denotes a system of arithmetic of 'Qualo-Peanic', hybrid, $\underline{\text { Q-subscripted, still unquantifiable qualifier units/ "monad"s }}$ which yet exhibit the 'Boolean, "idempotent" self-multiplication' which is also characteristic of the units of the classical arithmetic of the Diophantine monads [which Diophantus denoted by the "syncopated" abbreviation $\frac{\circ}{\mathbf{H}}$ ], and of Plato's «Arithmoi Monadikoi», but still partially interpretable as an arithmetic of metrical qualifiers, and which, via its Q-subscripting, first surfaces the 'principle of rules-system subsumption' and the 'principle of dis-entanglement' of "'fundamental units'" needed for a full-fledged arithmetic of dimensional analysis, and encompassing, for the first time, though in a 'pre-vestigial' fashion, multiple «arché" for multiple species of "'fundamental"' metrics.

combined, but 'disentangle-able' 'metrical qualifiers' are quantifiable, and its 'ontological qualifiers' are 'metrically quanto-qualifiable', its quantifier unit, and 'metrical and ontological qualifier units', are 'Peanic', its quantifier unit[y] is 'Boolean' in self-multiplication, its 'metrical and ontological qualifier' umits are 'contra-Boolean' in self-multiplication, and 'addable'/additively non-idempotent. Also, its 'metrical quanto-qualifier' sub-arithmetic instantiates a full-fledged, ideographical-algorithmic arithmetic of dimensional analysis, encompassing, in unified fashion, multiple «arché» for multiple species of "fundamental" metrical units/ «monad"s, and its zero division or additive-identity-division operation is well-defined/determinate. Consequently, the "singularities" of dynamical models can be calculated and 'semantified' as meaningful, accurate '[meta-] finite' oalues, thereby voercuming a key cause of "The Nonlinearity Barrier"', given the inherent connexion of degree $>1$ values of state-function unknowns in rate-of-change equations to finite-time, moveable pole, zero-division singularity. Each $\hat{G}_{M O N} \equiv \hat{\mathbf{G}}_{\underline{M U}} \equiv \underline{\underline{\mu}}{ }_{\underline{\prime}}$ 'meta-number' is thus able to express the momentaneous value of a 'state-variable' or a 'control-parameter' of a [nonlinear] dynamical system, with the 'ontological qualifier' "'factor'" denoting the 'vector component qualifier', the "kind", of the metric in question, with the 'metrical qualifier' "factor" denoting the dimensional/metrical unit of the metric in question, and with the 'quantifier' "factor"' denoting the momentaneous number of dimensional/metrical units of that metric manifest by the system in question, at the temporal moment in question. pedagogical decision that the increment to 'concept-ontology' produced by the 'self-reflexion' of $\underline{\underline{Q}}$ - of the first-order rules-system of 'ontological qualification' -- is $\mathbf{M}$ the rules-system of Metrical qualification and of an 'ideo-ontological meta-evolution' of 'metrical qualifiers' which is analogous to, or a special case/species of, ontological qualification and of the 'meta-evolution' of 'ontological qualifiers'.

The first Briefing presents the $\underline{\underline{\mathbf{Q}}}$ arithmetic. The second Briefing presents the $\underline{\underline{\mathbf{U}}}$ arithmetic. The third and final Briefing presents the $\hat{\underline{G}}_{\text {MON }}{ }^{\hat{\hat{\mathbf{a}}}_{\text {MU }}}$ or ${ }_{a} \underline{\underline{\mu}}$ arithmetic, including the build-up toward it via the successive conceptual superposition of the $\underline{\underline{M}}$ G $_{\text {MN }^{\prime}}$ and $\underline{\underline{G}}_{\underline{M Q}}$ arithmetics.

## Mathematical Logic, Dialectical Logic, and 'The Gödelian Ideo-Meta-Dynamic', or 'Gödelian Dialectic'.

Mathematical Logic is about 'evolving' a given mathematical system by deductively proving and thus "'birthing"' an ever greater 'population of theorems' of/within that system. The truth of such theorems is never absolute. It is always a truth from and relative to the axioms or postulates of that system; for the logical individuals/undefined primitives of the universe of discourse of that system -- for its postulated "ontology". It is also a truth arrived at by using the rules of inference of that system. All three major assumptional components of this relative truth -- axioms, individuals/primitives, and rules of inference -- are traditionally held to be fixed for all time. I.e., they are assumed to form a Parmenidean/early-Platonic 'ideo-onto-stasis', never an 'ideo-onto-dynamasis'.

Dialectical Logic encompasses this deductive methodology of formal logical, but also goes beyond it, picking up where it leaves off. Dialectical Logic is about 'meta-evolving' a progressive succession of increasingly-adequate axiom-sets and their corresponding mathematical 'ideo-systems'. The axioms, primitives, and rules of inference are not deduced. They are unproven assertions. In the beginning, they were held to be "self-evident", hence non-arbitrary. But, subsequently, this position has become untenable, at the very least since the discovery of the equi-consistency of Non-Euclidean Geometries vis-à-vis Euclidean Geometry, i.e., with the discoveries of formally independent/undecidable and mutually contrary competing candidate axioms, starting with the Euclidean Parallels Postulate, and its Non-Euclidean alternatives, and even more so with the systematization of the theory of independent/ undecidable propositions by Gödel and Cohen. Given that we have options -- given a spectrum of two or more candidate, mutually contrary, alternative axioms -- how does one "elect" the "best" candidate? It is this question -- one that Mathematical Logic leaves unaddressed -- that Dialectical Logic addresses. Even in the ancient days when "self-evidence" seemed like a tenable argument for justifying prevailing [e.g., the Euclidean] postulations, the Platonic dialectical tradition was not taken in:
"These conclusions point in particular towards that aspect of the dialectic which lay at the forefront of Plato's concern. He insisted upon two fundamental ideas: (1) that a starting point for rational argumentation cannot be merely assumed or postulated, but must itself be justified, and (2) that the modus operandi of a justification can be dialectical. Plato accordingly mooted the prospect of rising above a reliance on unreasoned first principles. He introduced a special device he called "dialectic" to overcome this dependence upon unquestioned axioms. It is worthwhile to see how he puts [this point - F.E.D.] in his own terms: There remain geometry and those other allied studies which, as we have said, do in some measure apprehend reality; but we observe that they cannot yield anything clearer than a dream-like vision of the real so long as they leave the assumptions they employ unquestioned and can give no account of them. If your premiss is something you do not really know and your conclusion and the intermediate steps are a tissue of things you do not really know, your reasoning may be consistent with itself, but how can it ever amount to knowledge? .. . So . . . the method of dialectic is the only one which takes this course, doing away with assumptions. . . Dialectic will stand as the coping-stone of the whole structure; there is no other study that deserves to be put above it. Plato's writings do not detail in explicit terms the exact nature of this crucial enterprise of dialectic. Presumably we are to gain our insight into its nature not so much by way of explanation as by way of example - the example of Plato's own practice in the dialogues. And what emerges is pretty much the sort of dialectical process envisaged in our present discussion: the comparative "cost-benefit" analysis of pros and cons of the proposed starting point in the face of competing alternatives". [Nicholas Rescher, Dialectics: A Controversy-Oriented Approach to the Theory of Knowledge, State University of New York Press [Albany, New York: 1977], pp. 48-49, bold italic emphasis added by F.E.D.].

In addressing the question of the systematic optimization of axiomatic election, neglected by Mathematical Logic, Dialectical Logic also demurs from the tacit 'Parmenidean Postulate' of eternal 'ideo-onto-stasis'. It admits the actuality of, and thereby discovers the unavoidable, inescapable necessity of, 'ideo-onto-dynamasis': that axioms-systems change with time, and also that they change due to psycho-historical causes, not just due to "eternal"-ideal/formal causes, and not just due to initial oversight and error in the "perception" of "absolute, eternal, immaterial truth". It is not enough to 'evolve' a single axiom-system via deductive derivation of new theorems, until, if ever, that axiom-system's axiom-set is exhausted of its implications. This 'evolution' will inevitably encounter, e.g., syntactically-correct but semantically transcendent unsolvable diophantine equations within undecidable propositions concerning the unsolvability of these equations [per Gödel's First Incompleteness theorem]. That is, this 'evolution' will encounter 'conceptual singularities', that can only be resolved favorably by, not only a change of axioms, but by a cumulative, progressive, «aufheben»'evolute' expansion of the axioms-set, as well as of its implied 'ideo-ontology'; that can only be resolved by adjunction of additional axioms which encompass the existential assertion and behavioral definition of, e.g., new kinds of numbers [which render the "unsolvable" equations solvable in all higher systems $\mathbf{I}$, and the undecidable propositions about their unsolvability decidable/provable, for that predecessor system, in all subsequent, higher systems, and] which higher kinds of numbers are 'modelable' as sets of higher "logical type", sets 'meta-fractally', 'evolutely', «aufheben»-containing, and thereby escalating beyond, the 'meta-fractal', 'quanto-qualitative' "scale"' of, the "[logical] types" of sets previously existentially asserted and described, by the previous axioms-set [per Gödel's Second Incompleteness theorem]. This 'exo-empirically', externally observed and 'intro[-in-spectively]-empirically', internally observed psychohistorical phenomenology is one which we term 'The Gödelian Ideo-Metadynamic', or 'Gödelian Dialectic '. Gödel described this "formal" 'meta-system'this "sequence", succession, or, in truth, «aufhebens-'progression-cumulum' of axiomatic [rules-]systems - as follows:
"Let $Z$ be the formal system that we obtain by supplementing the Peano axioms with the schema of definition by recursion (on one variable) and the logical rules of the restricted functional calculus. Hence $Z$ is to contain no variables other than variables for individuals (that is, natural mombers), and the principle of mathematical induction must therefore be formulated as a rule of inference [thereby maintaining $Z$ as a first-order formal theory - F.E.D.]. If we imagine that the system $Z$ is successively enlarged by the introduction of variables for classes of mombers, classes of classes of numbers, and so forth, together with the corresponding comprehension axioms, we obtain a sequence ... of formal systems that satisfy the assumptions mentioned above, and it turns out that the consistency... of any of these systems is prooable in all subsequent systems. Also, the undecidable propositions constructed for the proof of Theorem 1 [the "First Incompleteness Theorem" - F.E.D.] become decidable by the adjunction of higher types [higher logical types of sets, i.c., sets of the sets of lower logical type -- F.E.D.] and the corresponding axioms; however, in the higher systems we can construct other undecidable propositions by the same procedure...To be sure, all the propositions thus constructed are expressible in $\mathbf{Z}$ (hence are mumber-theoretic propositions); they are, however, not decidable in $Z$, but only in higher systems, for example, in that of analysis [i.e., in that of $\mathbf{R}$-- F.E.D.] [Kurt Gödel, On Completeness and Consistency (1931a), in J. van Heijenoort, ed., Frege and Gödel: Two Fundamental Texts in Mathematical Logic, Harvard U. Press [Cambridge: 1970], p. 108, bold italic emphasis added by F.E.D.I.

Summary: 'Psycho-archaeological' observors of the historical record and of the historical phenomenology of the axiomatic praxis will observe not a single axioms-system for each initial universe of discourse that has been so formalized, but a sequence of axioms-systems. That is, they will observe, for each sufficiently matured, developed such formalized subject-matter, an axioms-metasystem'. They will observe, accompanying the 'dynamic' of deductive, 'theorem-etical evolution', an ongoing 'meta-dynamic' -- the episodic, punctuated irruption of 'axiomatic revolutions', of axiomatic-systems 'meta-evolution'. This is the fundamental 'psycho-empirical', 'psycho-archaeological', and "'psycho-historical"' finding of Dialectics as 'Meta-Axiomatics'.

## Historical Dialectics, Systematic Dialectics, 'Meta-Systematic Dialectics', and Dialectical Ideography.

Consider the context of Hegel's works. Therein, what we mean by 'system-atic dialectics' is exemplified in the ideational-necessitarian exposition of the progression or 'progressive evolution' of categories and categorial transitions within the major 'systems'/ divisions of the Hegelian 'Meta-System' «Logiks, «Natur», and «Geist» [Spirit]. What we mean by 'meta-system-atic dialectic' is exemplified, if imperfectly, by the 'meta-evolutionary' transitions "'without"', and beyond/between, each of those divisions, from «Logik» to «Natur», and from «Natur» to "Geist» [plus, putatively, circularly from «Geist» back to "Logik»].

Consider also Marx's theory of human-social evolution-and-'meta-evolution'/'social-relations revolution'. Therein, what we mean by 'systematic dialectics' is exemplified by the "'meta-anatomical/synchronic"' categorial progressions and transitions within the System of Kapital, e.g., from Commodity to Money to «Kapital» as forms of Value, with each such form -- if we grasp human society itself as the collective subject/causal agent and also as the primary object/material of its own activity [hence an activity which can be characterized as a self-production/self-development] -- grasped as a human-social relation of [humansocietal self-[re-]]production. These categorial progressions provide a conceptual 'meta-anatomy' or categorial 'meta-physiology' of the capital-centered society; of a synchronic "slice/cross-section"" of the system of self-reproduction of «Kapital(s)». That is, they account for, primarily, the capacity for [an historically transitory] 'self-causation' or self-reproduction of the 'System of «Kapital(s)»', of the «Kapital(s)» social formation. These progressions should also locate the 'self-duality', or 'intra-duality' of the «Kapitals»system, including the 'essence-ial', internal, inherent, immanent grounds for its finality/entelechy as its ultimate, self-destined 'self-dis-causation' or 'self-dis-reproduction'/potential self-transcendence. But 'systematic dialectics' involves primarily a 'synchronic', "'slice-in/of-time ${ }^{\text {"I' }}$ exposition of a [transitorily] self-reproducing system.

What we mean by 'metasystem-atic dialectics' is exemplified by the "'psycho-historical", 'meta-evolutionary', and 'revolutionary'/'catastrophic', or singularity-like and human-social-ontology-expansive or "qualitative"/'meta-finite' diachronic transitions to outside/without/beyond/between social systems; from the systems of human-social relations of [humum-societal self-re-] production] of primitive-communal/kinship-based societies to those of servile [slavery/serfdom-based] societies, and from those of servile societies to those of "modern", capital-/wage-labor/sold-labor/alienated labor-centered societies, all as driven by the internal, endogenous self-growth of the "social forces of production"; of the level/magnitude of the human-social force of expanding human-societal self-re-production, that is, the self-induced growth of human-societal self-productivity; of the rate of [growth or of self-expansion of the] self-[re-]production of human society, including of 'memetic matter'; of 'human-socio-matter', of 'human-socio-mass', or of 'humansocial negentropy'. It is so exemplified when this 'diachronic meta-system' is presented as a conceptual, systems-progression as categorial-progression pedagogical exposition.

In summary, a "'system-atic dialectical"' theory dialectically 'exposits' a categorial progression comprehending primarily the self-reproduction of a system/sub-totality. A 'meta-system-atic dialectical' theory dialectically 'exposits' a 'meta-system', e.g., an historical or temporal progression of systems, as a categorial progression, both locating and scrutinizing the moment of non-self-reproduction or of self-non-reproduction of each system in that sequence; the moment where each such "self-organizing system" intrinsically passes over into a 'self-dis-organizing system' and into a qualitatively -- i.e., ontologically -- self-expanding 'self-re-organizing system'. Whereas "'systematic dialectics" emphasizes the momentaneous/synchronic "Being" of a given system, 'meta-systematic dialectics' emphasizes the moment of any given system's diachronic "Becoming", including of its eventual, 'self-un-Becoming', locating its moment of immanent self-revolution; of progressive, 'self-supercessive' self-dissolution / self-transformation; of self-transcendence; i.e., of "meta-system [self-]transition" [cf. Turchin]; of 'self-bifurcation', of '[self]-conversion singularity'; of 'meta-finite', 'meta-fractal', 'metadynamical', 'meta-monadic', 'meta-ontic', and 'meta-evolutionary' «autokinesis», or self-locomotion. Such a theory also theoretically anticipates/predicts and scrutinizes the transition-process itself: its transient, transitory phenomena, and their "laws". Each such "'qualitative leap"', or 'ontological leap', involves 'categorial change', that is, 'ontological change'; 'ontology-gain'. By 'ontological change', 'ontological growth', or 'ontology-change' we mean 'meta-finite', 'evolute'-helical, or «aufheben»-cumulative ontology net-expansion for successor systems vis-à-vis their predecessor systems within the total diachronic, or 'synchronicodiachronic', 'meta-system', or categorial-progression model of 'systems-progression'. Interior to the domain of 'human-social meta-systems', this ontology net expansion includes net expansion of the corresponding and accompanying ""psycho-historical" "universe of discourse". That is, it includes ideo-ontic expansion as well as expansion of the metrical repertoire necessary for adequate human [self-] descriptions of humanity's thusexpanded societal ontology and its phenomenology. Dialectical Ideography endeavors to discover/engineer a 'meta-system' of arithmetics fit to encode models of "historical dialectics", "systematic dialectics", and 'meta-systematic dialectics' alike, as illustrated herein.

## The Plot-'Line' Of This Story.

- The Plot Thickens. The $\mathbf{N}$ arithmetic is related to, though not identical to, the Platonic / Diophantine model/theory of the «Arithmoi Monadikoi», the arithmetic of "Dianoia», of «Verstand", of "The Understanding". The $\underline{\underline{0}}$ arithmetic is related to, though not identical to, the Platonic «Arithmoi Eidetikoi», the arithmetic of the «Eide», of "t $\delta \varepsilon \varepsilon$ "», of the ideas/concepts/categories of Reason; the arithmetic of «Vernunfts, the arithmetic of "'Higher Reasoning"', the arithmetic of «Dialektiké». In this progression, $\underline{\underline{\mathbf{N}}}$ is the implicitly dialectical «arché» [origin; controlling source; initial condition] of all rules-systems of number, and $\underline{\underline{\mathrm{Q}}}$ is the "archers of the explicit dialectical ideographies; of the explicitly dialectical arithmetics presented herein.
- The Plot Thickens Again. All of the systems of arithmetic in this 'meta-system' or sequence of systems of arithmetic, starting with $\underline{\underline{\mathbf{Q}}}$, are "Non-Standard" Models of the first-order Peano axioms for the Standard Natural Numbers, N. All are 'Peanic' in that their 'unit-qualifier' bases all conform to the first-order Peano axioms. Thus it appears that the 'dialectical ideography' implementation of the «Arithmoi Monadikoi» corresponds to the "Standard", outward, "'explicate"' face, and that the 'dialectical ideography' implementation of the "Arithmoi Eidetikoi» corresponds to the inward, occult or hidden, "implicate"', "Non-Standard" face, of the Janus-faced first-order rules-system of the "Natural" Numbers, which rules-system we denote by the symbol $\mathbf{\underline { \mathbf { N } }}$.
- The Plot Thickens Even More. Each of these "Non-Standard" "Natural" arithmetics also is or contains a contra-Boolean Arithmetic, and, hence, a contra-Boolean Algebra, i.e., each contains 'contra-Boolean' units
 models standard formal logic, this suggests that these "Non-Standard" Arithmetics might also model new, non-standard logics, qualitatively distinct from, and alternative to, Standard/Boolean/ Aristotelian/Formal logic. Should we not then expect that such alternative logics would turn out to be, or would include, precisely, dialectical $\operatorname{logic(s)}$ ?
- The Plot Grows Thicker Still. The Zermelo-Fraenkel Axioms for Set Theory are widely regarded by mathematicians as the foundation of "Standard" Mathematics. Work by Kurt Gödel and Paul Cohen circa mid 20th-century C.E. has established that the Generalized Continuum Hypothesis, a generalization of Georg Cantor's Continuum Hypothesis regarding transfinite cardinal quantities, is "independent" of/undecidable from the Axioms of Zermelo-Fraenkel Set Theory [just as Euclid's Parallels Postulate is "independent" of the other postulates of Euclidean Geometry], as is also the Zermelo-Fraenkel Axiom Of Choice, from the remaining axioms:

[^2]Both the Generalized Continuum Hypothesis and the Axiom Of Choice share a conceptual underpinning which can be characterized as 'reductionist', 'point-atomistic', or 'dimensionality-denying'. Joining the Set Theory Axioms with each consistent alternative to the 'Cantor Axiom' of the Generalized Continuum Hypothesis [not to mention with each consistent alternative to the Axiom Of Choice] yields yet a new, "Non-Standard", 'Non-Cantorian' Set Theory and Mathematics. Each such neo-mathematics is equally selfconsistent, yet qualitatively different from, the existing mathematics, just as the several Non-Euclidean Geometries differ qualitatively but self-consistently from Euclidean Geometry. Hypothesis: At least one of these Alternative Mathematics contains wherewithal for 'The Nonlinearity Breakthrough'. In Part III., we explore alternative foundations based upon a 'non-reductionist', 'non-point-atomistic' candidate replacement for the Generalized Continuum Hypothesis, which we term the 'Generalized Cumulum Hypothesis', for reasons which will emerge.

## - Pure-Quantitative Arithmetics, 'Pure-Qualitative Arithmetics', and 'Quanto-Qualitative' Arithmetics.

These "Non-Standard" "Natural" Arithmetics begin, with $\underline{\underline{\mathbf{Q}}}$, by bringing back the missing, little noticed, and neglected "half" of arithmetical and mathematical ideography, the qualitative "half", the "pure" 'qualifier' as opposed to the "pure" 'quantifier' "half". From there, they continue to burgeon, bringing about a redintegration of quantitative and qualitative ideographic language, of quantitative and qualitative computation, in the form of an «organon» of ontologically and metrically qualified quantifiers; of metrical and ontological qualifiers quantified, which we call 'Quanto-Qualitative Analysis' or 'Qualo-Quantitative Analysis', and, for reasons which will emerge, also 'A Posteriori Analysis', 'Synthetic Analysis', 'Realistic Analysis', or 'Re-Qualified Metafinite Analysis'. What these epithets point to is a species of mathematical analysis that respects the inductive, "a posteriori", "synthetic", and 'realistic' 'Principle Of Metafinity', and its implications; the principle which holds that 'infinity is non-empirical', or that 'only finite metrical values ever manifest'. It supports the 'semantification' of zero division and, therefore, of the "singularities" of especially the nonlinear dynamical systems' total [and partial] integrodifferential equations-models, and the rectification of the infinite error - of the 'infinity residuals' -- that arise from the attempted interpretations of such singularities in Standard, 'Unrealistic' and 'Unqualified' Analysis.

Our 'Meta-Systematic' Dialectic of $\mathbf{N}$ and the Historical Dialectic of the «Arché» of Written language. Denise Schmandt-Besserat, in the late 1970s, offered a theory of the 'proto-poiesis' and early 'meta-evolutions' of a prehistoric proto-writing technology, a theory that that combines conceptual coherence, subtle 'psycho-historical' insight, and rather massive empirical, 'psycho-artefactual' archaeological evidence. She sums up her theory in the following terms:
"In 1964, Pierre Amiet, conservateur en chef du department des Antiqités Orientales au Musée du Louvre, identified in the archives of Susa an archaic system of recording dating from the second half of the IVth Mill, B.C., and slightly predating the earliest tablets. The system consists of small clay tokens of geometric shapes mostly in the form of spheres, discs, cones, and tetrahedrons, found enclosed in [opaque - F.E.D.] clay envelopes in the shape of hollow clay balls (called bullae ...). The surface of the bullae usually bear seal impressions and sometimes marks indicating the number of tokens enclosed. Pierre Amiet interprets such bullae as representing a transaction, the tokens inside indicating the kinds of goods exchanged by their shapes and the quantity by their size and number. In the course of my recent study on the earliest uses of clay in the Middle East, I found that geometric tokens identical to those identified by Pierre Amiet in the bullae are found loose in most Middle Eastern sites of the IXth to the IId Mill, B.C. I have been intrigued by the striking similarity of the shapes of the tokens with some of the abstract signs from the Uruk tablets. In the present paper I will discuss four stages in the evolution of an early recording system based on the tokens which may have led to writing.

1. Ca. 8500 B.C. A three-dimensional system of recording is based on tokens.
2. Ca. 3250 B.C. Clay containers or bullae hold the tokens of a particular transaction.
3. Ca. 3250 B.C. Signs are impressed on the surfaces of the bullae [e.g., on their wet-clay, via tokens used as stamps, before firing - F.E.D.].
4. Ca. 3250-3100 B.C. Clay tablets [ $=$ 'meta-«bullae»', solid, no-longer-hollowed-oul clay slabs - F.E.D.] bear impressed and incised signs." [Denise Schmandt-Besserat, On The Origins of Writing, in Early Technologies, voL. 3, Undena Publications [Malibu, CA.: 1979], p. 42].

Schmandt-Besserat's 'psycho-archaeologically' reconstructed evolution and 'meta-evolution' of this putative recording system instantiates a 'psycho-historical dialectic' of 'quanto-qualification' that is deeply related to, though strikingly different from, the 'meta-systematic dialectic' of $\mathbf{N}$ exposited in these Briefings, and, in greater detail, in Part III., The Arithmetic of Meta-Evolution. The following passage from her book provides a brief immersion in the 'psycho-historical milieu' of the "cradle of civilization" as reconstructed through her discoveries:
"The content of these envelopes demonstrates that the Uruk VI accountants indicated quantities (how many) in a way radically different from ours. They did not show, as we do, " 5 " by a numeral. Instead, " 5 jars of oil" were translated by five tokens, each standing for " 1 jar of oil", as illustrated here.


Literally, the set of tokens meant "jar of oil, jar of oil, jar of oil, jar of oil, jar of oil". The token system had no symbols for abstract numbers such as " 5 " . . When tokens were replaced by their images impressed on the surface of an envelope or tablet, the resulting signs were already "more abstract" than the previous clay counters. Compared to three-dimensional clay counters, the two-dimensional markings represented commodities in greater abstraction since they could no longer be grasped in the hand and manipulated [mental "grasping" superseding manual "grasping" -- F.E.D.]. . The alignment of the markings on the face of a tablet also contributed to decontextualizing the data. Semantically, however, the impressed markings were identical to tokens: Each ideogram still fused together the concepts of nature/quantity (i.e., measure of oil and the number 1). . The accountants of Uruk IVa about 3100 B. C. invented the first numerals - signs encoding the concept of oneness, twoness, threeness, abstracted from any particular entity. This was not a small feat, since numerals are deemed to express some of the most abstract thoughts our minds are able to conceive. After all, "two" does not exist in [external, extra-mental - F.E.D.] nature, but only groups of two concrete items, such as two fingers, two people, two sheep, two fruits, two leaves, or even sets of heterogeneous items such as one fruit + one leaf, and so on. "Two" is the abstraction of the quality of twoness shared by such sets. The accountants of Uruk IVa can be credited with creating numerals and by doing so revolutionizing accounting and data manipulation. In fact, the Uruk IVa accountants devised two types of signs: numerals (symbols encoding abstract numbers) and pictographs (expressing commodities). Each type of sign was traced by a different technique pictographs were incised, whereas numerals were impressed, clearly standing out from the text. For example, a tablet from Uruk features two accounts of " 5 sheep" shown by the pictograph for "sheep" (a circle with a cross) and " 5 " appearing as five impressed wedges...The notion of number was finally dissociated from that of commodity. The numerals of the Uruk IVa tablets constitute the first evidence for the use of abstract counting and the creation of modern arithmetic. . The first numerals were not symbols specifically created for representing abstract numbers. Instead, they were the impressed signs formerly indicating units of goods [i.e., they began as constituents of the same "archén ontological category or 'ideo-onto' as the later, original "ideographs" or "pictographs": primitive undifferentiated unities of 'ontological qualifier', 'metrical qualifier', and of 'unit-quantifier', plus of whatever else is implicit in such unitary symbols - F.E.D.], such as measures of grain, endowed with a new numerical value. The wedge, which originally meant a small quantity of grain [a metrical unit of grain quantity then known as the "ban" - F.E.D.], now stood for 1 ; the circle, which represented a larger quantity of grain [a metrical unit of grain quantity then known as the "bariga" - F.E.D.], was 10; the large wedge, punched wedge, and large circle were greater numbers. ..'
"In fact, the impressed signs that came to represent numerals never lost their primary meaning. Instead, according to the context, they had either an abstract or a concrete value. For example, the wedge preceding a pictograph was read " 1 " ..., but alone it stood for a measure of grain. . This proved confusing to Sumerian accountants, who eventually eliminated the ambiguity by introducing a pictograph in the shape of an ear of grain (ATU 111/ ZATU 511). Abstract numbers, therefore, derived from the plain cones, spheres (cylinders and lenticular disks?) that were the most ancient tokens. The reason why these particular symbols became the first numerals can only be hypothesized. David E. Smith has remarked that in a number of societies the words for expressing numbers derived from concrete numerations of particularly frequent use. He cites languages that expressed "one, two, three" by number words that meant literally "one grain, two grains, three grains" or "one stone, two stones, three stones" or, like the Niues of the southern Pacific, "one fruit, two fruit, three fruit". It may be argued, therefore, that the first Sumerian abstract numbers derived from the grain and animal numerations because they were the most commonly used in Mesopotamia.

Grain, in particular, was not only the main staple but also the most usual means of exchange [key 'psycho-historical' insight: the primucy of the praxis of the emerging 'proto-money' or money-commodity in the splitting-off/ separation / bifurcation of 'pure quantifiers', and in the increasing polarization of human mentality as a whole toward the quantitative side throughout the epochs of increasing intra-social alienation; of the increasing re-foundation of social metabolism and social reproduction on "universal alienation" or umiocrsal selling - F.E.D.]. Furthermore, grain metrology constituted a unique gamut of signs of increasing magnitude that could be easily converted to signify mnits of abstract counting such as $1,6(?), 10,60,180$. The invention of numerals made a breach in but did not put an end to the age-old principle of one-to-one correspondence. It was a major break with the past that pictographs encoding commodities were no longer repeated as many times as the number of units involved. One-to-one correspondence continued governing the use of numerals, however. "Nine" was represented by nine wedges, fifty by five circles, and so on. For instance, the tablet of Godin Tepe Gid.73.295. . .bearing the notation " 33 jars of oil" displayed a single pictograph standing for "jar of oil" and expressed " 33 " by three impressed circles $(10+10+10)$ and three wedges $(1+1+1)$. This archaism, in turn, was perpetuated for centuries in the Sumero-Babylonian arithmetical system. In fact, one-to-one correspondence persisted in all numbering systems, including those of Greece and Rome, until the invention of the so-called arabic numerals in India about 700 B. C."
[D. Schmandt-Besserat, Before Writing, vol. I, From Connting to Cineiform, U. of TX. Press [Austin, TX: 1992], pp. 190-193, emphasis added by F.E.D.].
Schmandt-Besserat abstracts the multi-century 'meta-evolution' of this 'meta-system' of systems of proto-writing, in most concentrated fashion, in the following passage: "It was not by chance that the invention of pictography and phonetic writing coincided with that of numerals; instead, both were the result of abstract counting. The abstraction of the concept of quantity (how many) from that of quality, which merged inextricably in the token prototypes, made possible the beginning of writing. Once dissociated from any notion of mamber, the pictographs could evolve in their own separate way. The symbols formerly used for keeping accounts of goods could expand to communicate any subjects of human endeavor. As a result, items such as "the head of a man" or "mouth" that never had a token were expressed by a picture. True pictography, that is to say, concepts represented by their images, thus was the outcome of abstract counting. After pictography, writing crossed several new thresholds about 3000 B.C. in the Uruk III period. The abstraction of [continuous - F.E.D.] quantity (how much) [contimuous metrical units 'quanto-qualification'; counting of convention-stipulated units of continuous measure - F.E.D.] followed that of [discrete - F.E.D.] number (how many) ['candinometry'; counting of 'spatially' discrete ontological «monails» or units - F.E.D.]. Note, for example, that in Uruk V1 it took one token to indicate one jar of oil, or presumably, "one sila of oil" In Uruk IVa, the same was written with two signs, namely, "1" and the pictograph "sila of oil" [or, as we would hold, '
for ' 1 sila of or, more generally, for ' 1 standard metrical unit of whatever follows', and
"for "oil" - F.E.D.]. In Uruk III, however, each notion " 1 ","sila", "oil" was expressed separately, requiring a sequence of three signs.

Uruk VI


Uruk IVa

Finally, symbols could function phonetically, representing not objects but, in particular cases, sounds. The incentive to resort to phonetics was seemingly prompted by new administrative requirements for recording the names of donors/recipients of goods on the tablets. Individuals' names were transcribed by symbols meant to be read phonetically as a rebus. . This was the point of departure for a syllabary - symbols standing not for commodities or concepts but simply for the sounds they brought to mind. Pictography led to a syllabary, which was the true takeoff of writing. In the ancient Near East writing emerged from a conorting device. It is the main outcome of the invention of abstract counting. Tallying in one-to-one correspondence was superseded about 8000 B.C. by tokens of many shapes suited for concrete counting. Finally, writing emerged when abstract counting dissociated the concept of numbers from that of the commodity accounted. Writing resulted from a new way of handling data with an unprecedented [degree of] abstraction. Tallying in one-to-one correspondence coincided with a way of life and an economy based on hanting and gathering. Concrete connting with tokens was related to the rise of agriculture and an economy of redistribution. Complex tokens, envelopes, and impressed tablets were the consequences of urban development, the rise of industry, and the formation of the state. Pictographic and phonetic writing, about 3100 to 3000 B.C., however, seems independent of any socio-economic event. It was the outcome of a new threshold in cognitive development: abstract counting." [D. Schmandt-Besserat, ibid., pp. 194, emphasis added by F.E.D.].

We have illustrated this 'historical-dialectical meta-evolution' of the ancient Mesopotamian proto-writing language-praxis in the 'gram-ics' or 'graph-ics' which follow. Those graphics treat variously underscored versions of the three symbols cited above as 'archaeograms' and '"archén-grams' [indeed, as cognitive " $p s y$-glyphs"' [cf. Muses]] representative generically of the entirety of the three emergent symbol-categories, conceptual-categories, and symbolization sub-rules-systems noted in the quoted passage, rather than as symbols with the specific denotations described therein.

The singly-underscored symbols $\underline{\nabla}$, $\simeq$, and $\underline{\text {, }}$, denote and, by a kind of ideogramic "synecdoche", 'epitomize' for us, respectively, the entire 'spaces' or sets of ontological qualifier symbols, metrical qualifier symbols, and abstract quantifier symbols as a whole in the Uruk III epoch ancient Mesopotamian proto-writing system, whereas their 'underscoreless' counterparts, $\nabla$, and $\boldsymbol{\nabla}$. denote, respectively, oil, the sila unit of fluid volume measurement, and the abstract numeral 1, as of that epoch.

Likewise, in the end, ' $\underline{\underline{V}}$ ' denotes the rules-system of the category of pure quantifier symbols in general, ' $\bar{V}$ ' the rules-system of the category of kind of entity or pure 'ontological qualifier' symbols in general, and
 the rules-system of the category of pure 'metrical qualifier' symbols or units-of-measurement ["dimensional"] symbols in general. An anomaly in this graphic representation is that the symbol ' $\underline{\underline{\underline{V}}}$ ' stands, at first, for a rules-system which presupposes a primitive undifferentiated unity of quantifier and 'metrical qualifier' symbols in general, and later for one of pure quantifier symbols in general, after the splitting off of ' to denote a rules-system of purely metrical qualifier symbols in general. It would be notationally preferable to have three separate symbols, one for the primitive undifferentiated unity of quantifiers and metrical qualifiers, a different symbol for the post-bifurcation category of pure quantifier symbols, and yet another symbol for the post-bifurcation category of purely 'metrical qualifier' symbols, but such would be untrue to the actual historical, empirical 'psycho-archaeological sequence' we are modeling. Note how the historical dialectic of this proto-writing praxis differs markedly from the 'meta-systematic dialectic' of the Natural Numbers, $\underline{\mathbf{N}}$ "standard" and 'dialectical-nonstandard', as recounted in these Briefings. The 'meta-systematic dialectic of $\mathbf{\underline { \mathbf { N } }}$ takes as its «arché»/starting-point the first-order Peano-Postulates rules-system of "Natural" arithmetic, initially apprehended only as an arithmetic of pure quantifiers. The historical dialectic of Mesopotamian proto-writing takes, as its «arché", representations which are not yet even "two-dimensional" marks, cut into wet clay, but rather three-dimensional iconic-tactile symbols; fired-clay micro-effigies [e.g., of a jar of oil], herein denoted collectively by ' Nevertheless, these two distinct dialectical progressions, both involving concepts of 'qualifier' symbols versus 'quantifier' symbols, may prove mutually illuminating. Despite the marked differences between these two dialectical progressions, we may interpret the $\underline{\underline{\mathbf{Q}}}$ arithmetic so as to model this historical dialectic, and to calculate its 'ideo-onto-dynamics', via the following assignments:
$\hat{\mathbf{g}}_{1} \leftrightarrow \hat{\mathbf{N}}=$ The «arché» of this 'meta-system'; the pre-/proto-scribal rules-system for 3-dimensional tactile-iconic representation of primitive undifferentiated unities of 'onto-metrical qualifiers' and quantifiers, emphasizing the 'ontological qualifier' aspect of that unity

$\hat{\underline{G}}_{2} \leftrightarrow \quad \underline{\underline{\nabla}}=$ The scribal rules-system for, initially, the proto-written representation of generic metrical unit qualifiers, for the proto-written, " 2 -dimensional ", wet-clay impression-representation of primitive undifferentiated unit[ie]s of metrical dimensions or 'metrical unit-qualifiers', and "dimensionless" unit quantifiers

We then model, by the following 'qualitative computations' or 'ideo-ontological calculations', a sequence of practiced rules-systems as the historical 'meta-evolutions' of this ancient, proto-literacy symbolic 'meta-system'. In them, self-juxtaposition of symbols, e.g., 量 ], 'ontological self-multiplic[it]ation' or 'homogeneous ontological multiplication', may be interpreted as self-operation, self-reflexion, self-confrontation, 'self-«aufheben» self-negation', immanent critique or self-critique, self-differentiation, and self-hybridization of the 'self-besided' symbol, in this example. The juxtaposition or 'heterogeneous ontological multiplication' of already-differentiated signs may be interpreted as signifying their [partial] synthesis, hybridization, mutual unification or redintegration -


The calculated symbols above, interpreted as 'ideo-onto-computationally-derived' rules-systems, are defined as follows, via a method that we call 'qualitative factorization', 'ontological factorization' or 'intensional factorization' --

= Emergence of scribal sub-concepts/'sub-rules sub-systems' for using undifferentiated 'quanto-metrical quanto-qualifier' symbols together with 'ontological qualifier' symbols, i.e., for the purpose of the 'metrico-quantifying' of 'ontological qualifiers'; a system of rules relating 'metrico-qualo-quantifiers' like $\square$ and/with 'kind-qualifiers' like 1 , or $\nabla$.

 symbols in conjunction with now/newly "pure, abstract" quantifier symbols, i.e., for the quantifying of 'metrical qualifiers';
$\hat{\mathrm{g}}_{7} \leftrightarrow \underline{\underline{\hat{G}}}^{\text {A. Emergence of 'gram-matical' or 'gram-mathical' concepts/rules for notating now/newly fully-explicit/differentiated }}$
'metrical qualifier' symbols, "pure, abstract" quantifier symbols, and 'ontological qualifier' symbols, i.e., for the quantification of 'ontologically qualified metrical qualifiers', or for the 'ontological and metrical qualification' of quantifiers;


Denoting by $\underline{\mathbf{L}}_{\boldsymbol{z}}$ this sequence of proto-literate/proto-writing Linguistic rules-systems, our $\left\{\underline{\mathbf{L}}_{\mathrm{t}} \leftrightarrow \underline{\underline{Q}}_{\mathrm{v}}\right\}$ progression can, to its third epoch, be summarized as follows, with $\underline{L}_{0}=$ (:

$$
\begin{aligned}
& \underline{L}_{0}=\left[\underline{L}_{0}\right]^{0^{0}}=[\underline{\underline{\hat{t}}}]^{]^{0}}=[\underline{\underline{\hat{Q}}}]^{1}=[\underline{\underline{\hat{Q}}}] \quad \leftrightarrow \mathbb{E} \hat{\mathrm{a}}_{1} \mathbb{I} \quad=\underline{\mathrm{Q}}_{0} ;
\end{aligned}
$$

The detailed 'psycho-historical calculation', or 'computational psycho-archaeological reconstruction', of this linguistic 'meta-evolutionary sequence', per this model of it, and per the «aufheben» evolute product "multiplication" rule [and its non-distributive meristemal principle for heterogeneous multi-term products in $\underline{\underline{\mathbf{Q}}: ~ t h a t ~ o n l y ~ t h e ~ m o s t ~}$ $\rightarrow$-advanced operator term need operate upon each 'legacy' operand term], goes like this [epoch $\mathbf{0}$ to epoch 1] --

[epoch 1 to epoch 2] --


 [epoch $\mathbf{2}$ to epoch 3] --


Dialectical Ideograpiny
[in the minimally-interpreted $\underline{\underline{\mathrm{Q}}}$ arithmetic]:





In the above 'qualitative calculations' or 'ideo-ontological calculations', we are employing interpreted $\underline{\underline{Q}}$ as what we term an 'heuristic intensional calculus'. That is, we apply the arithmetical rules-system, the 'algorithmics' of $\underline{\mathbf{Q}}$, to the "intensions" or meanings of the interpreting symbols, whose 'extensions' are not explicitly specified.

An historical dialectic like that of the "arché» is, as we shall see in the sequel, best exposited "pedagogically" using term-by-term, step-by-step sequences-of-onto-symbols, which we notate as:


Hypothesis. The at least seemingly most questionable sub-transition in this model may be the second one, i.e., the assertion that $\boldsymbol{r} \underline{\underline{\underline{\nabla}}}=\underline{\underline{\nabla}}=\underline{\underline{\nabla}}^{2}=\underline{\underline{\nabla}} \underline{\underline{\underline{D}}}$; that the praxis of the sub-rules-system for 'pure metrical qualifiers', arises directly from the 'self-reflexion' and 'self-refluxion' of the praxis of the sub-rulessystem of 'metrico-qualo-quantifiers', $\underline{\underline{\nabla}}$. The case for this model is aided by noting that $\left.\underline{\underline{\nabla}}^{2}=\underline{\underline{\Delta}} \oplus \underline{\underline{\Delta}} \underline{\underline{\Delta}}\right]$,
 pervasive throughout all of its later, higher-degree 'self-iterations'. It aids further to recall that $\boldsymbol{\nabla}$ originally denoted not an abstract pure quantifier, but a 'primitive undifferentiated unity' of 'unit-quantifier', 'metrical qualifier', and 'ontological qualifier', i.e., "one ban of grain". Moreover, in this specific instance or application, when the ultimate, incised 'ontological qualifier', $\bar{\nabla}$, and the 'metrico-quantifier', 'metrical quanto-qualifier', or 'metrical qualo-quantifier', $\boldsymbol{V}$, bifurcated from one another, a boundary between two 'ideo-ontologically' distinct epochs in the 'meta-evolution' of this proto-written rules-system was congealed in the process, as evidenced in the 'psycho-archaeological horizon', or 'socio-sedimentary layer', denoted, in the passage quoted above, as "Uruk IVa". With this split, the former primitive undifferentiated unity of quantifier, 'metrical qualifier', and 'ontological qualifier', for example , meaning, "one sila of oil", split into 'pure-ontological qualifier' $\nabla$, meaning "oil", and $\nabla$, meaning "one sila". Thus, the symbol $\nabla$, after this conceptual bifurcation, initially denotes, at once, "univocally", "both" what we would now term the 'metrical «monad»', metrical unit, metrical unity, metrical quantum, metrical dimension, or 'metrical unit qualifier', "(one) sila", and also the abstract/generic unit quantifier, "one [anything]". That is, $\nabla$, after this first 'ideo-ontological' bifurcation, denotes a "'primitive undifferentiated unity'" of 'unit-quantifier' and 'metrical qualifier' -- metric, metrical unit, unit of measure, '"dimensional unit'", or 'dimensional «monad»' -- which it adopts 'chameleonically' from its operand, based upon the extant metrology convention, e.g., connoting "one sila" when its operand is $\nabla$, "one head" when its operand is the 'ontological qualifier' for sheep, etc., etc.

Thus denotes, when in juxtaposition with $\nabla$, the conventional "quantum" of oil, the sila, and also counts as "one" that conventionalized quantum or unit -- counts $\underline{a}$ sila -- and therein denotes a primitive undifferentiated unity of quantifier and 'metrical qualifier'. Thereby, the next ripening conceptual-semantic 'intra-duality', and the next 'self-bifurcation', will involve the splitting-off of "pure" quantifier from 'metrical qualifier', e.g., the split of $\bar{\nabla}$ into $\boldsymbol{\nabla}$ and $】 \underline{\underline{\nabla}}=\underline{\underline{\underline{\nabla}}}{ }^{2}=\underline{\underline{\nabla}} \oplus$; the «aufheben» self-negation / 'self-bifurcation' of $\underline{\underline{\nabla}}$. Thus, a "re-use economy/parsimony" of historical choices recorded in these 'psycho-artefacts' contributes some ambiguity here, because initially denotes one ban of grain, a primitive undifferentiated unity of 'ontological qualifier', 'metrical qualifier', and quantifier, then later [in most contexts] denotes a 'metrical qualo-quantifier', but then, later still, [in most contexts] denotes a pure quantifier. Similarly, ${ }^{f}$ and impressed $\nabla$ initially denote primitive undifferentiated unities of 'ontological qualifier', 'metrical qualifier', and quantifier, but, later, incised $\nabla$ denotes a pure 'ontological qualifier'. The case for this semantification of $\underline{\underline{\nabla}} \boldsymbol{\underline { V }} \oplus$ is further clarified if we consider 'psycho-archaeologically' the non-arbitrariness of the choice of $\boldsymbol{\nabla}$ from among all of the initial "'primitive undifferentiated unit[ie]s"' as the basis for the eventual impressed symbol for the "pure, dimensionless" unit quantifier: "Grain, in particular, was not only the main staple but also the most usual means of exchange" [ibid.].


[^0]:    Dialectical Ideosraply

[^1]:    Since, in these Briefings, we describe and instantiate 'arithmetical' calculations and 'algorithms' involving 'qualifier' 'numerals' of $\mathbf{3}$ distinct types, covering their interactions with one another as well as their interactions with standard numerical 'guantifiers', we use parenthetical cues, different kinds of parentheses, to connote different types of qualifier/quantifier contents and interactions, viz.:

[^2]:    "The two independence results [of Gödel and Cohen - F.E.D.] mean that in the Zermelo-Fraenkel system the axiom of choice and the continumu Inypothesis are mulecidable [in the full Godelian sense - F.E.D.]. . There are then mamy mathematics. There are numerous directions in which set theory (apart from other foundations of mathematics) can go. . As for the continuum hypothesis, here one ventures into the unknown and, whether one affirms or denies it, significant consequences are not known as yet. . Just as the work on the parallel axiom led to the parting of the ways for geometry, so Cohen's work on these two axioms about sets leads to a manifold parting of the ways for all of mathematics based especially on set theory, though it also affects other foundational approaches. It opens up several directions that mathematics can take but provides no obvious reason for preferring one over another." [Morris Kline, Mathematics: The Loss of Certainty, Oxford U. Press [NY: 1980], pp. 268-270, emboldened italics emphasis added by F.E.D.]

