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Dialectical Ideography

A Contribution to the Immanent Critique of Arithmetic

Volume I.: Prolegomena

I. A.: *Epitome*

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PROLEGOMENA: *Epitome*

Kernel

The heart of ideographic dialectics as set forth herein is a "conceptual meta-fractal". It can be expanded and explored on many different scales of exposition, as well as within many distinct contexts, points of view, or applications. A selection of applications are investigated in the main body of this treatise. Several scales of exposition are explored by sections located at its various extremities, viz. -- this opening section [micro-scale], the concluding **Condensations** section [small-scale], and the middle-most sub-section [medium scale]. Potentially larger-scales of exposition, as well as of application, yet to be completed, loom beyond this text. We hope readers who find compelling either this conception of dialectical ideography, or various of its contraries, several of which are also reconnoitered herein, will contribute to that latter scale of exposition and application. This inaugural section lays out the primary hypotheses of ***Dialectical Ideography*** in the briefest form offered herein. Meanings that the propositions arrayed below initially hold for you may mutate markedly as you read the main sections of this essay. Even so, we believe a capsule summary of the whole may prove useful to you, now, and later. Here it is. It outlines '***The Gödelian Dialectic***' – the dialectic of inherent axiomatic incompleteness; the syntactico-semantic dialectic of the solution of unsolvable equations -- *as mapped into human history*, as a project of the '''***meta-science***''' that we call '''***Cognitive Psycho-History***''':

A. Mathematics *'''meta-evolves meta-axiomatically'''*. Its '''*meta-evolution*''' cumulatively accrues new axioms, by «***aufheben***»/"conservative" "extensions", *punctuating and mediating a progression of '''psycho-historical''' cognitive crises.*
B. Each crisis involves -- either explicitly, or merely "*in effect*" -- the discovery of "unsolvable" [in]equations.
C. Such crises resolve by *expansion of the number concept*, of *number* '***ideo-ontology***', to admit new *kinds* of numbers.
D. These ***new kinds*** of numbers, ***with their new rules/axioms***, enable solution of those *previously* unsolvable equations.
E. '''***The Nonlinearity Barrier***''', the incapacity of modern mathematics to solve, in general, the sciences' nonlinear total and partial integrodifferential equations, *especially those which embody its most advanced conceptions of the "laws", or "habits", of nature*, constitutes the latest, and 333-year-protracted, *'''psycho-historical''' crisis of unsolvability.*
F. *As in the past, so presently, this crisis can be resolved by a further expansion of the* '***ideo-ontology***' *of arithmetic; of number.*
G. This requires the discernment of yet new *kinds* of [meta-]numbers, new '*qualities*' of "quantity", beyond those of the "*hyper-real*", "*Complex*", *Quaternion, Octonion, Clifford, Grassmannian, Boolean*, and *Cantorian* arithmetics, for example.
H. The higher degree terms which render the unsolved dynamical equations "nonlinear" essentially signify the "*self-reflexivity*" and '*self-refluxivity*' of the '*external*' *process-objects* that those equations describe, reflecting modes of «***autokinesis***», of *self-action, self-movement*, and *self-change*, rooted in subject/object '*intra-duality*'. The homologous process among '*internal*'/*mental process-objects*, forms the *paradoxes*, the "*insolubilia*" of formal logic and set theory.
I. Self-reflexivity, 'self-dialogue', ***self-activity***; the subject-/verb-/object-identical *'metafinite* ***meta-dynamic***' of *self-changing, self-developing, 'via-singularity-****self-bifurcating meta-systems***', is the *essence* of *dialectical process.*
J. The linearizing "*Fundamental Law of Thought*"/"*Law of Duality*" of Boole's *original* logic-algebra, $\mathbf{x^2 = x^1}$, akin to Cantor's $|\mathbf{R}^{n>2}| = |\mathbf{R}^2| = |\mathbf{R}^1| = \mathbf{c} = \aleph_1$, despite $\mathbf{R}^{n>2} \gtrless \ldots \gtrless \mathbf{R}^2 \gtrless \mathbf{R}^1$, works as *unitary axiom* for the mathematics which inherit '''***The Nonlinearity Barrier***''', *positing a reductionist, point-atomistic, onto-statical, fixed-points-only,* [≤]**1**-*attractor, monolithic/niche-less logic, a logic of equilibrium* [*linear*] '*anti-dynamics*' or '*pseudo-dynamics*'.
K. The partial *contradiction of reality* by *Parmenidean-Boolean logico-mathematical idealizations*, making them unfit to decode the nonlinear "laws", or "habits", of nature, may imply: (**1**) a kind of «*reductio ad absurdum*» / *empirical refutation* of the premise $\mathbf{x^2 = x^1}$, or (**2**) evidence of its "*independence*", or *Gödel-undecidability*, vis-à-vis any other axioms. This points to *new*, "*Non-Standard*", '*Non-Parmenidean*', '*Contra-Boolean*', '*Contra-Cantorian*' -- ***nonlinear***, that is, ***dialectical*** -- *logics/totality theories*, and to new, *'metafinite' arithmetics*, analogous to the *Non-Euclidean geometries* arising from various negations of Euclid's parallels postulate. Adding these *'ideo-increments'* to the *'multi-meta-ontic'*, '*meta-fractal ideo-cumulum*' of number *'ideo-ontology'* may render solvable these presently unsolvable, *because* ***nonlinear***, integrodifferential equations, *especially those which embody the most advanced conceptions of the "laws", or "habits", of nature, so far offered by modern science.*
L. The ***strong*** negation of Boole's axiom -- the ***inequation*** $\mathbf{x^2} \gtrless \mathbf{x^1}$, wherein the ideogram '$\gtrless$' signifies non-quantitative, *'''ontological'''*, ***qualitative inequality*** -- is solvable within certain new, 'contra-Boolean', 'contra-Cantorian', full-unit-interval *[onto-]logical arithmetics*. Their logics of '***metanumber*** *unit-qualifiers*', or '***dialector***' operators, provide a unified algorithmic «***mimesis***» for all of the key characters of ***dialectical***, '***metafinite-cumulum metadynamics***', and an alternative, non-Boolean architectonic for computer design. Their extension from the unit-interval realms of [onto-]logical '''qualification''' and quantification to full-multiplicity realms of *dimensionally-qualified* as well as *ontologically-qualified*, *'''arithmetical'''*, and *'''analytical'''*, quantification proper [as with Boole's logic, for linear dynamics, and its linear partial and total differential equations], may lead you to '''***The Nonlinearity Breakthrough***'''.

Taxonomic Level One Cosmos-History-Model – *Ontological Dynamics ['Onto-Dynamics'] of the Known Universe*.

1. Model: *'"Meta-Evolving"' 'Universe-Set' of Ontos* – *Self-Expanding Universal Qualities Sum*; Self-Growing Ontology.

1.a. Interpretation of $_N\underline{Q}$ for 'Level One' Cosmos '"Meta-Evolution"' – Core Assignments [$\mapsto$] of the $\mathfrak{Q}_k$ for *Non-Hybrid* ['*Self-Hybrid*'] 'Ontos'.

- **n** connotes the ***onto*** of pre-sub-atomic/pre-nuclear fields / "particles", [e.g., photons], plus anything prior / unknown to present science $\equiv \mathfrak{Q}_n \mapsto \mathfrak{Q}_1$
- **s** connotes the ***onto*** of sub-atomic fields / particles [e.g., protons] $\equiv \mathfrak{Q}_s \mapsto \mathfrak{Q}_2$
- **a** connotes the ***onto*** of atoms = '[meta-]*particles* [made up] *of* [sub-atomic] *particles*' $\equiv \mathfrak{Q}_a \mapsto \mathfrak{Q}_4$
- **m** connotes the ***onto*** of molecules = '[meta-]*atoms* [made up out] *of atoms*' $\equiv \mathfrak{Q}_m \mapsto \mathfrak{Q}_8$
- **p** denotes the ***onto*** of prokaryotic cells = '[meta-]*molecules* made up *of molecules*' $\equiv \mathfrak{Q}_p \mapsto \mathfrak{Q}_{16}$
- **e** denotes the ***onto*** of eukaryotic cells = '[meta-]*prokaryotes* made up *of prokaryotes*' $\equiv \mathfrak{Q}_e \mapsto \mathfrak{Q}_{32}$
- **b** denotes the ***onto*** of *meta-biota* = '[meta-]*eu-cells of eu-cells*', multi-cellular biota $\equiv \mathfrak{Q}_b \mapsto \mathfrak{Q}_{64}$
- **ℓ** denotes the ***onto*** of "animal societies", *proto-ℓ*anguage-based *'meta-metazoa'* $\equiv \mathfrak{Q}_\ell \mapsto \mathfrak{Q}_{128}$
- **h** denotes the ***onto*** of proto-human[oid] societies, *'meta-social meta-metazoa'* $\equiv \mathfrak{Q}_h \mapsto \mathfrak{Q}_{256}$

1.b. Interpretations for some '*culminant*'-*hybrid* , or 'grand uni-«*physis*»', Ontological Categories ['Ontos'].

- The maximum-subscript hybrid ***onto*** of the earlier *pre-galactic medium* $\sqsubset \mathfrak{Q}_{sn} \mapsto \mathfrak{Q}_3$
- The maximum-subscript hybrid ***onto*** of the later *intergalactic medium* $\sqsubset \mathfrak{Q}_{asn} \mapsto \mathfrak{Q}_7$
- The maximum-subscript hybrid ***onto*** of the early *interstellar medium* $\sqsubset \mathfrak{Q}_{masn} \mapsto \mathfrak{Q}_{15}$
- The max-subscript hybrid ***onto*** of initial *atmospheres* of typical planets $\sqsubset \mathfrak{Q}_{masn} \mapsto \mathfrak{Q}_{15}$
- The max-subscript hybrid ***onto*** of late Pliocene *ocean* of planet Earth $\sqsubset \mathfrak{Q}_{lbepmasn} \mapsto \mathfrak{Q}_{255}$
- The max-subscript hybrid ***onto*** of present *soils* of planet Earth $\sqsubset \partial\, \mathfrak{Q}_{hlbepmasn} \mapsto \partial\, \mathfrak{Q}_{511}$

1.c. First **3** *Self-Iterations* of the $\underline{Q}_\tau$, *interpreted* as specifications of the 'Level One' ontological content of the Cosmos as of "epoch"-index τ -- 'Meta-Dynamics' of the *Ontological [Meta-]State* of the Universe [per the rules, $\mathfrak{Q}_n \boxplus \mathfrak{Q}_n = \mathfrak{Q}_n$ ["*additive idempotency*"], & $\mathfrak{Q}_n \boxtimes \mathfrak{Q}_m = \mathfrak{Q}_n \boxplus \mathfrak{Q}_{n+m}$ ['*multiplicative **meta-potency***']].

τ	$\underline{Q}_\tau$	*Interpreted* Arithmetic: Application of the $_N\underline{Q}$ to Universe Modeled	*Un*interpreted $_N\underline{Q}$ Arithmetic
0	$\underline{Q}_0 = \langle \underline{n} \rangle^{2^0} = \underline{n} = \mathfrak{Q}_n \equiv$	stipulated origin; ***pre**-sub-atomic [ev]entities*	$\mapsto \mathfrak{Q}_1$
1	$\underline{Q}_1 = \langle \underline{n} \rangle^{2^1} = \langle \underline{n} \rangle^{2^0} \otimes \langle \underline{n} \rangle^{2^0} = \underline{n} \oplus \underline{n} = \underline{n} \oplus \underline{s} \equiv$	*sub-nuclears* ⊕ *sub-atomics*	$\mapsto \mathfrak{Q}_1 \boxplus \mathfrak{Q}_2$
2	$\underline{Q}_2 = \langle \underline{n} \oplus \underline{s} \rangle^2 = \langle \underline{n} \rangle^{2^2} = \langle \underline{n} \rangle^{2^1} \otimes \langle \underline{n} \rangle^{2^1} = \langle \underline{n} \oplus \underline{s} \rangle \otimes \langle \underline{n} \oplus \underline{s} \rangle = \underline{n} \oplus \underline{s} \oplus \mathfrak{Q}_{sn} \oplus \underline{a}$		$\mapsto \mathfrak{Q}_1 \boxplus \mathfrak{Q}_2 \boxplus \mathfrak{Q}_3 \boxplus \mathfrak{Q}_4$
3	$\underline{Q}_3 = \langle \underline{n} \oplus \underline{s} \oplus \mathfrak{Q}_{sn} \oplus \underline{a} \rangle^2 = \langle \underline{n} \rangle^{2^3} = \langle \underline{n} \rangle^{2^2} \langle \underline{n} \rangle^{2^2} = \underline{n} \oplus \underline{s} \oplus \mathfrak{Q}_{sn} \oplus \underline{a} \oplus \mathfrak{Q}_{an} \oplus \mathfrak{Q}_{as} \oplus \mathfrak{Q}_{asn} \oplus \underline{m}$		$\mapsto \mathfrak{Q}_1 \boxplus \ldots \boxplus \mathfrak{Q}_8$

. . .

1.d. The first three *'Self-Bifurcations'* of $\underline{\mathbf{Q}}_{\tau}$, expressed in phonogramic symbols.

τ $\underline{\mathbf{Q}}_{\tau}$ Interpreted Arithmetic: Application of the $_{\mathbf{N}}\underline{\mathbf{Q}}$ to the *Taxonomy Level One* Cosmos-History-Model.

0 $\underline{\mathbf{Q}}_0$ = *pre-nuclears*

1 $\underline{\mathbf{Q}}_1$ = *pre-nuclears* "of" *pre-nuclears* = *pre-nuclears*⟨*pre-nuclears*⟩ = ⟨*pre-nuclears*⟩^{2} = ⟨*pre-nuclears* + *sub-atomics*⟩

2 $\underline{\mathbf{Q}}_2$ = ⟨*pre-nuclears* + *sub-atomics*⟩^{2} = ⟨*pre-nuclears* + *sub-atomics* + hybrids⟨{ sub-atomics, pre-nuclears }⟩ + *atoms*⟩

3 $\underline{\mathbf{Q}}_3$ = ⟨ *pre-nuclears* + *sub-atomics* + hybrids⟨{ sub-atomics, pre-nuclears }⟩ + *atoms*⟩^{2} =
⟨*pre-nuclears* + *sub-atomics* + hybrids⟨{ sub-atomics, pre-nuclears }⟩ + *atoms* + hybrids⟨{ atoms, pre-nuclears }⟩ +
hybrids⟨{ atoms, sub-atomics }⟩ + hybrids⟨{ atoms, hybrids⟨{ sub-atomics, pre-nuclears }⟩ }⟩ + *molecules*⟩

Selected Components, Exemplary of «*aufheben*» 'Self-Meta-«*Monad*»-ization', or 'Self-Meta-*Unit*-ization' --

pre-nuclears "of" pre-nuclears = pre-nuclears⟨pre-nuclears⟩ = ⟨pre-nuclears⟩^{2} = ⟨pre-nuclears ⊕sub-atomics⟩,
i.e., sub-atomics = *meta-pre-nuclears* units/«*monads*», each one made up out of a heterogeneous multiplicity of pre-nuclear units;

sub-atomics "of" sub-atomics = sub-atomics⟨sub-atomics⟩ = ⟨sub-atomics⟩^{2} = ⟨sub-atomics ⊕atoms⟩,
i.e., atoms = *meta-sub-atomic* units/«*monads*», each one made up out of a heterogeneous multiplicity of sub-atomic units;

atoms "of" atoms = atoms⟨atoms⟩ = ⟨atoms⟩^{2} = ⟨atoms ⊕molecules⟩,
i.e., molecules = *meta-atom* units/«*monads*», each one made up out of a heterogeneous multiplicity of atom units;

molecules "of" molecules = molecules⟨molecules⟩ = ⟨molecules⟩^{2} = ⟨molecules ⊕prokaryotic cells⟩,
i.e., prokaryotic cells = *meta-molecule* units/«*monads*», each one made up out of a heterogeneous multiplicity of molecule units;

pro-cells "of" pro-cells = pro-cells⟨pro-cells⟩ = ⟨pro-cells⟩^{2} = ⟨pro-cells ⊕eukaryotic cells⟩,
i.e., eukaryotic cells = *meta-prokaryote* units/«*monads*», each one made up out of a heterogeneous multiplicity of prokaryote units;

eu-cells "of" eu-cells = eu-cells⟨eu-cells⟩ = ⟨eu-cells⟩^{2} = ⟨eu-cells ⊕multi-cellular organisms⟩,
i.e., multicellular organisms = *meta-eukaryote* units/«*monads*», each one made up out of a heterogeneous multiplicity of eukaryote units;

multi-cells "of" multi-cells = multi-cells⟨multi-cells⟩ = ⟨multi-cells⟩^{2} = ⟨multi-cellulars ⊕"animal societies"⟩,
i.e.," animal societies" = *meta*[-multi]*cellular*, "macro-cellular" organisms, each one made up out of multiple multicellular organisms;

ani-societies "of" ani-societies = ani-societies⟨ani-societies⟩ = ⟨ani-societies⟩^{2} = ⟨ani-societies ⊕ proto-human[oid] societies⟩,
i.e., proto-human[oid] societies = *'meta-animal-society'* meta-units/meta-«*monads*», each one made up out of a heterogeneous multiplicity of animal society units / «*monads*», via multi-animal-society 'social endosymbiosis', or 'social symbiogenesis' [mutual-/self-'"*domestication*"'].

2. Peano Compliance of the $_{\mathbf{N}}\underline{\mathbf{Q}}$ – The $_{\mathbf{N}}\underline{\mathbf{Q}}$ Language as a ***Dialectical Arithmetic*** of 'Gödelian Meta-Natural Meta-Numbers'.

2.a. The Five ***Dedekind-Peano Postulates*** for the Standard **N**atural Numbers [Peano's earlier version]:

P1: **1** is a [**N**atural] Number.
P2: The successor of any [**N**atural] Number is a[lso a] [**N**atural] Number.
P3: No two [**N**atural] Numbers have the same successor.
P4: **1** is not the successor of any [**N**atural] Number.
P5: If $\mathbf{P}^1$ denotes any unary Property, and if [**a.**] **1** has the property $\mathbf{P}^1$, and if [**b**.] whenever a [**N**atural] Number **n** has the property $\mathbf{P}^1$, then the successor of **n** also has the property $\mathbf{P}^1$, then every [**N**atural] Number has the property $\mathbf{P}^1$. [This axiom "quantifies" over properties/predicates, not just over individual "**N**atural" Numbers, **&** so is a ***2nd*** Order, not a ***1st*** Order axiom].

[reference: Reese, W. L., *Dictionary of Philosophy and Religion: Eastern and Western Thought*, Humanities Press [Atlantic Heights, NJ: **1987**], pp. **418-419**].

2.b. There is a version of the $_{\mathbf{N}}\underline{\mathbf{Q}}$ and $_{\mathbf{W}}\underline{\mathbf{Q}}$ 'evolute product', that we term the *'genealogical* product', that is commutative and associative, distributive over **Q**-addition, and whose 'meta-numbers' behave in many ways like the "**N**atural Numbers" or the "**W**hole Numbers", but with an ameliorated, readily-tractable form of '"zero division"' from the stage of $_{\mathbf{W}}\underline{\mathbf{Q}}$, onwards. However, what we call the '«*aufheben*» product' version of the 'evolute product' is non-commutative, and non-distributive over its analogue of the addition operation. In this the **Q** '«*aufheben*» product' rule is like the 'Power-Set Evolute Product' rule. The latter also yields a *'Seldon-Function'*; a '*meta*-exponential' *formula for / model of* the '"*Set Of All Sets*"', and the *'Set Of All Objects'*, which are both *'ideo*-[ev]entities'.

2.c. Compliance of the $_{N}\underline{Q}$ '**Natural** Dialectors' with the *First Order* ***Peano Postulates***, intended only for the Arithmetic of **N**atural Numbers:

Q1: $\mathbb{q}_1 \in {}_{N}\underline{Q}$.

Q2: if $\mathbb{q}_n \in {}_{N}\underline{Q}$, then also $\underline{s}[\![\mathbb{q}_n]\!] = \mathbb{q}_{s(n)} = \mathbb{q}_{n+1} \in {}_{N}\underline{Q}$.

Q3: for every $\mathbb{q}_m, \mathbb{q}_n \in {}_{N}\underline{Q}$, iff $m \neq n$, then also $\underline{s}[\![\mathbb{q}_m]\!] = \mathbb{q}_{m+1} \neq \underline{s}[\![\mathbb{q}_n]\!] = \mathbb{q}_{n+1}$.

Q4: There does not exist $\mathbb{q}_x \in {}_{N}\underline{Q}$ such that $\underline{s}[\![\mathbb{q}_x]\!] = \mathbb{q}_{s(x)} = \mathbb{q}_1$.

2.d. The '*Gödelian*' Character of the $_{N}\underline{Q} = \{ \mathbb{q}_n \mid n \in \mathbf{N} \}$; the '*intra-duality*' of **1**st order "**N**atural" Arithmetic:

[Dawson, John W. Jr., *Logical Dilemmas: The Life and Work of Kurt Gödel*, A. K. Peters (Wellesley, MA: **1997**), pp. **67-68**]: "Most discussions of Godel's proof,. . ., focus on its quasi-paradoxical nature. It is illuminating, however, to ignore the proof and ponder the implications of the theorems themselves. It is particularly enlightening to consider together both the [Gödel] completeness and [Gödel] incompleteness theorems and to clarify the terminology, since the names of the two theorems might wrongly be taken to imply their incompatibility. The confusion arises from the two different senses in which the term "complete" is used within logic. In the ***semantic*** sense, "complete" means "capable of proving whatever is valid", whereas in the ***syntactic*** sense, it means "capable of proving or refuting each sentence of the theory". Gödel's completeness theorem states that every (countable) ***first-order*** theory, whatever its non-logical axioms may be, is complete in the former sense: Its theorems coincide with the statements true in *all* models of its axioms. The [Gödel] incompleteness theorems, on the other hand, show that if formal number theory is consistent, it fails to be complete in the second sense. The incompleteness theorems hold also for ***higher-order*** formalizations of number theory [while the Gödel completeness theorem holds only for ***first-order*** formalizations – **F.*E.D.***]. If only ***first-order*** formalizations are considered, then the completeness theorem applies as well, and together they yield not a contradiction, but an interesting conclusion. Any sentence of arithmetic that is ***undecidable*** must be true in ***some*** models of ***Peano's axioms*** (lest it be formally refutable) and false in others (lest it be formally provable). ***In particular, there must be models of first-order Peano arithmetic whose elements do not "behave" the same as the natural numbers***. Such ***nonstandard models*** were ***unforeseen*** and ***unintended*** but they cannot be ignored, for their existence implies that *no first-order axiomatization of number theory can be adequate to the task of deriving as theorems* ***exactly*** *those statements that are true of the* ***natural numbers***." [***bold italic emphasis*** and square-bracketed commentary added by **F.*E.D.***]. There are connexions here to the Löwenheim-Skolem theorem [see Morris Kline, *Mathematics: The Loss of Certainty*, Oxford University Press [New York: **1980**], pp. **271-272**].

2.e. '*Meta*-Natural *Meta*-Number' Character of the $_{N}\underline{Q} = \{ \mathbb{q}_n \mid n \in \mathbf{N} \}$:

The $_{N}\underline{Q}$ pre-suppose the **N** -- for example, the **N** supply the subscripts, or "'denominators"', which distinguish the $_{N}\underline{Q}$ -- while the $_{N}\underline{Q}$ comply, too, with the first four, *first-order Peano Postulates* which also characterize the **N**; Thus the $_{N}\underline{Q}$ are '***meta***' to the **N**, and could only have been conceived, in the manner given here, subsequent to the conception of the **N**. Each $\mathbb{q}_n$ is a ***meta*-N *meta-****unit*** / ***meta-*«*monad*»**, each one made up from out of a *homogeneous* multiplicity of the **N** unit, **1** [*except* for the «*arché*» unit, $\mathbb{q}_1$], e.g., $\mathbb{q}_3 = \mathbb{q} \mid (1+1+1)$, the later denoting the '***self****-subsumption*' of the «***arithmos***» **1+1+1 = 3** to form $\mathbb{q}$ above/over **3**.

3. *Arithmetic Background*.

3.a. The '***Meta*-N**atural ***Meta***-Numbers' employed in the Model of **1.a.-d.**, above, denoted $\{ \mathbb{q}_n \}$, form an unbounded-above, ***potentially***-infinite "'Space"', or "Set", of "***q****ualitative units*" or '***q****ualitative* ***unities***', that is, of '***unit q****ualifiers*' or of '***un-quantifiable q****ualifiers*' which are higher forms -- and qualitatively distinct forms – analogous to "**R**eal unity", i.e., to the number "one", denoted **1**, and also to "imaginary" unity, denoted **i**, and to the unit-["length"]vectors, denoted variously as $\{\hat{\mathbf{e}}_k\}$, or as $\{ \hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}, \ldots \}$, of, e.g., the "orthonormal bases" of "vector spaces", such that, for $n \in \mathbf{N}$ --

$\underline{Q} \supset {}_{N}\underline{Q} = \{ \mathbb{q}_n \} = \{ \mathbb{q}_1, \mathbb{q}_2, \mathbb{q}_3, \ldots \}$ = The ***potentially***-infinite sub-space of the $\underline{Q}$ space; the sub-space involving only **N**atural Number subscripts/indexes / "'denominators"'.

3.b. The $\underline{\mathbf{Q}}_k$ Sub-Set/'Sub-Sum' of $\underline{\mathbf{Q}}$ as *Abbreviation* for '*Poly-Qualinomial*', *"Non-Amalgamative"* [cf. Musés] Finite Sums of 'consecutive' $\mathbb{T}_n$:

$$\underline{\mathbf{Q}}_k = [\![\mathbb{T}_1 \boxplus \ldots \boxplus \mathbb{T}_{2^k}]\!]$$

3.c. «*Aufheben*» '"***Meristemal***"' *Evolute* [*Self-*]*Bifurcation Product Rule* for '[*Self-*]*Multiplication of* [*Ontic*] *Qualifiers*'.

Given $\mathbf{k, l, m, n} \in \mathbf{N}$:

$\mathbb{T}_m[\![\mathbb{T}_n]\!]$ = conservation of $\mathbb{T}_n$ $\boxplus$ $\mathbb{T}_m$-annulment/elevation of $\mathbb{T}_n$ = $\mathbb{T}_n \boxplus \mathbb{T}_{m+n}$

$\mathbb{T}_n[\![\mathbb{T}_n]\!]$ = conservation of $\mathbb{T}_n$ $\boxplus$ $\mathbb{T}_n$-annulment/elevation of $\mathbb{T}_n$ = $\mathbb{T}_n \boxplus \mathbb{T}_{n+n}$

= $\mathbb{T}_n \boxplus \square\mathbb{T}_n$ = $\mathbb{T}_n \boxplus \mathbb{T}_{2n}$

if $n > k > l > m$: $\mathbb{T}_a[\![\mathbb{T}_k \boxplus \mathbb{T}_l \boxplus \mathbb{T}_m \boxplus \mathbb{T}_n]\!] = [\![\mathbb{T}_k \boxplus \mathbb{T}_l \boxplus \mathbb{T}_m \boxplus \mathbb{T}_n]\!] \boxplus \mathbb{T}_{a+n}$

if $n > k > l > m$: $[\![\mathbb{T}_a \boxplus \mathbb{T}_b]\!][\![\mathbb{T}_k \boxplus \mathbb{T}_l \boxplus \mathbb{T}_m \boxplus \mathbb{T}_n]\!] = [\![\mathbb{T}_k \boxplus \mathbb{T}_l \boxplus \mathbb{T}_m \boxplus \mathbb{T}_n]\!] \boxplus$

$= \mathbb{T}_a[\![\mathbb{T}_n]\!] \boxplus \mathbb{T}_b[\![\mathbb{T}_n]\!]$

$= \mathbb{T}_k \boxplus \mathbb{T}_l \boxplus \mathbb{T}_m \boxplus \mathbb{T}_n \boxplus \mathbb{T}_{a+n} \boxplus \mathbb{T}_{b+n}$

if $n > m$: $[\![\mathbb{T}_m \boxplus \mathbb{T}_n]\!][\![\mathbb{T}_m \boxplus \mathbb{T}_n]\!] = [\![\mathbb{T}_m \boxplus \mathbb{T}_n]\!]^2 = \mathbb{T}_m \boxplus \mathbb{T}_n \boxplus \mathbb{T}_{m+n} \boxplus \mathbb{T}_{n+n}$

3.d. Ideograms of Relation / Relational Symbols.

analogy: $\underline{\mathbf{Fa}} \asymp \underline{\mathbf{Gb}}$ denotes '$\underline{\mathbf{Fa}}$ *is analogous to* $\underline{\mathbf{Gb}}$'.

qualitative inequality: $\underline{\mathbf{a}} \lessgtr \underline{\mathbf{b}}$ denotes '$\underline{\mathbf{a}}$ is *non-quantitatively different from* or is *qualitatively different from* $\underline{\mathbf{b}}$'.

['''*meta-evolutionary*'''] total order: $\underline{\mathbf{a}} \longrightarrow \underline{\mathbf{b}}$ denotes "$\underline{\mathbf{b}}$ *is a successor of* $\underline{\mathbf{a}}$"; or "$\underline{\mathbf{a}}$ *is a predecessor of* $\underline{\mathbf{b}}$";

[qualitative/ontological total order] $\mathbb{T}_m \longrightarrow \mathbb{T}_n$ implies $\mathbf{n > m}$; $\mathbf{n > m}$ implies $\mathbb{T}_m \longrightarrow \mathbb{T}_n$;

[*Non*-quantitative, *'Qualo-Peanic Progression'*] $\mathbb{T}_n \longrightarrow \mathbf{s}[\![\mathbb{T}_n]\!] = \mathbb{T}_{s(n)} = \mathbb{T}_{n+1}$.

3.e. *Idempotent Rule* for 'Addition of [Ontic] Qualities' ['*unquantifiability*' or '***non-additivity***' of *'ontic qualifiers'*].

$$[\![\mathbb{T}_n \boxplus \mathbb{T}_n = \mathbb{T}_n]\!] \asymp [\![\{\mathbb{T}_n, \mathbb{T}_n\} = \{\mathbb{T}_n\}]\!].$$

J. O. Urmson, *The Greek Philosophical Vocabulary*, Gerald Duckworth & Co., Ltd. [London: **1990**], pp. **31-32** [***emphasis*** added by **F.E.D.**]: "***arithmos***: number; ***arithmêtikê***; the science of number. Zero was unknown as a number and one also was not counted as a number, the first number being the *duas* – two. From the Pythagoreans, *ton arithmon nomizontes arkhên einai* – who consider number to be the first principle (Ar. ***Met.* 986a15**) – number played a great part in metaphysics, especially in Plato's unwritten doctrines, involving obscure distinctions of e.g. *sumblêtoi* and *asumblêtoi* – addible and *non-addible numbers*."

[Gunawardena, Jeremy, Ed., *Idempotency*, Cambridge University Press [Cambridge, U.K.: **1998**], pp. **1**; **28**]: "The word ***idempotency*** signifies the study of ***semirings*** in which *the* ***addition*** *operation is* ***idempotent***: $a + a = a$. ... Interest in such structures arose in the 1950s through the observation that certain problems of discrete optimisation could be ***linearised*** over suitable ***idempotent*** semirings. ... More recently, intriguing new connections have emerged with automata theory, discrete event systems, nonexpansive mappings, ***nonlinear*** *partial differential equations*, optimisation theory, and large deviations...The phrase ***idempotent analysis*** first appears in the work of Kolokoltsov and Maslov... *It may seem implausible that* ***idempotency*** *has anything to say about differential equations*... However, ***remarkable advances have taken place in our understanding of nonlinear partial differential equations which enable us to give meaning to solutions...which may not be differentiable anywhere***" [**bold** *italic* ***emphasis*** **added by F.E.D.**].

The analogue of the **N** addition operation for 'Boolean meta-numbers', i.e., for the arithmetic of the "Boolean algebra" of formal logic, is also "*idempotent*": $0_B + 0_B = 0_B$ -- and, more '"non-standardly"', w.r.t. **N**, and w.r.t. $\mathbf{W}$ -- $1_B + 1_B = 1_B$, versus $1 + 1 = 2$.

3.f. The *"Non-Amalgamation" Rule* for 'Addition of [Ontic] Qualities / Qualifiers' [cf. Musés].

for $n, m, x \in \mathbf{N}$, if $n \neq m$: $\underline{\mathbb{Q}}_n \boxplus \underline{\mathbb{Q}}_m = \underline{\mathbb{Q}}_n \boxplus \underline{\mathbb{Q}}_m \asymp$ "**apples**" ⊞ "**oranges**", i.e.,

if $n \neq m$: $\nexists \underline{\mathbb{Q}}_x \in {}_{\mathbf{N}}\underline{\mathbb{Q}}$ such that $\underline{\mathbb{Q}}_x = \underline{\mathbb{Q}}_n \boxplus \underline{\mathbb{Q}}_m$

3.g. The *'self-bifurcation index'*, τ. The 'meta-time-like' index τ represents the count of cumulative *self-reflexions* constituting $\underline{\mathbb{Q}}_\tau$. It also counts the *'self-bifurcations'* in $\underline{\mathbb{Q}}_\tau$ as ontological *'meta-state-vector'* or *'meta-state meta-vector'*, and as 'Ontological *Universe-Set*' or 'Universal *Sum* of [Ontological] Qualities'. It is, too, the iteration-count of *self-expansions* of the *'ontological possibility-space'* of the 'Universe' so modeled.

3.h. The 'Onto-Dynamical', *Pure-Qualitative 'Meta-Evolution Equation'*, or *'Self-Involution Equation'*.

$$\underline{\mathbb{Q}}_{\tau+1} = \underline{\mathbb{Q}}_\tau[\![\underline{\mathbb{Q}}_\tau]\!] = \underline{\mathbb{Q}}_\tau^{\,2} = \underline{\mathbb{Q}}_\tau \boxplus \square\underline{\mathbb{Q}}_\tau \gtrless \underline{\mathbb{Q}}_\tau ;$$

$$\underline{\mathbb{Q}}_{\tau+1} = [\![\underline{\mathbb{Q}}_1 \boxplus \ldots \boxplus \underline{\mathbb{Q}}_{2^{\tau+1}}]\!] \asymp \{\underline{\mathbb{Q}}_1, \ldots, \underline{\mathbb{Q}}_{2^{\tau+1}}\} \asymp [\![\underline{\mathbb{Q}}_1^{\,1} \boxplus \ldots \boxplus \underline{\mathbb{Q}}_1^{\,2^{\tau+1}}]\!] = [\![\underline{\mathbb{Q}}_1]\!]^{2^{\tau+1}}.$$

3.i. The 'Onto-Dynamical' *Generating Equation* -- "Closed Form Solution" of *'The Meta-Evolution Equation'*.

$\underline{\mathbb{Q}}_\tau = \underline{\mathbb{Q}}_0^{\,2^\tau}$, under the pairwise expansion convention $= [\![\underline{\mathbb{Q}}_1 \boxplus \ldots \boxplus \underline{\mathbb{Q}}_{2^\tau}]\!] \asymp \{\underline{\mathbb{Q}}_1, \ldots, \underline{\mathbb{Q}}_{2^\tau}\}$.

Note that the expression of this *'Seldon Function'* solution-function involves a kind of new operation -- an arithmetical operation which we call 'meta-exponentiation' -- involving two 'tiers' of 'superscription', with the "independent variable", τ, situated at the second level of 'superscription'. If we model the '"meta-evolution"' of arithmetical operations one-sidedly, ignoring the "operands", "arguments", or 'nomia' which these operations "combine" -- the 'mathematical nouns' which these 'mathematical verbs' transform -- we find the following *dialectical*, *'meta-monadological'*, $\underline{\mathbb{Q}}$ *'Qualo-Peanic'* sequence of paired -- and inverse -- conceptual developments, of which the last is still barely emergent -- and still unnamed -- at present:

addition(addition) = addition^2 = addition ⊕ Δaddition = addition ⊕ multiplication; addition $\gtrless$ multiplication;
subtraction(subtraction) = subtraction^2 = subtraction ⊕ Δsubtraction = subtraction ⊕ division; subtraction $\gtrless$ division.

multiplication(multiplication) = multiplication^2 = multiplication ⊕ Δmultiplication = multiplication ⊕ exponentiation;
division(division) = division^2 = division ⊕ Δdivision = division ⊕ nth-root-extraction; division $\gtrless$ nth-root-extraction.

exponentiation(exponentiation) = exponentiation^2 = exponentiation ⊕ Δexponentiation = exponentiation ⊕ *meta-exponentiation*;
nth-rooting(nth-rooting) = nth-rooting^2 = nth-rooting + Δnth-rooting = nth-rooting ⊕ meta-rooting; nth-rooting $\gtrless$ meta-rooting.

Thus, we can define the successive arithmetical 'pure operations' as follows:

Each multiplication ≡ a 'meta-addition' made up out of a homogeneous multiplicity of 'additions';
Each division ≡ a 'meta-subtraction' made up out of a homogeneous multiplicity of 'subtractions';

Each exponentiation ≡ a 'meta-multiplication' made up out of a homogeneous multiplicity of 'multiplications';
Each root-extraction ≡ a 'meta-division' made up out of a homogeneous multiplicity of 'divisions';

Each *'meta-exponentiation'* ≡ a 'meta-exponentiation' made up out of a homogeneous multiplicity of 'exponentiations';
Each 'meta-rooting' ≡ a 'meta-root-extraction' or 'meta-rooting' composed of a homogeneous multiplicity of 'root-extractions'.

3.j. 'Gödel-numbering', 'multiplicative subscript rule' version of the *'«aufheben» evolute product'*, $[\![{}_{\mathbf{g}}\underline{\mathbb{Q}}]\!]$ --

[$_{\mathbf{g}}\underline{\mathbb{Q}}$'s 'strong' *non*-commutativity reduces the higher-degree 'ontos' *'"confounding"'* that afflicts the 'additive subscript rule' version of $\underline{\mathbb{Q}}$]

$${}_{\mathbf{g}}\underline{\mathbb{Q}}: \; [j, k \in \mathbf{N} \;\&\; j < k] \Rightarrow {}_{\mathbf{g}}\underline{\mathbb{Q}}_j[\![{}_{\mathbf{g}}\underline{\mathbb{Q}}_k]\!] = {}_{\mathbf{g}}\underline{\mathbb{Q}}_k \boxplus {}_{\mathbf{g}}\underline{\mathbb{Q}}_{p(j)^j * p(k)^k} \gtrless {}_{\mathbf{g}}\underline{\mathbb{Q}}_k[\![{}_{\mathbf{g}}\underline{\mathbb{Q}}_j]\!] = {}_{\mathbf{g}}\underline{\mathbb{Q}}_j \boxplus {}_{\mathbf{g}}\underline{\mathbb{Q}}_{p(j)^k * p(k)^j}.$$

3.k. Finite Difference Equations describing the growth of the 'meta-population' count of 'ontos' in generic ${}_{\chi}\underline{\mathbb{Q}}_\tau$, as a function of τ and χ.

for the 'additive subscript rule': $N_{\mathbb{Q}}[\![{}_{\mathbf{N}}\underline{\mathbb{Q}}_{\tau+1}]\!] = 2^{\tau+1}$; for the 'Gödelian' rule: $N_{\mathbb{Q}}[\![{}_{\mathbf{g}}\underline{\mathbb{Q}}_{\tau+1}]\!] = {}_{\mathbf{g}}N_{\tau+1} = 1 + {}_{\mathbf{g}}N_\tau^{\,2}$, ${}_{\mathbf{g}}N_0 = 1$,

the latter modeling the veritable "*Combinatoric Explosion*" in the potential of ${}_{\mathbf{g}}\underline{\mathbb{Q}}$ to express *'ontic' / ontological distinctions* as τ increases.

'***Taxonomic Level***' in this context refers to a 'meta-fractal ***scaling***' of **Q**-based or **U**-based *'onto-dynamical'* universe-[*of-discourse*] *'meta-models'*. A *'meta-model'* models transitions from one *model*ed epoch to the next.

The '***meta-fractal scaling***' associated with this concept of '***Taxonomic Level***' is primarily a *synchronic* scaling, as contrasted with the *diachronic* '***meta-fractal scaling***' associated with the 'self-bifurcation index', τ, in the *generic* designation of an *'onto-dynamic pure-qualitative universe meta-model'*, $\{ {}^{n}_{u}\underline{Q}_{\tau} \}$, or *'quanto-qualitative meta-model'*, $\{ {}^{n}_{u}\underline{U}_{\tau} \}$.

A *first-taxonomic-level* 'onto-dynamical' universe 'meta-model' is the one that describes the 'sequence of appearance' or 'ordinality of genesis' of each of the known *'"ontological categories"'*, or '*ontos*', constituent of the top-of-scale, *maximally-known* context of the given universe of discourse, per the *first-level component*, denoted by ${}^{1}_{u}\partial$, of the modeler's ***taxonomy***, or ***principle of ontological-categorial part[ition]ing***, $\{ {}^{n}_{u}\partial \}$, for **u**niverse of discourse **u**, for the *'Taxonomic Level'* **n**.

We designate a *first-taxonomic-level* **Q**-based such *'meta-model'* by $\{ {}^{1}_{\forall}\underline{Q}_{\tau} \}$, where the 'pre-sub-script' $\forall$ denotes the ***universe*** of discourse -- here the maximal context of *all* known universes of discourse - where the 'pre-super-script' **1** denotes the '***Taxonomic Level***' or synchronic '***depth-into-detail***' of the model, here the minimal such depth, and where τ denotes, again, the "independent variable" '***self-bifurcation index***', or '***meta-evolutionary epoch(s) counter***'.

A *second-taxonomic-level* 'onto-dynamical' universe *'meta-model'* is a dialectical-ideographic description of the 'onto-dynamics' of the ***sub-ontology*** of just one of the *first-taxonomic-level* 'ontos'. Such a ***sub-ontology*** is determined by applying the modeler's part[*ition*]ing principle to the *sub*-categories of that *first-taxonomic-level* *'"ontological category"'*; i.e., to identifying, and building a *second-taxonomic-level* '"Seldon Function"' around, the «***arché***» '***sub-onto***' of that *first-taxonomic-level* 'onto', with that *first-taxonomic-level* 'onto' taken as *a* [*sub-*] ***universe***[*-of-discourse*]***-unto-itself***.

Thus, suppose '${}^{1}_{\forall}\underline{\mathbf{atoms}}$' or '${}^{1}_{\forall}\underline{\mathbf{a}}$' names one of our *first-taxonomic-level* 'ontos' of our universal *'meta-model'*, given our ontological taxonomy, or principle of ontological-categorial part[*ition*]ing. Then, either a **Q**-based, or a **U**-based, 'onto-dynamical' *'meta-model'* of the epochs of cosmological and/or of stellar ***atomic nucleosynthesis***, describing the "filling-in" of the periodic table of the chemical elements, starting from an initial, «***arché***» **a**tomic 'sub-onto', of, say, Hydrogen, denoted ${}^{2}_{a}\underline{H}$, followed, in natural-historical *'"order of appearance"'*, by Helium, Lithium, Carbon, Oxygen, and Nitrogen, denoted ${}^{2}_{a}\underline{He}$, and ${}^{2}_{a}\underline{Li}$, etc., would exemplify a ***second-taxonomic-level model***; a model of the ***sub-universe*** constituted by the 'onto-dynamical', progressive, cumulative self-genesis of the '***sub-ontos***' of the *'Taxonomy-Level-One'* '***onto***' ${}^{1}_{\forall}\underline{a}$.

We designate a **Q**-based such *'meta-model'* by $\{ {}^{2}_{a}\underline{Q}_{\tau} \}$, where ${}^{1}_{\forall}\mathbf{a}$ denotes the [sub^{1}-]***universe*** of discourse -- in this case the sub-universe of ${}^{1}_{\forall}\underline{\mathbf{atoms}}$ -- where **2** denotes this *'meta-model's'* '***Taxonomic Level***', and where τ denotes the '***self-bifurcation index***' value for the stage of self-development in question for *this* sub-universe. Given that the *'"*[sub-]*ontological category"'* of Hydrogen **a**toms, denoted by ${}^{2}_{a}\underline{H}$, is the «***arché***» ontic category for the [***sub-***]***universe***[*-of-discourse*] of ${}^{1}_{\forall}\underline{\mathbf{atoms}}$, we would look for a Seldon Function based around that «***arché***» ontic category as the solution-function for the *'meta-model'* describing the *'"meta-evolution"'* of that [sub-]universe, *'"interpreting"'/'"assigning"'*, e.g., the generic «***arché***» *'ontological qualifier meta-number'*, denoted by $\mathbb{I}_{\lfloor_1}$, to ${}^{2}_{a}\underline{H}$, or ${}^{2}_{a}\underline{\mathbb{Q}}_{\flat H}$, and forming, our *'Taxonomy-Level-Two 'meta-model'* of the epochs of *cosmological/stellar nucleosynthesis* thusly: $${}^{2}_{a}\underline{Q}_{\tau} = \langle {}^{2}_{a}\underline{Q}_{0} \rangle^{2^{\tau}} = \langle {}^{2}_{a}\underline{H} \rangle^{2^{\tau}} = \langle {}^{2}_{a}\underline{\mathbb{Q}}_{\flat H} \rangle^{2^{\tau}} \mapsto [\![\mathbb{I}_{\lfloor_1}]\!]^{2^{\tau}} = \underline{Q}_{\tau}.$$

Likewise, a **Q**-based or **U**-based 'onto-dynamical' *'meta-model'* of the *sub-ontology* of the Hydrogen atom -- e.g., of the natural-historical emergence of ***isotopes*** of Hydrogen, such as Deuterium and Tritium, which constitute two *'"*ontological category*"'* components of our *sub-sub-ontology* of our first-level 'onto' of "**a**toms" or "chemical elements" -- is termed a '***third-taxonomic-level*** [sub-]universe *meta-model*'; a model of that *sub-sub-universe* constituted by the '*sub-ontos*' of the '*sub-onto*' ${}^{2}_{a}\underline{H}$, which are, likewise, among the '*sub-sub-ontos*' of the ${}^{1}_{\forall}\underline{a}$ 'onto' of our *first-taxonomic-level* ontology.

We designate a **Q**-based such *'meta-model'* by $\{ {}^{3}_{H}\underline{Q}_{\tau} \}$, where *'pre-subscript'* **H** denotes the [*sub*^{2}-]***universe*** of discourse, in this case, the *sub-sub*-universe, or *sub*^{2}-universe[-of-discourse], of the isotopic «***species***» of the '**Hydrogen**' atoms «*genos*», where the *'pre-super-script'* **3** denotes the *'meta-model's'* '***Taxonomic Level***', *relative to* '***Taxonomy-Level-One***', and where τ denotes the '***self-bifurcation index***' value for *this* *sub*^{2}-universe.

The Productive Force of Language

This treatise sets forth a new, 'engineered' ideographic language -- an explicit system of ideographical language-rules -- one which is designed to help surmount some limitations of contemporary '''Parmenidean'''/'''Boolean'''/'''Cantorian''' set theories, and arithmetics, and of the rest of the mathematics that they engender.

Language is a technology. Language, however intangible, is also a ***productive force***, in Marx's sense. Advances in language can increase human-social productivity - the ***self-productivity of humanity***. As with any ***force***, and as with any advance in human-***social*** [self-re-]***productive force***, such advances can ***accelerate*** human social '''***meta-evolution***'''.

Linguistic objects, language artefacts -- especially ideographic ones -- can encode 'human programs', *'programs for humans'*, or *'praxis algorithms'*. Such formulae -- ideographically-recorded recipes for changing reality -- specify *sequences of actions* by organized groups of instrumented human subjects -- systems of *mental* plus physical, 'instru-*mental operations*' as means designed to achieve a pre-targeted end. By such *'procedures'*, in the form of controlled experiments, the dynamical and *'poly-qualitative'* stuffs of reality are brought to reveal their natures. By such cooperative labor, they can be molded to the service of humanities' continued construction of the cosmos -- to that part of the 'self-building' of the universe which is mediated through humankinds.

Innovations that bring gains in 'human ***self***-productivity' and 'social ***self***-reproductivity' are changes in behavior, '''additions''' of new behaviors, changes in activity, *innovations in action*. The '''recipes''' for such behavioral innovations or 'activity-innovations' are encoded, transmitted, and, in part, *discovered*, via human languages.

By '''human society''', as an '''ontological category''' -- as an «***arithmos***» -- we mean an «***arithmos***», or '''assemblage of units''', made up out of individual human societies as its units, or «***monads***». Each individual human society unit, or «***monad***», in that category/«***arithmos***», is, in turn, in[side] itself, a 'sub-«***arithmos***»'. Each individual '''human society''' contains ['$\sqsubset$'] '''animal societies''' as its 'sub-«***monads***»', or '''sub-units'''. That is, we *'ontologically categorize'* a '''human society''' in terms of our *'self-meta-«**monad**»-ization* paradigm' for the concretization of 'the ***self***-«***aufheben***» process' that defines the '''***dialectic of nature***''' throughout, to-date, per our extant knowledge of *the history of nature*. We therefore define an *individual* '''human society''', as itself forming a kind of unit, or «***monad***». Each such *'''human society'''* is a *'**meta**-animal-society'*, i.e., an '''animal society''' raised to the '''***second degree***''', or '''***squared***''', such that each individual *'''human society'''* unit is made up out of a heterogeneous multiplicity of '''animal societies'''. It is the *'mutual internalization'* of multiple, genomically-heterogeneous '''animal societies''' -- e.g., **humans** ⊕ **dogs** ⊕ **cattle** ⊕ **fowl** ⊕ ... that constitutes *the new 'interiority'*; that creates the new layer, level, and *'meta-fractal scale'* of cosmological, ontological *'''meta-evolution'''*, that we call '''human society''', or '''meta-society''', from out of the '''lower''' [less-inclusive] layer/level/scale of the '''animal society''' ontology. Thus, we do ***not*** see as merely incidental, or contingent, the deep, 'meta-social' alliances among multiple '''merely social''' «***species***» -- including one key, human[oid] '''social-animal''' «***species***» with advanced genomic, neurological and vocal adaptation that persists because it leverages the unprecedented survival advantages of 'sociality', and including also «***species***» of plant units, or plant «***monads***», which, while characteristically lacking the motile, neuro-muscular development that would lead us to classify their assemblages/«***arithmoi***» as '''societies''', nonetheless manifest their growth in spatially self-densifying, "grassland", etc., "***communities***". Thus, wherever the phrase '''human society''' occurs in this text, keep in mind its '***meta-social***' meaning.

This essay is framed by a network of *cumulative, 'onto-dynamic', 'meta-evolution equations'*, of the following form:

$$\mathbb{X}_{\tau+\Delta\tau} = \mathbb{X}_{\tau} \text{ "of" } \mathbb{X}_{\tau} = \mathbb{X}_{\tau}\langle\mathbb{X}_{\tau}\rangle = \mathbb{X}_{\tau}\circledast\mathbb{X}_{\tau} = \mathbb{X}_{\tau}^{2} = \sqcap\mathbb{X}_{\tau} = \mathbb{X}_{\tau} \oplus \Delta\mathbb{X}_{\tau} = \mathbb{X}_{\tau+\Delta\tau} \not\simeq \mathbb{X}_{\tau}.$$

Therein, $\mathbb{X}_{\tau}$ denotes the τth epoch of development of the '''meta-evolving''' [ev]entity, $\mathbb{X}$; the sign '$\not\simeq$' signifies that $\mathbb{X}_{\tau}$ differs ***non-quantitatively***, that is, ***qualitatively - ontologically*** -- from the *new part* of what the model *'meta-evolution equation'*, $\mathbb{X}_{\tau}^{2} = \mathbb{X}_{\tau+\Delta\tau}$, denotes, namely $\Delta\mathbb{X}_{\tau}$; τ denotes the *self-transformation index* value for that τth epoch [or 'system'] of the total *'meta-system'*, or *'systems-progression'*, $\mathbb{X}$. That totality is identified with the "set" or 'sequence' of all its '''meta-evolutionary epochs''' [or '''systems'''], from birth $[@\,\tau = \alpha]$, to death $[@\,\tau = \Omega] -- \mathbb{X} = \{\mathbb{X}_{\tau}\}$, $\alpha \leq \tau \leq \Omega$.

Members of this set of stages, or of '''epochs''', for such a *'self-meta-evolving'* [ev]entity, $\{\mathbb{X}_{\tau}\}$, are numbers of new kind, *'meta-numbers'*; "quantities" *of new quality*: '***ontological qualifiers***', each one connoting a different ***ontological category***.

Perhaps most crucial of all of these '***self***-iterative' expressions to date is the case which assigns the τth stage of $\mathbb{X}$ to consciousness [sentience]: $\mathbb{X}_{\tau} \leftrightarrow$ **consciousness**, and

consciousness *"of"* **consciousness** = ⟨**consciousness**⟩⊛⟨**consciousness**⟩ = ⟨**consciousness**⟩^{2} =

the *self-interaction*, *self-reflexion*, or *self-refluxion* of **consciousness** or of **sentience** = $\sqcap$⟨**consciousness**⟩

consciousness ⊕ Δ⟨**consciousness**⟩ = **self-consciousness** ≄ **consciousness**.

Hypothesis: The emergence of spoken language is central to a *'self-bifurcation'* within ancient Terran proto-human[oid] 'meta-meta-meta-zoan' societies which gave rise to *Homo sapiens*.

We use the term "***cooperative labor***" to designate the process of the production of vital ***social-reproductive 'exo-artefacts'*** and also of '***endo-artefacts***', or "memes", as enacted by inter-communicating, goal-sharing groups of dexterous human[oid]s. These include all of the '*self-reflexive artefacts*' -- the ***self-****artefacts* of the ***self-****domestication* and ***self-****cultivation* of the laboring subjects -- increased dexterity, skill-honing, and deepened empirical knowledge and insight; emergence of *new powers, new needs, new products, new relationships, new concepts*, and *new increments of language 'phenomo-ontology'*.

We term, by "***universal labor***", deliberate efforts to form "*universal knowledge*" or "*universal science*" -- deeper, nature-wide and society-wide principles of action, potentially applicable to cooperative labor by *all* groups of laboring subjects; the activity of knowledge-production, whether in its incipient phases, or in full-bloom.

Human spoken language is the primal example of the technological forces of human social production, the original and originating such *human-society-productive* force, or *human-society-reproducing* force. First and foremost, human language is productive of human society -- of association, of the nonlinear, superadditive, cooperative effects of cooperative labor, of the synergies of dialogic conceptual collaboration in universal labor; of the emergence of knowledge-based human social reproductive praxis.

Human society itself, as a whole is, in turn, the greatest of the human-social productive forces, and the foundation for all of the rest. Emerging language is productive of greater awareness, and, beyond a critical point, of emerging *self*-awareness, and, later, of acceleratory growth in self-awareness.

Within the 'meta-dynamical meta-system' of human[oid] societies are found several qualitatively distinct stages in the '''meta-evolution''' of language, both prior to, within, and, we hold, beyond the present stage of Terran human social development. ***Dialectical Ideography*** is an attempted precursor of the next, *conditionally necessary* stage of human[oid] linguistic ''meta-evolution''' -- necessary if Terran human evolution, and *'''meta-evolution'''*, is to continue much further.

Language is the foundation of the 'human[oid]-social' phases of cosmic '''meta-evolution'''. By the 'human[oid]-social' phases, we mean those in which animal societies made up out of metazoan multicellular biota, eventually including proto-human[oid] populations, form synergetic associations, coalitions which begin harnessing the enormous nonlinear potential energy of the cooperative interaction of metazoan individuals, and, later, of the language-mediated self-conscious self-interaction of proto-human[oid] individuals.

Of all the human social forces of production, language is the premier, but not only originally: also *recurrently*. Every milestone step in self-humanizing social formation is accompanied by major mutations in language.

A new language, or a new step in the *'''meta-evolution'''* of languages, is not just a new tool *of communication*; it is also a new tool *of cognition, of thought;* a new «***organon***».

A change *of language* is also a change *of mind*. A change in language affects the conceptual processes, the thought patterns, the mental dynamics of the mind that uses it; that iterates its use; that *practices* it habitually.

A language is a cognitive *instrumentality* that reacts upon and mutates the *mentality* of the mind that wields it.

Throughout Terran human prehistory to date – in Marx's sense -- qualitative leaps in the level of the human social forces of human societal self-expanding self-re-production – i.e., in the level of ***human-social***, or of *'**meta-social**'*, 'self-productivity', or of human-societal 'self-reproductivity' -- correlate with corresponding advances in this primary human social technology: with advances in the media of "natural" languages, including with advances in that most consciously crafted of the so-called "artificial" languages: mathematics.

Past [st]ages of emerging Terran human self-identity, of humans' *interiorized self-models;* of their *self-consciousness*, become '''fossilized''', in human language, and in its related 'psycho-artefacts', including its mathematics. But of late human-societal *'''meta-evolution'''* has reached that threshold where the sustenance of advancing human self-identity necessitates an individual and collective *'**self-bifurcation**'*, a "quantum leap" in the quality of that self-identity, a moral and spiritual forward leap tied to the cognitive qualitative leap of *'''The Nonlinearity Breakthrough'''*.

'Psycho-Archaeology': Reconstructing the "'*Psycho-History*'" of our Deep Past. Remnants of ancient writing, "'fossils'" of former thought, instances of inscription from antiquity -- linguistic, symbolic, and iconographic artefacts, even three-dimensional clay tokens -- are the primary data of *'**Psycho-Archaeology**'*. They *partially* reveal, via still-extant '***object***-ivities' of great temporal depth; of deep 'durative' extension, how past, surpassed social inter***subject***ivities thought about ***objects*** -- external *and internal* ***objects***; sensuous *and conceptual* ***objects***. Such psychological remains thus also reveal, at least *indirectly*, how the human[oid] ***subject***ivities of those by-gone epochs thought about themselves; about their own identities. They thereby help to reveal the ***subject*** -- the state of human ***subject***ivity, the form of human self-identity, the level of human self-consciousness -- of the people of their times of origin. Such '***object***-ive', '***psyche***-ological' survivals may thus serve as veritable barometers of the degree of emergence of human social individuality, self-awareness, and self-reflection at each such '*psycho-archaeological horizon*' so excavated.

We scrutinize more than mere arbitrary rules of grammar when we ponder the patterns of past human-linguistic "'*meta-evolution*'". We excavate the human psyche. Ragged remains of written documents, stone inscriptions, carvings, and shapes in clay -- even snatches of oral tradition, still extant from the deeps of time -- are not just the beginnings of "'*recorded **pre**history*'". Records of past language may tell us much beyond what they are ostensively about. Linguistic artefacts, semiotic artefacts, are 'psych***e***-artefacts'; 'psych***o***-artefacts'. They record the social "'*meta-evolution*'" of the human mind; of ***the human "'phenome"'***. They register implicitly the world-views of the past. They do so in ways sometimes far surpassing explicit written testimony, in part because the information they give was given unguardedly.

Look backwards -- into the *historical* temporal direction -- toward the ultimate origin of human language! Today's articulated noun-versus-verb dichotomy diminishes in degree in direct proportion to the depth of your vision's penetration of that past. At the tribal stage of linguistic "'*meta-evolution*'", the two sides of this dualism *merge*. Our "noun" and "verb" opposites *converge* into a single grammatical category, from which they *diverged* in the forward sense of time. This divergence, in the history since then, has become marked. Today, many people tend to take it as *ontologically* axiomatic, as self-evident. Many Terrans today think as if their internal and external *worlds*, not just their *words*, are divided into nouns and verbs. In the history by which their ancestors led them here, an ancestral and unifying perception of reality as fluid reality-*process* was lost.

Contemporary humans are *re*-awakening to a sense of a 'self-animate' reality like that which their ancestors relinquished along that way. They are spiraling back, helically, at a higher level, in these latter days of the 'Dim Ages' of human prehistory, to echoes and "'higher-octave'" resonances of those former forms of perception, qualitatively new but '***meta-fractally*** ***similar***.

The human future, if Terran humanity is to have a future, promises to bare some 'meta-fractal' kinship to that deep past. Let us therefore delve deeper into this *'psycho-archaeological assemblage'*. What does it suggest to you about that future? What clues does it provide toward solving this riddle of the noun/verb-dichotomizing drive of humanity's historical-linguistic "'*meta-evolution*'"? What clues does it offer toward *'**The Nonlinearity Breakthrough**'*?

Foreshadowings of an Immanent Critique of "Natural" Language. Benjamin Lee Whorf has given us one of the more striking accounts of the ontological and 'psyche-ological' influences of grammar, in his "Principle of Linguistic Relativity". According to this principle, languages with fundamentally different grammars unconsciously inculcate different logics [and different ontologies]. Each such language would tend to a different mathematics and a different science should the culture bearing it develop to mathematico-scientific levels of literacy [absent other impacts of the social relations of that self-development]. In his *Science and Linguistics*, Whorf described a tribal language with the following remarkable grammatical trait:

"*... in Nootka, a language of Vancouver island, all words seem to us to be **verbs**, but really there are no classes 1 [nouns] and 2 [verbs]; we have, as it were, a **monistic** view of nature that gives us **only one class of word for all kinds of events**. "A house occurs" or "it houses" is the way of saying house, exactly like "a flame occurs" or "it burns".*"[1]

Elsewhere on the subject of Nootka, Whorf wrote: "*... Nootka has **no parts of speech**; the simplest utterance is a sentence treating of some **event** or **event-complex**. Long sentences are sentences of sentences (complex sentences), not just sentences of words.*"[2] [*emphasis added* by **F.E.D.**].

This Amerindian language implies a dynamical world-picture, in which "things" are grasped as "events". Or rather, the grammar makes no radical distinction between "entities" and "events". Its world-picture contains only what we might call *'event-entities'* or *'events-as-entities'* -- *'''**eventities**'''*[3], for short!

Ernest Fenollosa was another early appreciator of this 'psyche-ological' dimension of language. His beautiful essay, *The Chinese Written Character as a Medium for Poetry*, finds in the ancient roots of the Chinese language a logic of noun-verbs or verb-nouns similar to that which Whorf found in the Nootkan language:

*"...Chinese notation is something much more than arbitrary symbols. It is based upon a vivid shorthand picture of the **operations** of nature...The earliest forms of these characters were pictorial...the great number of these **ideographic** roots carry in them the verbal idea of **action**. It might be thought that a picture is naturally the picture of a thing, and that therefore the root ideas of Chinese are what grammar calls nouns. But examination shows that the large number of the primitive Chinese characters (even the so-called radicals) are shorthand pictures of **actions** or **processes**."*[4] [***emphasis*** added by **F.E.D.**].

Fenollosa noted explicitly the ontological bias of noun-verb cloven languages: *"**A true noun, an isolated thing, does not exist in nature.** Things are only terminal points, or rather the meeting points of actions, cross-sections cut through actions, snap-shots. **Neither can a true verb, an abstract motion, be possible in nature.** The eye sees **noun and verb as one**: things in motion, motion in things, and so the Chinese conception tends to represent them."*[5] [***emphasis*** added by **F.E.D.**].

Fenollosa traces the derivation of nouns [and other *presently distinguishable* parts of speech] from verbs in ancient Chinese. He also notes that [what we call] *'''**eventity ontology**'''* is not confined to ancient China [any more than to ancient Amerindia]: *"In the **derivation of nouns from verbs**, the Chinese language is forestalled by the Aryan. Almost all Sanskrit roots, which seem to underlie European languages, are **primitive verbs**, which express **characteristic actions** of visible nature. **The verb must be the primary fact of nature**, since motion and change are all that we can recognize in her."*[6] [***emphasis*** added by **F.E.D.**].

Heraclitus was an Ionian philosopher of the pre-Socratic period. He remains to this day, in the few fragments of his work that survived the last Dark Ages, the most striking articulator of the '''eventity''' world-picture implicit in such 'unitary grammars'. To him we owe such apposite adages as "You cannot step twice into the same river"; "Everything flows and nothing abides, everything gives way and nothing stays fixed", and, "The sun is new each day". Though the pithiness of phrase may be all his own, the world-view he expressed was not a purely personal or unprecedented achievement. His way was prepared-for by the linguistic, grammatical patterns of early Greek:

*"**The distinction among parts of speech is less pronounced in the Greek language than in the Latin and its Western successors**...in general, our contemporary Western languages keep a fairly steadfast distinction among the three [word] types -- nouns standing for things, adjectives standing for qualities, and verbs standing for actions and events...**in the thought of Heraclitus, abetted by the comparative fluidity of the Greek language, the linguistic distinction and correspondingly the ontological distinction are somewhat less firm.**"*[7] [***emphasis*** added by **F.E.D.**].

A guiding thread in our ongoing discovery/design of the *'meta-arithmetics'* denoted by **Ω**, **U**, **μ**, and beyond, is the imperative to inaugurate an ***ideographic dialectical language*** based explicitly upon these insights into the *unitary, noun/verb pre-unified, subject-verb-object-identical,* ***'eventity-ontology'***-based grammars of early "natural" languages. We call the sum of these insights -- in the context of "artificial", designed ideographies -- the *'Principles Of Operatorial Ideography'*.

The Linguistic Approach to Mathematical Foundations: Homeomorphic Defect; Linguistic Overhead Costs. An upshot of this study of 'Psycho-Archaeology' -- of the patterns of *'''meta-evolution'''* of linguistic artefacts and other 'psycho-artefacts' from the ancient past which it reveals -- is the perception of opportunities for deliberate, conscious, felicitous design of *future* 'psycho-artefacts'. ***Dialectical Ideography*** often addresses potential enhancements in the engineering of physical artefacts, via a deepening appreciation of the nonlinear, or dialectical, nature of nature. But ***Dialectical Ideography*** is, directly, a work of *'Linguistic Engineering'* and of *'Conceptual Engineering'*. It is based, in part, upon insights drawn from 'Psycho-Archaeological' studies.

This essay is an effort to craft, so as to exemplify -- to engineer by *immanent critique*, that is, by the 'self-reflexion' of the existing mathematical language -- a new *'phono-picto-ideographic'* mathematical language, and the conceptual processes which undergird it, and which it, in turn, facilitates. We seek to help you to design a language, and a conceptual praxis, more apt to the dynamics, and the *'meta-dynamics'*, that Terran humanity now confronts in its expanding experience of the cosmos. This *experiential expansion*, in *inward as well as outward* experience, has been induced, for Terran humanity, principally by its own expanding social praxis – by its tendentially accelerating social self-reproduction on an expanding scale -- to date, and especially of late.

The next volume, entitled ***The '''Meta-Evolution''' of the "Standard" Arithmetics***, and the volumes following, especially volume **III**., entitled ***The "Non-Standard" Arithmetics of '''Meta-Evolution'''***, make evident that our approach to questions of the foundations of mathematics is neither that of logicism, nor that of formalism, nor that of constructivism, though it incorporates aspects of each. It is *a linguistic approach*, analyzing mathematics as a system of linguistic artefacts, and as a system of collective, cultural, memetic, *'''human-phenomic'''* 'psycho-artefacts'.

What Is "Mathematics"? We explore, in the next volume, entitled ***The '''Meta-Evolution''' of the "Standard" Arithmetics***, our proposed definition of mathematics as '***ideometry via phono-picto-ideography***'. By '***ideometry***' we mean ***the measurement of ideas***. 'Measurement' is here taken in a most expanded sense, and in a *quanto-qualitative* sense. 'Ideometry' encompasses any and all communicable and coherent ***accounting for*** ideas. Mathematics has covered, in its development to date, only a small part of the content intended by the term '***ideometry***'. But, we hold, '***ideometry***' is the destiny, the essence, and the future appearance; the *entelechy*, of what we know today as mathematics. Mathematics is, in essence, and therefore always was, temporary appearances notwithstanding, ***a science of the qualitative***, of the qualitative, including, but *not limited to*, the ***quality*** of *cardinality*, usually seen today as "*pure quantity*", the *apparent opposite* of all [*other*] "*quality*".

'**Homeomorphic Defect**'. Mathematical language, as a so-called "artificial language", has long been the object of more sustained conscious *'linguistic engineering'*, of more deliberate syntactico-semantic "notational design", than have so-called "natural languages". Nevertheless, unconscious / unintentional and semi-conscious features abound in the "grammar" of the "standard" mathematics of today. Such features are especially evident in the evolving, and *'''meta-evolving'''*, "grammar" of ideogramic mathematical language -- its *'ideogrammar'* -- and in the semi-conscious "ontological commitments" which that *'ideogrammar'* entails. The emergence of the "operator" ideogram is a crucial case in point.

Each such linguistic package of ontological commitments, each such mathematical *'meta-model'* of "what there is", can only be a *homeo*morphism, not an *iso*morphism -- a one-to-many rather than a one-to-one mapping of that which it is intended to map. Each is thus inherently a limiting, omissive, biased, and, consequently, inherently and ineluctably problematic *abstraction from* actuality. Each is, at best, an *in*complete representation, and a "***useful fiction***".

That is, every such homeomorphism exhibits '***homeomorphic defect***'. There are gaps in its description. Some features of the actuality modeled are left out of the model almost entirely. Others are partially deformed in its imperfect mirror. Some of its features may be "overheads" of its intrinsic apparatus, "artifacts" or 'extranea' which have no counterpart in the actuality which the model is intended to mime. The distribution of good and poor coverages varies among alternative mathematical 'sub-languages'. But all exhibit such uneven coverage, suggesting a kind of ' "law" of the conservation of incompleteness'. This imposes a burden, a cost, upon users of any such language. Complex auxiliary contraptions may, at certain junctures, need to be built around each to compensate for its 'onto-linguistic deficiencies'. That of ***Dialectical Ideography*** is no exception.

We call such costs '***Linguistic Overhead***'.

Examples abound. *The operation of counting is itself a useful, coherent fiction, abstracting from qualitative and quantitative uniquenesses and variabilities of each specimen of a given kind of object; treating, e.g., any pair of apples as "two" "**different**" yet **effectively identical units** of "apple"; any triple of oranges as "three" "**different**" yet **effectively identical units** of "orange". Whereas, in truth, no "two" apples, oranges, etc. can ever be alike in all detail.*

Per Gödel's Incompleteness Theorems, finitely altering axioms of mathematical systems may change the content of "undecidable" Gödel formulae -- 'non-deducible theorems' -- but cannot eliminate them. Ontologies presupposing timeless, static, 'nounic', pre-existent, as if from eternity *separately self-subsistent* individuals and classes in *Principia Mathematica* cause paradoxes *of dynamism*, of *inescapable auto-mobility*, which deduce to self-contradictory propositions, requiring contra-intuitive contraptions, like, e.g., Bertrand Russell's "ramified types hierarchy", to suppress them.

Calculations from Newtonian gravitic ontologies of "force acting over distance" contradict observation under extreme parameters of gravitic intensity, velocity, etc., where those from Einsteinian ontologies of "geometrodynamics" agree far better. Yet the Newtonian gravitic languages are apt *enough*, and, at present, more conveniently computable than the General Relativistic, for most cases involving non-extreme parameter-values.

Ontological commitments that mathematicize "elementary quanta" as "points" cause "*singularities*", in the computed values of particle "self-energies", etc. -- unwanted zero-division infinities -- requiring vast "renormalization" contraptions to remove them. Mathematicizing those "quanta" as *linear waves* entails complementary descriptive advantages and disadvantages vis-à-vis as *atomistic point-particles*. Mathematically languaging the same as "quantum fields", as "superstrings", or as "'M-branes'" introduces, in each case, a different distribution of linguistic overhead pros and cons. The actualities that physics calls "elementary quanta" are no doubt better metaphorized mathematically as *neither* linear waves *nor* point particles *nor* strings *nor* "membranes" [*nor* "M-branes" *nor* "p-branes"]. But until 'onto-linguistic *meta-models*' with less net defects are achieved, physicists are stuck with these high linguistic overhead burdens.

Here, emerging **3+**-dimensional *nonlinear wave*, or *toroidal and hyper-toroidal vortex* concepts may provide major conceptual and descriptive advantages vis-à-vis the requirements of an expanding human-societal self-reproductive praxis.

Consider the relative ease with which mathematics presently solves general *linear* -- and perturbationally, etc. *linearized nonlinear* -- integrodifferential equations, though the resulting solutions generally give highly omissive descriptions of the objective processes that they attempt to model.

Consider the typically great difficulty with which this mathematics solves the few *nonlinear* integrodifferential equations mastered in the last **~333** years, and the "mathematical intractability" of the most important, "natural law"-formulating instances of these nonlinear integrodifferential equations for *this* mathematics.

We hold that this "'***Nonlinearity Barrier***'", too is, *in part*, a problem of 'Linguistic Overhead'. Syntactical as well as semantical features of contemporary mathematical language and '*ideogrammar*', reflecting ultimately Parmenidean, Boolean / Cantorian, crypto-statical, linear / atomistic / 'equilibriumistic' "ontological commitments", help erect this barrier. They do so because they are inapt to 'experience-able' and measurable actualities for the kinds of problems posed by emerging Terran human social self-reproductive praxis.

Of course, linguistic maladies are reflections of underlying conceptual maladies. Conceptual maladies, in turn, reflect underlying '***self***-conceptual' maladies, maladies of human self-identity. 'Psycho-archaeological analysis' can help to illuminate the historical roots of these maladies, and can also intimate mental medicines that might help in healing them.

Thus, we hold that '***The Nonlinearity Breakthrough***' will be, in part, a linguistic achievement. But also that this achievement will have cultural, 'psycho-anthropological', "'psycho-historical'" roots. The requisite linguistic breakthrough is predicated upon a parallel breakthrough in *'conceptual engineering'*. And, that conceptual breakthrough is predicated upon a co-evolving breakthrough in human *'self-conception'*, or self-identity.

Sciences of Objectivity, Sciences of Subjectivity, and Mathematics. The foregoing raises questions regarding the scientific status of mathematics. Is mathematics an *empirical, experience-disciplined* body of theory? Are its doctrines constrained by any requirements of experiential conformity and by an experimental practice? Is its "subject matter" an *'object-ive'* subject matter in any sense?

We hold that Mathematics is ***not***, in itself, a *science of* ***external*** *'object-ivity'*, such as are physics and chemistry.

Its key objects are not exterior, *external-sense objects*. Mathematicians and logicians study inward objects, *idea-objects*, ***idealizations***. They work with 'interior', mental objects; *'internal-sense objects'*; *'mind-sense objects'*, as perceived by the mind's 'inwardly-directed eye'. Such objects can be "inspected" sensuously, but only by "introspection". Such objects are "seen" and manipulated by means of intro-reflective thought, though thought often aided by special forms of writing activity. Such writing renders internal objects, *in*visible to our outer eyes, as externally visible ideographs, [picto-] graphs, and phonograms.

We hold that mathematics is a science, but one of *'**internal** objectivity'*: ***a science of subjectivity***, a science of the regularities of the *'[idea-]object-ive'* phenomena experienced in humanity's internal, mental worlds. By 'subjective' here we do not mean "illusory", "arbitrary", or "merely a matter of taste". We mean *inter*subjective.

Mathematical findings represent *'cognitive-psychohistorical' regularities* of conceptual phenomena, of the phenomenologies of ideas, upon which entire communities of 'internal self-observers' may concur. By 'knowledge', we mean verifiable, reproducible findings. In consensus cases, each mathematician constructs "the same" concepts, and comes to essentially the same results. The 'internal matter'; the ***'idea-matter'***, which mathematicians work is a refractory material, however fluid it may seem in comparison to external, physical matter. The mentally met behaviors of idea-objects are highly constrained -- by interconnection, by coherence, and by consistency. Mathematics is partly *discovery*, not invention / free construction only.

Indeed, we see mathematics as an experiential, ***experimental*** *science*. Its laboratories exist in mental space. Its instruments are mind tools, facilitated by writing -- in dialects peculiar to mathematicians -- and by computer simulators. Computer programs emulate -- and extend in scale -- the mental and written implementation of algorithmic experiments. Einstein's examples point to the power of *«gedanken» experiments* in the physical sciences. But those sciences rely finally upon external, physical experiments. The experiments *fundamental* to mathematics are *thought experiments*. The role of rigorous logical proof in mathematics might seem to belie any notion of true experimentation in that science. That sense is largely a leftover of bygone beliefs in mathematical theorems as *absolute* truths. Most mathematicians today recognize mathematical truths to be *relative* truths[8]. Their theorems are true, first and foremost, *relative* to the assumptions and definitions from which they follow.

The explicit, unproven bases of mathematical systems include logically independent/non-redundant axioms. These axioms are typically modifiable in ways which yield alternative, incommensurable, qualitatively distinct systems - *but systems which exhibit an equivalent degree of logical validity*. Such alternative systems also often offer unprecedented utility for *modeling previously neglected or untheorised aspects of human experience*. For geometry, the case of Euclid's parallels postulate -- *long presumed the only option possible* -- is exemplary. Tries at *«reductio ad absurdum»* proof of the parallels postulate produced surprise. Far from all of its negations breeding absurdities, some yielded consistent alternative systems, the *"non-Euclidean geometries". One of these founds General Relativity, Einstein's theory of the cosmic grawitic field*; of the theorized non-flat geometry of the *physical*-universal space-time continuum, today's best-fit model for physical space.

For arithmetic**+**, Gödel's Incompleteness Theorems demonstrated, via a '''nonlinear''' ['''self-reflexive''', 'self-refluxive', self-applying, self-implicative] logical formula, that formal-deductive codifications/axiomatizations of cardinal arithmetic, or more, inherently exclude portions of their own relative truth from formal demonstrability. There must exist arithmetic expressions, 'meta-demonstrably' true under their axioms, that cannot be formally deduced from those axioms, or from any of those axioms' "countable" extensions, via the prescribed methods of deduction. "Uncovered" domains of "undecidability" hide within the world of every such formal mathematical theory, untouchable by its axiomatizations. There must always be formulas which are "undecidable" - which cannot be established as either true or false *from the axioms*, using the known rules of inference.

Mathematicians and logicians demonstrated, during this century, in addition to the coherence of *non-Euclidean geometries*, that of alternative, *non-Aristotelian logics*, of non-Cantorian Transfinite Theories, of "Non-Standard" models of "Natural" arithmetic, of "Non-Standard Analysis" and of Category-theoretic "Topos" Theory [the latter two of which, in different ways, expand the arithmetical underpinnings of calculus to *rigorously* include "infinitesimals"]. They demonstrated the independence of the Axiom of Choice and of the Cantor-inspired Generalized Continuum Hypothesis, if taken as an additional axiom, from the other Axioms of Zermelo-Fraenkel "Standard Set Theory", opening prospects of "*Non-Standard*" *Set Theories*, of alternative formulations of the "Real" 'continuum', and of the properties of the "Irrational" "Real" numbers, whose cardinality, per Cantor's Continuum Hypothesis, is $\mathfrak{c} = \aleph_1 = 2^{\aleph_0}$.

We have so far propounded only the perils of *explicit* postulation. But mathematical practitioners are also inescapably hostage to semi-conscious, *implicit* assumptions, as the history of the subject so richly reveals. Such are typically teased into explicitude only as the protracted result of continued exploration, of new and unexpected findings, of conceptual paradoxes and crises, and of propositional contradictions.

In the actual practice of axiomatic-system development, the "trial" axioms of incipient axioms-systems incur modification as a reflux of their consequences.

It is now known that "mathematical truth" is not monolithic. Plurality, 'alternativity' abounds. Axiomatizations of major portions of mathematics are complex. Consequences of "tweaking" axioms are not readily foreseen intuitively by mere inspection. This plurality, and these high degrees of "axiomatic" complexity, point up an inescapably exploratory and experimental essence of mathematics which was actually operative all along. The independent axioms of a given axiomatic 'ideo-system' are like the control parameters of a mathematical '*meta-dynamical meta-system*'. Shifts in such parameters -- changes in their formulation, even if sometimes seemingly slight -- may produce "***bifurcations***", quantum leaps, qualitative transformations in the 'meta-semantics', and in the axiomatically-asserted *'ideo-ontologies'*, of that *'axiomatic meta-system'*, or *system**s**-sequence*. Also, unrecognized assumptions, not axiomatically explicit, may harbor as yet undiscovered doors to ever vaster realms of mathematico-conceptual, linguistic, ***and technological*** possibility. Thus, the various «***species***» of the «***genos***» that we name '*meta-axiomatic meta-dynamics*' may unearth *conceptual '''buried treasure'''*.

Cooperative Labor and Universal labor: Language as Foundation of Both. Human languages constitute an inherently social material; the primal, non-privatisable form of social property and common-wealth.

To recapitulate:

Language is the primary medium, first for *cooperative labor* and, second, for *universal labor*.

Cooperative Labor names the '"nonlinear"' social process whereby human[oid] social individuals, in concert with their '***meta-societal*** *co-endo-symbionts*' – domesticated herd/social animals and plant communities -- work in concert to reproduce themselves and their *'meta-societal'* existence, including their *'meta-societal'* organization and division of labor. *Cooperative labor* is the activity of a human[oid] meta-society's ***self****-reproduction*, subsuming biological/genomic reproduction in the '"complex unity"' of a combined, *'phenomic-genomic'* self-reproduction of human society.

Universal Labor names the process of the revelation -- especially through reflection upon the results of deliberately designed experiences, or *experiments* -- of society-wide or nature-wide principles of human praxis.

These principles, when embodied in the intensions, tools, and procedures appropriated by the cooperative labor-process, increase the *meta-societal productivity* -- the '***self-re-productive force***' that ***accelerates*** human society's ***quanto-qualitatively growing***, self-expanding self-production of *'meta-socio-mass'* -- of cooperative social-reproductive labor, contributing to the quantitative *and qualitative* growth of the 'socially endosymbiotic', [*proto-*]*human*[*oid*]*s-led 'meta-animal-societal'* populations.

Hypothesis. ■ The dynamics and *'meta-dynamics'* of the *'meta-dynamical meta-system'* named 'human[oid] society' are such that such emerging planetary *'meta-societies'* regularly become *trading societies*, ***buying-and-selling*** *'meta-societies'*. They develop social metabolisms based upon emergent "markets", regularized exchanges, or inter-mutual / multi-mutual ***sales*** -- *mutual* ***alienations*** -- of the products of human[/animal/plant] labor, that is, these 'meta-societies' develop human-societal *'meta-metabolisms'* based upon ***the praxis of alienation*** and of ***production for alienation***. This leads, at length, with the growth of the social-productive forces, as expressed in a growing plethora and surfeit of products of many kinds, to the emergence of monetized exchange praxis. One consequence is the emergence of cardinal arithmetic and of the entire mathematics that flows from it. After a while, *'ideo-ontologically'*, this mathematics appears to its users to be one of "pure quantity". This appearance fits the indelible perception and dominating paradigm of monetized commodity exchange: qualitatively different objects equated, for purposes of trade, by qualitatively identical units of the prevailing money-commodity, that is, of price, and thus apparently by "quantity alone" -- by quantities counted [later] in units of [metal or paper] currency, ***units of seemingly far-removed, vacuous, intangible, arbitrary, or even nonexistent quality***. The further immanent self-elaboration of this exchange-praxis leads, at length, for *'meta-societies'* which survive their 'meta-Darwinian *planetary selection tests'*, to deepening syntheses within the dialectic of cooperative labor and universal labor. These emerging syntheses, which we call *universal cooperative labor*, orchestrate 'social phase transitions'.

'Knowledge-capital-based' or 'science-capital-based' praxes of continually expanding 'meta-societal' self-reproduction arise. Past dualistic, *'antithesistic'* distinctions of consumption vs. production; of *producing* skilled labor-power/"human capital", vs. *consuming* it; of *advancing* science vs. *applying* science to production; and among doing science, doing art, and doing labor, become increasingly obsolete.

These transitions are also characterized by passages of pluralities of populations from the *formal operations* to the *'"dialectical operations"'* stages of adult cognitive development, accompanied by covariant mutations in human languages, including in the languages of mathematics.

These linguistic mutations typically include mathematical mutations which redintegrate quantitative and qualitative description -- the advent of computable *qualitative algorithms*, and, thence, of an integrated, '***quanto-qualitative***' algorithmics. We hold these mutations to be part and parcel of *'The Nonlinearity Breakthrough'*. ■

'Pictogramy', 'Ideogramy', 'Phonogramy' and the "Phylogenetic" Tree Of Written Language Genealogy. Three major lineages of written symbolization diverged from the primal emergence of writing -- pictography, ideography, and 'phongramy'. 'Phonogramy' -- written language encoded by means of phonetic alphabets -- has long since swept the field for the core reaches of literature, in the Terran Occidental civilizational Hemisphere at least. There remain, however, *'''evolute''' survivals* and outgrowths of both pictography and ideography within the dominance of 'phonogramy'. The most important survival and continuing outgrowth of ideography is found in the *'''meta-evolution'''* of arithmetical symbolization, and of mathematical symbolization generally.

Leibniz's Dream. Leibniz co-discovered, with Newton, what has become the *'meta-meristem'* of Terran humanity's ideographic languages to-date, "the calculus". This "calculus", or "analysis", can be seen as a "higher algebra", one which includes putatively infinitary, "transcendental", "limit" processes. "The calculus" expands classical algebra, augmenting its repertoire of "mutually-inverse operations" -- addition versus subtraction, multiplication versus division, exponentiation versus root-extraction.

This calculus adds the mutually-inverse operations of "differentiation" versus "integration".

This "calculus" is the ideographic language in which are written down the unsolved nonlinear total and partial integro-differential equations that so far constitute this humanity's highest expressions of nature's "laws".

In his more theo-philosophical work, Leibniz hypothesized an *'''evolute'''*, cumulative cosmos, one characterized by *continuous creation* -- at least at the *meta*physical level:

"For Leibniz, creation was continuous, in the sense that *God conserves created monads* and *produces them continuously* by a kind of emanation, as we produce our thoughts. Thus to Johann Schulenburg . . . in Bremen he remarks that, in the binary system, he had found a very beautiful picture of the continuous creation of things out of nothing and their dependence on God for their continued existence."[9]

This image of the metaphysical cosmos as a *continuous* and ***cumulative*** Creation-*process* is reminiscent of, and partially resonant with, that of a *'''**meta-dynamical**''' physical* universe as *'**self-developing process**'*; as *a continually, and ontically self-enriching '**multi-meta-ontic cumulum**'*, as projected by the *'**onto-dynamical** meta-models'* of ***Dialectical Ideography***.

Leibniz was also perhaps the most prominent -- if not the earliest or most published -- proponent of "symbolic logic", that is, of an *ideographic*, algebraic, "mathematical" formulation of Aristotelian syllogism, and beyond, *launching his key writings on this, his almost life-long project, in* **1666**, *in the same year that Newton first discovered the Calculus.*

George Boole's "*Fundamental Law Of [formal-logical] Thought*" was expressed, in Boole's original algebra, by the equation $\mathbf{x^2 = x}$, an [algebraically] *nonlinear* equation that asserts a *total inconsequentiality of nonlinearity* for formal logic; an *absolute reduction* of nonlinearity to linearity in that logic. The *maximal negation* of this proposition, $\mathbf{x^2 \neq x}$, founds the *«**arché**» dialectical arithmetic* of *'onto-logic'* presented herein. Boole's *"fundamental law of thought"* was presaged ideogramically by Leibniz, in such forms as $\mathbf{AA = A}$; as $\mathbf{A + A = A}$; and as $\mathbf{A \oplus A = A}$. Formal-logic's *law of double negation* -- in Boole's ideography, $\mathbf{1 - (1 - x) = 0 + x = x}$ -- was expressed by Leibniz in *partially-ideographic form*: "Not-not-**A** is the same as **A**".[10]

However, Leibniz's goals in the domain of a "mathematics of reasoning" went far beyond those of a syllogistic ideography, and well beyond even the remarkable achievements of contemporary Terran symbolic logic.

Leibniz, from early youth, was captured by the vision of a universal science - a 'meta-science', 'made up out of' the many sciences, a 'science *of* sciences' [a '**science**^{2}' - a "*science squared*" or "*science of the second degree*"] -- founded, in part, upon a new, ideographic language. Some components of Leibniz's vision include:

(1) An Encyclopedia Of Fundamental Concepts; a Dictionary of Primitive Ideas;
(2) An Alphabet Of Thought, based upon the Encyclopedia's Fundamental Concepts;
(3) A Mathematics Of Reasoning [*Mathesis Rationis; Mathesis Universalis; Calculus Ratiocinator*], employing the Alphabet Of Thought in algebraic fashion;
(4) A Universal "'Algebra"', i.e., a Universal '*Character*-istic', or General Algorithm and Notation, employing '*Characters*' [***Characteristica Universalis***; *Characteristica Generalis; Calculus Universalis; Calculus Philosophicus*] for solving the expressions of the Mathematics Of Reasoning and of the *General Science* or *Science Of Sciences;*
(5) A World Language, based on the Mathematics Of Reasoning and its Universal Characteristic(s).

A "*characteristica*" or "***character*-istic", for some sub-domain of human knowledge, is an '''***algebra***''', a system of notation using "***characters***", or "***letters***", as ideographic symbols descriptive of the problems or states-of-affairs of that sub-domain, together with a set of rules/algorithms for transforming statements of problems written in that character-notation into statements of their solutions in that same notation. The term also carries the connotation that this character-notation, or '''algebra''', is to be 'apt' or "character***istic***" of the phenomena it encodes, in a kind of ideographical sense of *onomatopoeia*: formation of the symbolic expressions in imitation of the mentally-seen phenomenologies of the ideas that they are to symbolize.

A "*universal* ***character-istic***" is thus a ***character-language***, or '''***algebra***''', applicable to *all* domains of knowledge.

Viewed *sub specie* Leibniz's vision, our mission in this essay is to put before you a veritable ***Dialectical Characteristic*** which, we claim, also qualifies as a ***Universal Characteristic***.

Leibniz's Dream has been fulfilled, in part, by the later work of Boole, Jevons, Schröder, and Peirce in the *'Arithmetic Of Logic'*, and the *"Algebra Of Logic"*, and by the related development of binary digital computers; by the work of Frege, Peano, Whitehead, and Russell in symbolic formal/mathematical logic; by the work of Cantor, Zermelo, Fraenkel, Gödel, and others in Set Theory, and by the developments of "meta-mathematical" ideography, and of Category Theory.

Yet, to our reading of Leibniz, all of these developments fall far short of Leibniz's full vision!

This essay can be seen as an exploration of a particular candidate for a '''Universal Characteristic'''. It seeks to envision, and to encompass in human thought and language, a universal theory of «***arithmoi***»; a universal principle of '***Meta-Monadology***'; of '*self*-iterated', ***dialectical*** - '***self-«aufheben»***' - and escalating/recurrent, '***meta-fractal***-*ogenic*', '*self*-meta-«***monad***»-izations'; of '*meta-dynamics*', or of '***meta-fractal***', '***self-«aufheben»***' '*self-subsumptions*', '''*self-involutions*''', or '*self-incorporations*' -- revolutionary '''*meta-evolutions*''', and their inherent 'neo-qualitatio-genetic', 'neo-onto-genetic', *inter*-epochal irruptions, punctuating and interrupting the quantitative continua of *intra*-epochal "evolutions" or "dynamics"; ever adding new 'ontos' - new ontological categories -- and their new «***monads***», to the *'multi-meta-ontic'*, *'multi-meta-monadic'*, *'meta-fractal* ***cumula***' of the cosmos, a cosmos characterized by ***quanto-qualitatively-scaled self-similarity*** '***helicity***'.

This principle is intended to encapsulate the '''*meta-dynamic*''' that we call '*via singularity self-bifurcation*'. '*Self-bifurcation*' refers to *self-induced* '*metafinite* [*self-*]*conversion-singularities*'; to '*quanto-qualitative*' *self-transformations*; to the *qualitatively, ontologically self-expanding* [or *self-contracting*] *self-reproductions* of the *self-developing-and-mutually-developing process-entities*, or '''*eventities*''', which populate our universe, we humans included. It codifies a universal pattern of '''nonlinearity''', of *self*-action, of *self*-reflexion, of *self*-refluxion -- of the *self-propelling*, *self-accelerating* return of action to and upon the source of that action -- in a way which *qualitatively, ontologically* changes that source, and thus also changes all subsequent actions of/emitted by that source.

That candidate *Characteristica* can be formulated as follows: any dialectical [ev]entity, call it $\diamondsuit X$, metamorphoses itself -- its '*system-identity*', its '''*meta-state*''' -- in due course, by virtue of the *cumulative* consequences of its own *activity* upon the materials modeled by the "control parameters" of its model, *which control that activity*, into, equivalently, '$\diamondsuit X$ *of* $\diamondsuit X$', $\diamondsuit X\langle\diamondsuit X\rangle$, $\diamondsuit X^2$, $\diamondsuit X \diamond \diamondsuit X$, $\diamondsuit X \diamondsuit X$, or $\sqcap\diamondsuit X$, all of which equal $\diamondsuit X$ again, but also *plus* something ***qualitatively***, ***ontologically different*** from $\diamondsuit X$, though born[e] from out of $\diamondsuit X$, namely, what we denote by $\triangle\diamondsuit X$. The resulting '''non-homogeneous''', '''non-amalgamative''' sum/'''*superposition*'''/'***cumulum***' of $\diamondsuit X$ and $\triangle\diamondsuit X$, is, in part [in part $\triangle\diamondsuit X$] different *in* ***quality***, from the starting point from which, by self-action, it emerged, namely, from what we denote by $\diamondsuit X$. In fully-ideographic shorthand, using the ideogram '$\rightarrow$' to stand for '''***becomes***''', or '''transforms *itself* into''':

$$\diamondsuit X \blacksquare \diamondsuit X^1 \rightarrow \diamondsuit X \textit{ of } \diamondsuit X \blacksquare \diamondsuit X\langle\diamondsuit X\rangle \blacksquare \sqcap\langle\diamondsuit X\rangle \blacksquare \diamondsuit X^2 = \langle \diamondsuit X^1 \diamond \triangle\diamondsuit X^1\rangle \neq \diamondsuit X^1, \text{ because } \triangle\diamondsuit X^1 \neq \diamondsuit X^1.$$

The 'meta-finite difference operation', $\triangle$, above, denotes 'pure-***qualitative***', ***ontological*** difference, not mere quantitative difference. The "pure-quantitative" finite difference operator, Δ is, thereby, extended, all the way to denoting its *opposite*, namely, [meta-]*finite purely-****qualitative*** *difference*; 'incremental ontology'; 'ontological incrementation'; the '''addition''' of a new *ontological category*; of a new *ontological quality*; of a new '''***determination***''', denoted $\triangle\diamondsuit X$, to the pre-existing ['*cumulum* of'] ontological category(y)(ies), denoted by $\diamondsuit X$.

The above rendition of our candidate Universal Characteristic is radically incomplete. It treats ✕ as if it were a *totality-unto-itself*, as if an isolated universe; as if it were something completely *self*-determined, *internally determined*, determined by its *'internity'* alone; uninfluenced by its "environment" – by its *'externity'* -- which consists of *other* [ev]entities. The *'linearistic bias'*, on the contrary, sees entities as mainly *externally determined*.

This *'non-linearistic bias'* is an antidotal counter-bias to that prevailing, *'linearistic bias'*, which tends to see only *external causation*; only the moment of *other-determination*, but not that of *self-determination*, in the *'cumulum'* of changes observed in any dialectical *[ev]entity* -- *especially* in the so-called "*inanimate*", "*nonliving*" ones.

We will counteract this bias, by bringing such "external causes" into focus, in conjunction with the '''internal causes''' further on.

Hypothesis: ■ The next rebirthing burst of Terran human-social [meta-]evolution -- if it is to happen at all -- entails a quantum leap in the form and content of Terran human-social self-identity. ■

'Psycho-anthropologically', this will be both reflected in and catalyzed by corresponding revolutions in the human languages, and in the media of communication which embody that language, including expected outgrowths of the *'Omni-Com'* -- that ***omni***bus global ***com***munications/***com***merce public utility and infrastructure, presently known as the "Internet", and as the "World Wide Web".

Dialectical Ideography aims to articulate and instantiate certain aspects of that conditionally necessary leap.

That leap involves a **3**-fold *'meta-social' phase transition*, in **(1)** cosmological and socio-historical self-awareness, in **(2)** «***organon***», or technology/methodology, of thought-praxis, and in **(3)** linguistic technology.

This deep 'self-bifurcation' in human self-conception, and linguistic tool-kit, is key to vast proliferations of new technologies for energy production and distribution, metallurgy and other materials' sciences, medicine, "environmental" amelioration / internalization of "externalities" [including of the 'ecological depreciation' costs of human-social self-reproduction], global 'econo-ecological' self-management, and space-faring.

This next 'self-bifurcation' also impends the *democratic self-management* of the expected, *potentially-destabilizing* emergences, and emergencies, associated with new and unprecedented human-social praxes involving:

(a) ***marked extension of human individual longevity*** -- of crucial value to a globalizing capitalist economy shifting increasingly to a 'knowledge-capital' or 'science-capital' basis, in the former 'First World', where an ever intensifying "demographic transition" increasingly impends negative rates of population growth. Presently, this exists in multi-dimensional tension with consequences of the suppression of that *'democratic transition'*-enabled and -enabling *"demographic transition"* in the rest of the world, where poverty has been perpetuated *and deepened* by First-World-sponsored rapacious tyrannies so horrific that they once made even Stalinist "Communism" look like a *relatively* attractive alternative, and which thus served as the primary recruiting force that kept the moldering corpse of that "Communism" seemingly-alive for so long.

(b) *human-**phenome**-mediated **genomic self-intervention*** -- genetic *self-re-engineering* of the human species;

(c) ***cyborg-prosthetics***, or ***cyborg-bionics***, a *hybrid* of **(d)** and **(b)**, and;

(d) ***android*** ***[and non-android] robotics***.

This 'self-bifurcation' forms part of the grounding for Earth's first ***planet-wide*** 'meta-societal' ***renaissance***, and for its first truly global and truly post-prehistoric human civilization: ***human*** --

$$\langle {}^{1}_{\vee}\underline{h}\rangle \blacksquare \langle {}^{1}_{\vee}\circledcirc_{\flat h}\rangle \longmapsto [\![\overline{\sqcup}_{256}]\!]; \quad \sqsubset \quad \langle {}^{1}_{\vee}\circledcirc_{\flat n}\rangle^{2^{8+}}$$

-- and '''*meta-human*''' --

$$\triangle\langle {}^{1}_{\vee}\underline{h}\rangle \blacksquare \triangle\langle {}^{1}_{\vee}\circledcirc_{\flat h}\rangle \blacksquare \langle {}^{1}_{\vee}\triangle\underline{h}\rangle \blacksquare \langle {}^{1}_{\vee}\triangle\circledcirc_{\flat h}\rangle \blacksquare \langle {}^{1}_{\vee}\circledcirc_{\flat hh}\rangle \longmapsto [\![\overline{\sqcup}_{512}]\!]; \quad \sqsubset \quad \langle {}^{1}_{\vee}\circledcirc_{\flat n}\rangle^{2^{9+}}.$$

Endnotes: Volume I., PROLEGOMENA, Sec. A., *Epitome*

[Note: All **bold** *italics emphasis*, and all **bold** *italics* underscore ***emphasis***, below, has been added by **F.E.D.**]

Epitome

1. Benjamin Lee Whorf, "Science and Linguistics" in *Language, Thought, and Reality*, M.I.T. Press [Cambridge: **1956**], pp. **215-216**.

2. Benjamin Lee Whorf, "Languages and Logics", *loc. cit.*, p. **242**.

3. Archie J. Bahm, *Polarity, Dialectic, and Organicity*, World Books [Albuquerque, NM: **1988**], p. **190**; also pp. **192**, **200**, **264**, **& 265**.

4. Ernest Fenollosa, "The Chinese Written Character as a Medium for Poetry", in *Prose Keys to Modern Poetry*, edited by Karl Shapiro, Harper & Row [New York: **1962**], pp. **140-141**.

5. *Ibid.*, p. **141**.

6. *Ibid.*, p. **146**.

7. Phillip Wheelwright, *Heraclitus*, Atheneum [New York: **1968**], pp. **29**; **37**.

8. Morris Kline, *Mathematics, The Loss of Certainty*, Oxford University Press [New York: **1980**], pp. **269-277**.

9. E. J. Aiton, *Leibniz, A Biography*, Adam Higler Ltd. [Accord, USA: **1985**], p. **207**.

10. G. H. R. Parkinson, *Leibniz Logical Papers*, Clarendon Press [Oxford, U.K.: **1966**] -- Respectively, *ibid.*, pp. **90**, **93**, and **142**; p.**124**; p. **132**; and pp. **47**, **54**, **69**).

[16. A Study in ***the Calculus of Real Addition*** (after 1690), pp. **132**; **142-143**]: Watch him thinking about these issues, in ways which massively anticipate Boole as well as many others --

(**132**): *"Axiom* 2. **A ⊕ A = A**. If nothing new is added, nothing new is made; i.e. ***repetition changes nothing here***. (For although four coins and ***another*** four are eight coins, four coins and ***the same*** four already counted are not.)"

(**142-144**): *"Note to axioms* ***1*** *and* **2.** As ***general algebra*** [*speciosa generalis*] is ***merely the representation and treatment of combinations by signs***, and as various ***laws of combination*** can be ***discovered***, the result of this is that various methods of computation arise. Here, however, ***no account is taken of the variation which consists in a change of order alone***, and **AB** is the same for us as **BA**. Next, ***no account is taken here of repetition***; i.e. **AA** is the same for us as **A**. Consequently, ***whenever these laws are observed, the present calculus can be applied***. It is evident that this is observed in ***the composition of absolute concepts***, where ***no account is taken of order or of repetition***. Thus it is the same to say 'hot and bright' as to say 'bright and hot', and to speak of 'hot fire' or 'white milk', with the poets, is a ***pleonasm***; 'white milk' is simply 'milk', and 'rational man' -- i.e. 'rational animal which is rational' -- is simply 'rational animal'. It is the same ***when certain determinate things are said to exist in things***: ***real addition*** of the ***same things*** is ***vain repetition***. When two and two are said to make four, ***the latter two must be different from the former***. If they were ***the same, nothing new would result***; it would be just as if, for a joke, I wanted to make six eggs out of three by first counting three eggs, then taking away one and counting the remaining two, and finally taking one away again and counting the remaining one. But in ***the calculus of numbers and magnitudes***, **A**, **B**, or other signs do not stand for ***a certain thing***, but for ***any thing*** of the same number of congruent parts. For ***any*** two feet are signified by **2**, ***if a foot is the unit of measure***, whence **2 + 2** ***makes something new***, **4**, and **3** by **3** ***makes something new***, **9**; for it is ***presupposed*** that what are used are ***always different*** (though of the ***same magnitude***) [*and of the same '**metrical quality**', "**unit** of **measure**", or '*metrical monad*' -- F.E.D.*].

The situation is different in the case of certain things, for example lines. Let it be assumed that something moveable describes the straight line **RY ⊕ YX= RYX**, or, **P ⊕ B = L**, going from **R** to **X**. Then let us assume that the same thing goes back from **X** to **Y** and stays there; then, although it twice describes **YX** or **B**, it ***produces*** nothing else than if it had described **YX** once. So '**L ⊕ B**' is the same as **L**, i.e., '**P ⊕ B ⊕ B**', or, '**RY ⊕ YX ⊕ XY**' is the same as '**RX ⊕ YX**'. This caution is of great importance in estimating the magnitude of things which are ***generated by the magnitude of the motion*** of those things which generate or describe. For care must be taken that, in describing, one thing does not choose as its own path the track of another, or that one part of the describer does not succeed to the place of another; or there must be a subtraction, so that there is no reduplication. It is also evident from this that, according to the concept which we are using here, ***components can by their magnitudes constitute a magnitude which is greater than that of the thing which they compose***.

Hence the ***composition of things*** and ***of magnitudes*** differs widely. For example, if a straight line **L**, or **RX**, has two parts, **A**, or **RS**, and **B**, or **YX**, either of which is greater than half of **RX** -- e.g., if **RX** is five feet, **RS** four feet and **YX** three feet -- it is evident that the magnitudes of these parts will constitute a magnitude of seven feet, ***greater than the magnitude of the whole***. Yet the straight lines **RS** and **YX** compose nothing other than **RX**, i.e., **RS ⊕ YX = RX**. This is why I here designate this ***real addition*** by ⊕, as the ***addition of magnitudes*** is designated by **+**. Finally: when, in ***real addition***, one is ***concerned with the actual generation of things, it makes a great difference what the order is*** -- for the foundations are laid before the house is built. But in ***the mental formation of things*** the result is the same, no matter which ingredient we consider first (although one method of consideration may be more useful than another), so ***the order does not make any change in the thing which is produced***. In due course order also will be considered; for the moment, however, '**RY ⊕ YS ⊕ SX**' is the same as '**YS ⊕ RY ⊕ SX**'."

[15. A Study in the Plus-Minus Calculus ('A not inelegant Specimen of Abstract Proof') (after 1690), p. **124**]:
"*Axiom* 1. If the ***same term is taken with itself, nothing new is constituted***; i.e., **A + A = A**.

Note. It is true that, in the case of numbers, **2 + 2** makes **4**, or 'two coins added to two coins make four coins' but then the two which are added are ***other than*** the previous ones. ***If they were the same, nothing new would emerge***, and it would be as if we wished for a joke to make six eggs out of three, by first counting three eggs, then removing one and counting the remainder, two, and then removing one again and counting the remainder, one."

[10. Bases of A ***Logical Calculus*** (2 August 1690), p. **93**]:
"(3) **A = AA**; i.e. the ***multiplication of a letter by itself is here without effect***."

[9. The Primary Bases of a ***Logical Calculus*** (1 August 1690), p. **90**]:
"(5) **A =** (not-(not-**A**))."
"(6) **AA = A**."

[7. General Inquiries about ***the Analysis of Concepts*** and of Truths (1686), p. **56**]:

"(18) From the nature of the symbolism **A**, **AA**, **AAA** &c. coincide -- or 'man', 'man man' and 'man man man'. So if anyone should be called both a man and an animal, by analyzing 'man' into 'rational animal' he will be called equally a rational animal and an animal, i.e., a rational animal."

"(26) We must note something else about ***this calculus*** which we should have stated earlier: namely, that what is generally asserted or concluded, not as an hypothesis, about any letters which have not yet been used, is to be understood of any number of other letters. So if **A = AA**, it will also be possible to say **B = BB**."

(p. **47**): "Not-not-**A** is the same as **A**."
(p. **54**): "Not-not-**A** and **A** coincide; so if not-**A** and **B** coincide, not-**B** and **A** will also coincide."
(p. **69**): "(96) Not-not-**A** = **A**."

"Early in February **1664** Leibniz graduated as Master of Philosophy with a dissertation *Specimen quaestionum philosophicarum ex jure collectarum* . . . Among the theorems are the following:

1 If the hypothesis is posed, the thesis follows
2 If the thesis is suppressed, the hypothesis is suppressed

Leibniz notes that a hypothetical judgment affirms nothing categorically, neither the hypothesis nor the thesis. In application to law, he considers the case of a law subject to a certain condition. If this condition is impossible, the law is null. If the condition is necessary (and therefore certainly satisfied), the law is absolute. If the condition is contingent or uncertain, the law is conditional. These results are set out in the following table, which is remarkable for the numerical values of **0**, **1**, and **1/2** given to laws which are null, absolute and uncertain respectively. The symbol **1/2**, he notes, stands for ***some fraction between*** **0** and **1** . . . . ***There is just a suggestion here of a calculus of probabilities***. However, neither this novel idea, nor that of conditional judgments depending on other judgments (that is, the secondary judgments ***introduced again by George Boole in the nineteenth century***) appear again in Leibniz's logical writings . . ."

Anna Teresa Tymieniecka, *Leibniz' Cosmological Synthesis*, Van Gorcum Ltd. [Assen, The Netherlands: **1964**]:

(p. **45**): "Leibniz confesses to having believed in his youth in atoms. His main reason for rejecting them was their uniformity. ***As a uniform plurality they could not account for the variety of phenomena***; as the basic buildingstones of the phenomenal world, they could not explain the unity of its varied structures; ***nor could they function as a ground for an infinitely graduated continuity of phenomena***." [cf. Plato's «***arithmoi monadikoi***» versus his «***arithmoi eidetikoi***» per: Jacob Klein, *Greek Mathematical Thought and the Origin of Algebra*, Dover [New York: **1992**], pp. **61-99**.].

On Leibniz' conception of a "*Characteristica Universalis*", see:

(i) N. I. Styazhkin, *History of Mathematical Logic from Leibnitz to Peano*, M. I. T. Press [Cambridge: **1969**] pp. **56-92** [*et passim*.];

(ii) C. I. Lewis and C. H. Langford, "History of Symbolic Logic" in James R. Newman, ed., *The World of Mathematics*, vol. **III**.

(iii) Morris Kline, *Mathematical Thought from Ancient to Modern Times*, vol. **3**, [New York: Oxford University Press, **1972**], pp. **1187-1188**.

The conception that ideographic or "symbolical" algebras are not limited to "purely quantitative" interpretation; that algebraical symbols can represent other than ["real"] numbers; that: for example, they may directly represent mental processes, "***mental operations***", is also found to a highly developed degree in:

(iv) George Boole, *The Mathematical Analysis of Logic: Being An Essay Towards A Calculus of Deductive Reasoning*, Barnes & Noble [New York: **1965**], pp. **3-7**.
See also: George Boole, *An Investigation Of The Laws Of Thought On Which Are Founded The Mathematical Theories Of Logic and Probabilities*, Dover [New York: **1958**], pp. **5-17**.