SUPPLEMENTS TO THE F.*E*.*D*. INTRODUCTORY LETTER

An Introduction to Dialectical Arithmetic:

A Primer for *Dialectical Ideography*.

Supplement A. *Foundations of Dialectical Arithmetic*.

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Zoom-In Map of *Supplement A*.

Supplement A.: *Foundations of Dialectical Arithmetic*

■ **The Foundations of [Quanto-]Q*ualitative Arithmetic* and the Missing 'Half' of Math**. The key to the whole development of *Dialectical Arithmetic* is the relation we denote by a 'neo-ideo-gram' of our own 'coinage', but one which carries forward the implicit 'picto-ideo-*gram*-matical' rules of the predecessor 'picto-ideograms', namely '$\not\gtreqless$', wherein the vertical line or "slash", |, drawn all through the stacked '>', '=' [*reduced to* '–' *as per convention with* ≥ & ≤], & '<' signs, indicates their conjoint denial. Therefore, our new 'picto-ideogram' '$\not\gtreqless$' denotes a relation of '***non***-*quantitative inequality*', i.e., that of a '***non***-*quantitative difference*' or '***qualitative*** *difference*' between two 'relata'.

This new symbol [re-]marks a portal into a vast but hitherto largely unnoted universe of arithmetical ideas, and of new mathematical insights generally, through which the human mind that considers it may conduct itself. The key that unlocks this doorway is the noticing of something that -- though already silently pervasive even in its present 'un-development' and 'unnoticed-ness' -- goes, for the most part, as if unseen, or which is shunned and avoided in those fleeing moments in which it may obtrude into the prevailing consensus consciousness regarding mathematics. This key is the becoming [*re*-]sensitized to an aspect of thought-reality to which an entire civilizational and cultural mentality has become increasingly *de*sensitized. It is the opening to and the willing confrontation with the difficulties and the challenges -- the gauntlet -- which the systematic admission of this facet of mathematical and conceptual experience throws down. This key is also the fleeing no more from the, at first, seemingly overwhelming demands that accompany such a re-sensitization. It is to allow the force of that 'demandingness' to ignite a new and accelerated *irruption of ideas*; of new arithmetical/mathematical 'inventions/discoveries', a new outbreak of '***ideo-onto-dynamasis***'.

A Note on Notation. We delimit *major hypotheses* -- typically textual, and denoted generically here by ellipsis dots, '...' -- as follows: ■ ... ■ [*though most of the material not so enclosed also remains conjectural*], vs. ***theorems***, derived deductively from explicit premises, via ▐... ▌. *Single quote-marks* enclose '***self-quotes***' of our *own coinages; double quote-marks,* ***exact quotes*** of *others*. *Triple quote-marks* enclose *paraphrased quotes* of *others*. Double 'angle' marks, «...», enclose words of languages other than English, whether transliterated or rendered in their native alphabets. Additional notation is defined below as and where the need for it arises.

Genesis of a Symbol for Qualitative Inequality. The implicit ubiquity of the '$\not\gtreqless$' relation, the relation of '***qualitative inequality***', is perhaps most readily 'explicitized' initially via the various colloquialisms regarding "apples vs. oranges", or, per our 'symbolizational' conventions, in the '*in*equation': '**apples** $\not\gtreqless$ **oranges**'. In these colloquialisms, the 'seeings', the critiques, and the caveats or warnings that found *Dialectical Arithmetic* are already emergent and manifest in the collective consciousness and "natural language" of contemporary Terran humanity. Clearly, our perception of the inequalities between a particular apple and a particular orange, as between the "ontological categories" or "classes of *kinds* of things" called '**apples**' versus '**oranges**', is immediately not a matter of "*greater than*" or "*lesser than*." The inequality '**apples** ≠ **oranges**' is clearly not intended to assert that '**apples** $\gtrless$ **oranges**'. *Neither* are '**apples** > **oranges**', *nor* are '**apples** = **oranges**', *nor* are '**apples** < **oranges**'. We may more compactly summarize this triple series of denials by writing ¬[**apples** $\gtreqless$ **oranges**], with the ' ¬ ' ideogram denoting 'not', and, more compactly still, by introducing the 'slash-denial' of '$\gtreqless$' as the 'combineal' ideogram '$\not\gtreqless$', and by writing out the expression '**apples** $\not\gtreqless$ **oranges**'. The foregoing summary assertion thus provides us with a new ideographic sign of relationship to work with and to contemplate: '$\not\gtreqless$'. The perceived difference between the category we denote by '**apples**' and the category we denote by '**oranges**' is, in its totality, and in its unity, a difference of *kind*, a difference of ***quality***, not a difference of ***quantity***. This difference is ***not*** one of *differing quantities* of ***the same*** *quality*, of the *same* ***kind*** of thing. Inequalities like '**two apples** ≠ **four apples**' present *a difference of* ***quantity*** *within* the *same* ***quality***, namely, within the 'quality' or 'ontological category' of 'apple-ness'. They stay '*within the* ***same quality***'; because they contrast different '*full-multiplicity* ***quantifiers***', **two** vs. **four** in this example, which are being applied to the same '*ontological* ***qualifier***', here, to the '*ontological* ***qualifier***' or '*kind* ***qualifier***' denoted by '**apples**'. Thus, we here have '**2 apples** $\gtrless$ **4 apples**', and, indeed, '**2 apples** < **4 apples**'.

So likewise does an inequality like '**two oranges** $\neq$ **four oranges**', but *not* an inequality like '**two apples** $\neq$ **two oranges**'. The ***qualitative difference*** presented by the latter is what we call an '***ontological***' *difference*, given our perspective of 'totality-ism' or '*ontological* ***non****-reductionism*' [i.e., we do not '"reduce"' both apples and oranges to '"mere collections of quarks and other "elementary" "particles" '"]. Thus, our phonetic ['phonogramic', or 'phoneticographic'] word-symbols '**apples**' and '**oranges**' denote, in this perspective, '***ontological categories***' or, for short, '***ontos***'. They are *not* two of *a* kind. They are ***of two kinds***, two *different* kinds.

They denote two *different* '*ontos*'. They are ***ontologically different*** from one another. They ***differ ontologically***, when compared in ***duo*** quantity, as above, just as when compared in *any other non-zero* ***quantity***. The expression '**two apples** $\neq$ **two oranges**' is an "***inhomogeneous***"/"***dimensionally heterogeneous***" one. In short, the two 'relata' differ ***non****-quantitatively*. They differ ***ontologically***.

But ***how*** could one construct an 'arithmetic' on the foundation of the above observations; one that would be inclusive of such '*ontological*' and *qualitative* observations and relations? Moreover, ***why*** would one want to or need to? Part of the answer to the latter question, we hold, arises from observing the vast immanence, the immense but silent, unheralded ubiquity of the '$\overset{\not>}{\underset{\not<}{\neq}}$' relation even in *contemporary* mathematics; even in the mathematics already extant.

A Psycho-Historical [Self-]Experiment that You Can Perform [Only Once]: The '"Mesmerism"' Of Monetary Value. We attribute the '***Psycho-Historical Force***', or '***Ψ-Force***', driving '***The Elision of the Qualifiers***' in modern arithmetic, and in modern mathematics in general, to '***The Mesmerism of Exchange-Value***'. 'Qualifiers' -- ontological, metrical, and otherwise -- have gone missing from explicit representation, from explicit written symbolization, in our '"purely-quantitative"' mathematical notations, though they remain, of course -- and necessarily so -- there, but hidden in implicitude. The 'qualifiers' have been elided from both our ideography and our ideas. There is a psycho-historical experiment that you can perform here and now, using your own inculcated, acculturated, exchange-value-permeated «***mentalité***», if your social self-identity is a psycho-historical 'self-specimen' of the prevailing social consciousness in this regard. But it is a 'self-experiment' which you can perform fully only once, because you will have changed your self -- your cognition -- after you first perform it, as a consequence of performing it. That is, performing this reflection, you may experience a shock of 're-cognition', a cognitive expansion or regeneration -- a healing from a kind of partial blindness; a restoration of a kind of insight, or even, possibly, the seed of a cognitive revolution. Consider the following propositions:

Yes, **1 = 1 = 1 = . . . = 1**, but also **1 cm.** $\overset{\not>}{\underset{\not<}{\neq}}$ **1 gm.** $\overset{\not>}{\underset{\not<}{\neq}}$ **1 sec.** ...

Yes, **3 gm.** $\neq$ **2 gm.**, and **3 gm.** > **2 gm.**; so too **3 cm.** $\neq$ **2 cm.**, and **3 cm.** > **2 cm.**,

but also **3 gm.** $\neq$ **2 cm.**, and **3 gm.** $\not>$ **2 cm.**, and **3 gm.** $\overset{\not>}{\underset{\not<}{\neq}}$ **2 cm.**

The latter clauses, involving the '***qualitative inequality***' ***relation***, denoted '$\overset{\not>}{\underset{\not<}{\neq}}$', are true, yet theirs is a truth that is very difficult for many modern humans -- yet not so difficult for typical ancient, e.g., Hellenistic humans -- to notice, or to discern initially at first explicit encounter. The symbols '**cm.**', '**gm.**', and '**sec.**' do not denote "*quantities*" or '*quantifiers*'. They denote '***qualifiers***' -- '***metrical*** ***qualifiers***'. They denote qualitative ***units*** -- or 'monads' -- ***of measure***[ment], for different '"dimensions"' of our experience of reality, namely, the spatial extent/distance, weight or mass, and time '"dimensions"'. Thus, expressions like '**2 cm.**', '**3 gm.**', and '**1 sec.**' are ***not*** "purely quantitative" expressions. They are '***quanto-qualitative***' expressions.

The habitual and habituating experience of money-mediated exchange, exchange of exchange-values for exchange-values, seems to equate, in its '"interchanges of equivalents"', qualitatively different, heterogeneous, goods/commodities, by means of currency units of value which increasingly appear to be [especially after the emergence of ***paper*** *money*] 'quality-less'; '"purely quantitative"'. The quality behind value -- both its ***real ontological qualifier*** and its ***real metrical qualifier*** -- are '***social noumena***' in the sense that, for '"alienated"' ['sold'], and '"self-alienating"' ['self-selling'] humanity, they are unknowable sensually, via immediate appearances at '"the surface of society"'. They become knowable only via intuition/theory/social science/critique. The human roots of monetary value, in the '"time-binding"' of living human time as creative-productive life-activity, constrained to the expanded reproduction of capital-value, and metrically qualified in temporal units, remains veiled for humanity-in-alienation.

The Ubiquity of Qualitative Inequality -- The Already Extant, But Latent Ubiquity of Qualitative Difference in Standard Mathematics and Mathematical Logic. So far, we have been grounding our encounter with the '$\overset{\not>}{\underset{\not<}{\neq}}$' relation by reference to 'ontological qualities', 'predicates', or "intensions" like 'apple-ness' and 'orange-ness'. We have referenced them by using our standard 'phonetic-literal', word, or 'phono-gram-ic' symbols like '**apples**' and '**oranges**', which we call 'ontological ***qualifiers***', to stand for the perceptible qualities of our experiences of *real* apples and *real* oranges. Perhaps then, our easiest «***entrée***» into an encounter with the mathematics of '$\overset{\not>}{\underset{\not<}{\neq}}$' -- with the ubiquity of the '$\overset{\not>}{\underset{\not<}{\neq}}$' relation even within the mathematics *presently* officially recognized as such -- is to consider the mathematics of logic; the '"calculus of predicates"' that is, the "first-order predicate calculus" of modern "*symbolic*" formal logic, or '*ideographic* formal [ostensively ***non-dialectical***] logic'.

Indeed, in general, any two distinct symbolic or ideographic predicates bear this '$\overset{\not>}{\underset{\not<}{\neq}}$' relation to one another. If the "intension" of a "unary" predicate symbol, or '"one-place"' predicate symbol, $\mathbf{G}^1$, is the color Green, and if that of another unary predicate symbol, $\mathbf{S}^1$, is the quality of human taste-perception known as Sweet, then $\mathbf{G}^1 \not> \mathbf{S}^1$ and $\mathbf{G}^1 \not\equiv \mathbf{S}^1$ and $\mathbf{G}^1 \not< \mathbf{S}^1$, so ∴, in summary, $\mathbf{G}^1 \overset{\not>}{\underset{\not<}{\neq}} \mathbf{S}^1$.

Now of course, things look different in terms of the "extensions" of the different "intensions" of these two '"predicate letters"' -- *the set of all* ***green*** "objects" [or *of all* ***green*** "logical individuals"] in our "universe of discourse", relative to *the set of all* ***sweet*** objects or *of all* ***sweet*** "individuals" in that universe. The "cardinality", the count of the membership of the "extension" of this "intension", $\mathbf{G}^1$, here denoted $|\ \mathbf{G}^1\ |$, may be greater than, or less than, or *maybe* even exactly equal to the "cardinality" of the "extension" of the other "intension", $\mathbf{S}^1$. We may symbolize this by asserting [employing the Fregean/Russellian '$\vdash$.' as an '***assertion without proof sign***' [$\vdash$]/*parentheses-substitute* or ***enclosure pre-sign*** [.]']: $\vdash . \ |\ \mathbf{G}^1\ | \ \gtrless \ |\ \mathbf{S}^1\ |$.

Note that, in this predicate calculus, the juxtaposition of a unary predicate symbol to the symbol for a logical individual denotes the *attribution* of the predicate 'intended' by that predicate-symbol to the individual 'intended' by that individual-symbol: $\mathbf{X}^1\mathbf{x}$ means that '$\mathbf{x}$ is an $\mathbf{X}$' or, in extensional terms, that $x \in X$; that ***x is an "∈lement" of*** the set or extension, denoted X, of $\mathbf{X}$ [wherein the *ideogramic* '$\mathbf{e} \in \mathbf{S}$' abbreviates for the *phonogramic* statement '**e** *is an element of the Set* **S**']. Thus, if **g** denotes a particular logical individual within our universe of discourse -- so that ***g*** denotes a "member" or "element" of the "extension" of that universe, of its "universal set", ***U*** -- which exhibits, among its attributes, that of '''green-ness''', of looking green to us, and if **s** denotes another such logical individual, one which exhibits the quality of '''sweet-ness''', i.e., which tastes sweet to us, then $\vdash .\ \mathbf{G}^1\mathbf{g}$, is a true assertion for us, as is $\vdash .\ \mathbf{S}^1\mathbf{s}$. '''**Reducing**''' these quite distinct assertions to their "truth-***values***", we may write $\| \mathbf{G}^1\mathbf{g} \| = \| \mathbf{S}^1\mathbf{s} \| = .\mathrm{T}.$, in which the symbol '**.T.**' denotes the "truth-***value***", or '*logical meta-number* ***value***' "True", corresponding to the value '$\mathbf{1_B}$' in Boolean 'logical arithmetic'. But the predicate *concept* or "intension" denoted $\mathbf{G}^1$, is, in itself, ***qualitatively different*** in its denotation from that of the "intension" denoted $\mathbf{S}^1$. The assertion denoted $\vdash .\ \neg[\mathbf{G}^1 \gtreqless \mathbf{S}^1]\ \&\ [\mathbf{G}^1 \neq \mathbf{S}^1]$ is a true assertion ['$\neg$' stands for '*not*']. Moreover, the truth of $\neg[\mathbf{G}^1 \gtreqless \mathbf{S}^1]$ does not imply that $\mathbf{G}^1 = \mathbf{S}^1$. There is another possibility: $\vdash .\ \mathbf{G}^1 \underset{\nless}{\overset{\ngtr}{\neq}} \mathbf{S}^1$. The recognition of this ***fourth***, *qualitative* relational possibility, '$\underset{\nless}{\overset{\ngtr}{\neq}}$', in addition to the conventional ***three***, '**>**', '**=**', **&** '**<**', thus expands or "extends" the '*pure-quantitative*' "***trichotomy principle***", which ***explicitly*** pervades contemporary mathematics, to a presently ***implicit*** and '*quanto-qualitative*' '***tetra-chotomy principle***'. The ***trichotomy principle*** holds that, for any constituents **x**, **y** of the **R**eal numbers, $\mathbb{R}$, or of any other such "*totally-ordered*" ['*rectilinear*'] "*number-set*"/"*number-space*", *always either* $\mathbf{x} > \mathbf{y}$, *or* $\mathbf{x} = \mathbf{y}$, *or* $\mathbf{x} < \mathbf{y}$. In summary, given that $\mathbf{x}, \mathbf{y} \in$ "Numbers" with "*total order*", then, per the ***trichotomy principle***, we will have ***only*** the three possibilities expressed ideogramically by $\mathbf{x} \gtreqless \mathbf{y}$, wherein the relation symbol '$\gtreqless$' is formed by '*stacking*' the symbols **>**, **=**, **&** **<** atop one another in "totem pole" fashion to form a new, single symbol. But, as we have just elaborated, there is a fourth possibility, beyond this "set" of three relations, that set being denoted by the "extension" $\{ >, =, < \}$ [Note: In fairly standard fashion, we use 'curly parentheses' or "braces" to enclose symbols which specify the content of a "set" or "space"]; a possibility which we denote by the 'stack' $\underset{\nless}{\overset{\ngtr}{\neq}}$, short for $[\ \ngtr\ \&\ \neq\ \&\ \nless\ ]$. I.e., *it is useful to assume* that the '$\neq$' relation '***genus***' comes in [at least] two distinct [sub-]varieties or '***species***', namely '$\gtreqless$' & '$\underset{\nless}{\overset{\ngtr}{\neq}}$', the latter being a '***neti neti neti***' or '**not & not & not**' category of relationship: **not** '**>**' and **not** '**=**' and not '**<**'.

Arithmetical Monads, 'Boolean' versus 'contra-Boolean'. Consider the standard 'hybrid' arithmetic of "real" and "imaginary" numbers, the so-called "**C**omplex" arithmetic of $\mathbb{C}$, with its *two* distinct arithmetical *units* or *monads*, $+1 \equiv \mathbf{r}$ and $+\sqrt{-1} \equiv +\mathbf{i}$. Again, we have that $+\mathbf{i} \ngtr +1$ and that $+\mathbf{i} \neq +1$ and $+\mathbf{i} \nless +1$. Thus, we write $+\mathbf{i}\ \underset{\nless}{\overset{\ngtr}{\neq}}\ +1$, which relation, '*analytical-geometrically*', maps to one of the mutual perpendicularity, orthogonality, or '''linear independence''' of the associated 'directed unit intervals' $[0, +\mathbf{i}] \perp [0, +1]$. Note also that, $+\mathbf{i}^2 \underset{\nless}{\overset{\ngtr}{\neq}} +\mathbf{i}$, as $+\mathbf{i}^2 \equiv -1$, & $-1 \underset{\nless}{\overset{\ngtr}{\neq}} +\mathbf{i}$, and also $[0, -1] \perp [0, +\mathbf{i}]$. Numbers whose operations, as with **0** and **+1**, are described by the rule $\mathbf{x}^2 = \mathbf{x}$ $[0^2 = 0;\ 1^2 = 1]$ of Boole's "*Fundamental Law of Thought*", we call '***Boolean numbers***'. Numbers which, on the other hand, are described by the 'contra-rule' $\mathbf{x}^2 \neq \mathbf{x}$, i.e., $\mathbf{x}^2 \gtreqless \mathbf{x}$ or $\underline{\mathbf{x}}^2 \underset{\nless}{\overset{\ngtr}{\neq}} \underline{\mathbf{x}}$, we call '***contra-Boolean numbers***'. Those whose *operatorial behavior* is described by the latter, $\underline{\mathbf{x}}^2 \underset{\nless}{\overset{\ngtr}{\neq}} \underline{\mathbf{x}}$, we term '***strongly contra-Boolean***'.

Thus, already $\mathbf{i} \in \mathbb{C}$ exemplifies such '***strongly contra-Boolean***' numbers/operators. Moreover, these 'contra-Boolean' **i**-numbers are needed for the "closure" of ordinary algebra – for the general solvability of ordinary algebraic equations, and for the "Fundamental Theorem of Algebra" to be true. However, as we shall see, the '***convolute***' **i**-numbers, along with the rest of the '***convolute***' "hypernumbers" – the "quaternions", "octonions", 'Musean' hypernumbers, etc. – constitute but the barest beginnings and the leanest intimations of the vast potential realm of '***contra-Boolean numbers/arithmetics***' and of their operatorial logics, especially of the conceptually possible realms of the '***evolute***' "hypernumbers", hitherto largely uncharted.

Arithmetical Monads, 'Convolute' versus 'Evolute': An Arithmetic of the «*Aufheben*» Operation. The predicates '***convolute***' and '***evolute***' as employed herein are metaphoric for some of the differing spiral motifs among seashells. In a '*convolute*' spiral shell, succeeding whorls of the shell cover-over and hide from view all of the preceding whorls, because all whorls remain in and expand into the same, 'horizontal' plane. In an '***evolute***' spiral shell, succeeding whorls rise up, vertically, out of the horizontal planes of the past whorls, so that the *succeeding* along with all of the *preceding* whorls both remain uncovered and in-view together, at each stage of the growth of the shell and of its inhabitant. This 'evolute' shell growth pattern provides a useful metaphor for the "'conservation moment'" and movement of the core-dialectical «***aufheben***» operation.

The product of two [*hyper*]*number* units mutually applied, or of a single unit [*hyper*]*number* applied to itself, we call '***convolute***', or "***linear***" [see M. Kline, ***Mathematical Thought From Ancient to Modern Times***, vol. **2**, Oxford U. Press [NY: **1972**], p. **793**], and call the [*hyper*]*numbers* involved '***convolute***', or "***linear*** [***hyper***]***numbers***", if $\underline{\mathbf{x}} \gtrless \underline{\mathbf{y}} \gtrless \underline{\mathbf{z}}$, and $\underline{\mathbf{x}} \cdot \underline{\mathbf{y}} = \underline{\mathbf{z}}$, or $\underline{\mathbf{y}} \cdot \underline{\mathbf{y}} = \underline{\mathbf{y}}^2 = \underline{\mathbf{z}}$, and neither $\underline{\mathbf{x}}$ nor $\underline{\mathbf{y}}$ is additively '*visible in*' $\underline{\mathbf{z}}$, meaning that neither $\underline{\mathbf{x}}$ nor $\underline{\mathbf{y}}$ is an *explicit* ***additive*** *part of* $\underline{\mathbf{z}}$. Here, both $\underline{\mathbf{x}}$ and $\underline{\mathbf{y}}$ in $\underline{\mathbf{x}} \cdot \underline{\mathbf{y}}$, and $\underline{\mathbf{y}}$ alone in $\underline{\mathbf{y}} \cdot \underline{\mathbf{y}}$, disappear into $\underline{\mathbf{z}}$. Thus **i** is a 'convolute' "hypernumber", viz.: $\mathbf{i} \cdot \mathbf{i} = -\mathbf{1}$. On the other hand, if $\underline{\mathbf{x}} \gtrless \underline{\mathbf{y}} \gtrless \underline{\mathbf{f}}[\,\underline{\mathbf{x}}, \underline{\mathbf{y}}\,] \gtrless \underline{\mathbf{x}}$, and $\underline{\mathbf{x}} \cdot \underline{\mathbf{y}} = \underline{\mathbf{z}} = \underline{\mathbf{y}} + \underline{\mathbf{f}}[\,\underline{\mathbf{x}}, \underline{\mathbf{y}}\,]$, or if $\underline{\mathbf{y}} \cdot \underline{\mathbf{y}} = \underline{\mathbf{y}}^2 = \underline{\mathbf{z}} = \underline{\mathbf{y}} + \underline{\mathbf{f}}[\,\underline{\mathbf{y}}, \underline{\mathbf{y}}\,]$, i.e., such that the operand, $\underline{\mathbf{y}}$, re-appears ***additively*** in the product, then we term $\underline{\mathbf{x}}$ and $\underline{\mathbf{y}}$ '***evolute***' "[hyper]numbers", and their product an '***evolute***', or "'***nonlinear***'", '***product***'.

Note that, in the case above, the evolute hypernumbers' mutual operation or self-operation models the Hegelian «***aufheben***»-*negation* principle of ***conservation*** *with*[*in*] ***transformation***. This is because the "multiplicand" or "operand" or "argument" is 'conserved' within the "product" it forms via this mutual interaction with the "multiplicator" or "operator" or "function" which was applied to it. The operand or argument is conserved via an additive reappearance, a 'heterogeneous addition' or "non-amalgamative addition" to the qualitatively unequal, transformational other part of the product or result, «*á la*» the addition in $\mathbf{+1 + (+i) = 1 + i}$.

The 'Tetra-Chotomy' Principle: A Generalization of the "'Trichotomy Principle'". Thus, the set of possible arithmetical/mathematical relations expands by one element, from that of the standard *trichotomy principle*, $\{\, >, =, < \,\}$, a "set" of "cardinality three", to a "*non-standard*" "set" of "cardinality four", $\{\, >, =, <, \gtrless \,\}$.

The cardinality of the "set" $\{\, >, =, < \,\}$ is, indeed, three, $|\{\, >, =, < \,\}| = \mathbf{3}$, and is thus less than that of $\{\, >, =, <, \gtrless \,\}$, i.e., $\vdash . |\{\, >, =, < \,\}| = \mathbf{3} < \mathbf{4} = |\{\, >, =, <, \gtrless \,\}|$. However, note that that these "*sets*" *themselves* bear the '$\gtrless$' relation to one another: $\vdash . \{\, >, =, < \,\} \;\gtrless\; \{\, >, =, <, \gtrless \,\}$.

This is so, even though, in «***aufheben***» fashion, the "set" $\{\, >, =, <, \gtrless \,\}$ "includes", or "contains" [as a ***subset***, *not* as an *element*], and "***conserves***", as it also [ac]***cumulates*** *beyond*, or '***contentally***', '***qualitatively***', and ***ideo-ontologically***' '*exceeds*', '*supersedes*', and '*surpasses*', the "set" $\{\, >, =, < \,\}$.

We may symbolize such relations of one set's inclusion or '**C**ontain-ment' in another set by formulae like: **⊢ . { >, =, < } C { >, =, <, ≹ }**, or, standardly, by **⊢ . { >, =, < } ⊂ { >, =, <, ≹ }**. But **F.*E*.*D*.** also employs a non-standard symbol, '⊏', to formulate more general kinds of '*containment*', including "['***evolute***'] ***conservation***", which might be generically characterized as '***system/sub-system containment***', or as «*aufheben*» '***successor/predecessor containment***', as in

'*predecessor-system* ⊏ *successor-system*',

so that we may also write: **⊢ . { >, =, < } ⊏ { >, =, <, ≹ }**. Our meaning for '⊏' includes, but also exceeds, the standard meaning of '⊂'. Thus, **⊢ . {a} ⊄ { {{a}, {b}}, {{c}, {d}}, ... }**; & indeed, **⊢ . {a} ∉ { {{a}, {b}}, {{c}, {d}}, ... }**, but **⊢ . {a} ⊏ { {{a}, {b}}, {{c}, {d}}, ... }**.

So, we may also write: **⊢ . '⊂' ⊏ '⊏'**: i.e., asserting that the '⊂' *type of containment* is also *wholly* '⊏'-*contained* in the '⊏' *type of containment*. But, **⊢ . '⊏' ⊄ '⊂'**, and **⊢ ¬['⊏' ⊏ '⊂']**.

Note that, therefore, **{ >, =, <, ≹ }** is, indeed, '"bigger"' than **{ >, =, < }**, but in a very different sense from that in which **4** (= |{ >, =, <, ≹ }|) is '"bigger"' than **3** (= |{ >, =, < }|). The domain of relations denoted by **{ >, =, <, ≹ }** is ***conceptually***, and '***ideo-ontologically***' ***bigger*** than the domain of relations denoted by **{ >, =, < }**. In fact, the relations which are constituent of these two domains bear the '≹' relation to one-another:

⊢ ['>' ≹ '='];   **⊢ ['>' ≹ '<']**;   **⊢ ['>' ≹ '≹']**;

⊢ ['=' ≹ '<'];   **⊢ ['=' ≹ '≹']**,

&

⊢ ['<' ≹ '≹'].

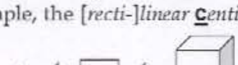
Note too that the two kinds of inequality mutually bear the '≹' relation: **⊢ ['$\gtrless$' ≹ '≹']**, *and* that the '***meta**-relation*' or '***relation*** [made up out] ***of*** [multiple] ***relations***', denoted by '$\gtreqless$', and which comprehends the entire gamut of "*purely*" *quantitative* relations, bears the '≹' relation to that other '***meta***-relation' *made up out of multiple relations*, namely, the relation '≹' of qualitative inequality itself: **⊢ ['$\gtreqless$' ≹ '≹']**. [Not only do *relations have relations; meta-relations also have* [meta-]*relations, and* '≹' *is typically among those relations*].

So ubiquitous is ***the qualitative relation*** that *even* ***the quantitative relations*** bear it -- bear ***the qualitative relation*** -- among themselves, to one another.

Metrical Monads: The 'contra-Boolean', 'Convolute', and *To-This-Day* Still "Syncopated", Arithmetic of Dimensional Analysis. We can see that { >, =, <, $\ngtrless$ } is '''more''' than { >, =, < }, but in a very different sense from that in which **4** is '''more''' than **3**. It is just so, in the domain of ***units of measure***, the domain of the units or "monads" of the "***physical dimensions***" which our quantifiers may count or quantify when applied concretely. This is the domain of what physicists term "***dimensional analysis***". In it, a similar distinction abounds. We can see that, e.g., in the metrical sub-domain of units of *length*, one square centimeter is different than and '''more''' than one linear centimeter, and one cubic centimeter is different than and '''more''' than one square centimeter. Yet the former sense of 'excession' is a very different one from that in which *three linear* **c**enti**m**eters exceeds *two linear* **c**enti**m**eters, and from that in which *two linear* **c**enti**m**eters exceeds *one linear* **c**enti**m**eter. I.e., denoting *linear* **c**enti**m**eter by '**cm.**^{1}' via "syncopation" [abbreviation], we have –

, or, **(3)·[cm.^{1}] > (2)·[cm.^{1}] > (1)·[cm.^{1}]**;

all "*quantitative*" relations, but between '*quanto-qualitative*' terms, products of the '***generalized multiplication***' of a '*quantifier*' [e.g., **(3)**, **(2)**, or **(1)**] '''into''' a '*qualifier*' [e.g., **[cm.^{1}]**], via different counts of *the same unit*, the same '***metrical monad***' or '***metrical qualifier***', namely, in this example, the [*recti-*]*linear* **c**enti**m**eter.

However, we also have -- , or: **(1)[cm.^{1}]** $\ngtrless$ **(1)[cm.^{2}]** $\ngtrless$ **(1)[cm.^{3}]**.

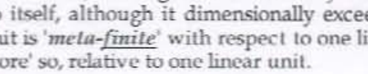
Indeed, by some idealizations, **cm.^{1}** $\sqsubset$ **cm.^{2}** $\sqsubset$ **cm.^{3}**, in the sense that there are an "***infinite***" number of **cm.**-long "*line-segments*" contained in one square-centimeter '*plane-segment*', and an "***infinite***" number of square-centimeter '*plane-segments*' contained in one **cubic-cm.** '*solid-segment*' or '*volume-segment*'. Here also we encounter an intimation of the '***meta-finite***' resolution of those mathematical idealizations that seem to surface "singularities", and other 'unphysical' "infinities" as outcomes to ontology-changing, 'onto-dynamical' operations. While square units clearly do, in the sense of dimensionality, "exceed" and "transcend" linear units, and while cube units, in turn, "dimensionally exceed" and "transcend" square units, such that the former may seem, in some idealizations, "transfinite" with respect to the latter, the former always ***remain finite*** in the 'self-relative' sense. The 'idea-onto' of one square unit still means just "one unit", *not "infinity"*. The 'ideo-onto' of one cubic unit still means just "one unit", *not "infinity"*. One square unit is ***finite*** relative to itself, although it dimensionally exceeds [and yet also in a sense contains] one linear unit. One cubical unit is ***finite*** relative to itself, although it dimensionally exceeds [and yet in a sense also contains] one square unit. We therefore say that one square unit is '***meta-finite***' with respect to one linear unit, and that one cubic unit is '***meta-finite***' relative to one square unit, as also, even 'more' so, relative to one linear unit.

The 'self-multiplication' or 'self-product-*tion*' of our unit of length, '**cm.**', i.e., **cm. × cm.** or **cm.^{2}**, produces a new monad; adds a new '''***dimension***'''; irrupts ***new ontology***, creates a new '***ontological category***', or '*onto*', of measure-ment, a new and higher 'species' of the 'genus' of metrical unit(s)(y), of metrical monads; is itself a new, higher unit in its own right, a higher-level unit, a unit of higher dimensionality, a "**2**-dimensional" unit rather than a "**1**-dimensional" unit, a unit of dimensionality **2** rather than of dimensionality **1**, in short, a unit of *area*, the '**sq. cm.**'. Thus, the '**sq. cm.**' is, in our terms, per our '***Method Of Flexions***', i.e., of '*generalized multiplication*', '*generalized operation*', or '*generalized function-ing-s*', a '***meta***-unit' or '***meta***-monad', a 'unit of higher degree', of degree **2**, rather than of degree **1**; a unit of ***area*** *made up from / as product of* ("two") units of ***length***, or via the ***self-multiplication***, *self-operation*, '*self*-**de-*flection***', '*self-flexion*', '*auto-flexion*', '*re-flexion*', or '*self-reflexion*' -- connoting the ***bending*** ["*flex-ion*"] ***back upon*** ['*re*'] ***self*** -- of a ("one single") unit of length. Likewise and next in order, the '*re-multiplication*' or '*co-product-tion*' of that new unit, of the '**sq. cm.**' or '**cm.^{2}**', with '**cm.^{1}**', i.e.,

$$\mathbf{cm.}^1 \times \mathbf{cm.}^2 \;=\; \mathbf{cm.}^{1+2} \;=\; \mathbf{cm.}^3 \;\ngtrless\; \mathbf{cm.}^2,\ \mathbf{cm.}^1,$$

yields further ***new ontology***; yields yet a new monad, higher still; is itself a new unit in its own right, a unit of higher-level, a unit of higher degree, a unit of higher dimensionality, a unit of dimensionality **3**, in short, a *solid* unit, the '**cu. cm.**'. The '**cu. cm.**' is thus, again, in our terms -- in terms of the '***Method Of Flexions***' -- a '***meta***-unit', that is, a '***meta***-monad', a 'unit of higher degree', of degree **3**, rather than of degree **2** or of degree **1**; a unit of '*solidity*' or of '*capacity*' / "*volume*", *made up from* / out of / *as product of* a unit of length and a unit of area, or by way of the *mutual* operation, '''*co-operation*''', '[*co*]-*flexion*', or '*allo-flexion*' [***mutual "bending"***; *mutual* '**de-*flection***'; ***mutual alteration***] of those two kinds of [mutually ***qualitatively*** *distinct*] units or monads, to yield a third new kind of unit, distinct in kind or quality or '***dimensionality-ontology***' from both of its "parent" units; therefore ***qualitatively*** *unequal* to each of them.

Indeed, the '''multi-dimensional''', 'qualitatively scaled', 'ontologically scaled', and 'upward-scaling' succession/progression/series, or ***heterogeneous***/'''***non-amalgamative***''' ***sum***, a conceptual, geometrical 'sum' which can be depicted as --



-- exemplifies what we call a '***qualo-fractal***' or a '***meta-finite***', '***meta-fractal***', '***consecuum-cumulum***', wherein the '⊕' sign denotes an operation which generalizes that denoted, in standard arithmetic, by the '+' sign, to encompass not only '''purely-quantitative''', standard-arithmetical addition, but also '***qualitative addition***' -- 'inhomogeneous' or 'heterogeneous', 'non-amalgamative', and therefore '***non-reductionist*** ***addition***. The expressions ', **(10)[cm.^{1}]**, **(10)[cm.^{2}]**, and **(10)[cm.^{3}]**, each describe an «***arithmoi***», or '''assemblage of units/monads'''. However, the sum '**(10)[cm.^{1}] ⊕ (10)[cm.^{2}] ⊕ (10)[cm.^{3}]**' denotes a '***meta***-assemblage ***of assemblages***'; ***of*** «***arithmoi***», involving different ***kinds*** of units/monads. We term the latter a '***meta***-«***arithmoi***»' or '***cumulum***'.

We also notice here, in passing, for future reference, that "physical dimensional units" or '*metrical qualifiers*', the "dimensional monads" of "dimensional analysis", are '*strongly contra-Boolean*', e.g.,

$$\text{cm.} \times \text{cm.} = \text{cm.}^1 \cdot \text{cm.}^1 = \text{cm.}^1[\text{cm.}^1] = \text{cm.}^{1+1} = \text{cm.}^2 \neq \text{cm.}^1.$$

Notice also that they are '*convolute*' rather than '*evolute*' in their [generalized-]multiplicative behavior, operator [operand], or *function*-al behavior, i.e., in their '*self*-[re-]*flexion*' or '*auto-flexion*', and '*other*[de]-*flexion*', or '*allo-flexion*'.

The '*qualitativity*' of metrical units, metrical 'monads', or metrical 'qualifiers', is noted, in a recent exegesis of the work of James Clerk Maxwell, one of the pioneers of "dimensional analysis" for physics, as part of his development of electromagnetic field theory, in the following terms: "... Maxwell's line of thought here is representative of a very elegant and very powerful *mode of reasoning in which he pioneered*, termed *dimensional analysis*. In this ***remarkable algebra***, ***symbols are used to represent not numbers but*** the *concepts* [the '***metrical qualifier***' concepts -- **F.E.D.**] of which the *numbers* are the *measures* [the '*generic "pure" quantifiers*' -- **F.E.D.**]. In this sense, *dimensional analysis penetrates to the very elements of a physical system*... The "*dimensions*" to which the name refers seem to be *the very parameters of the cosmos itself, as human science has at any point been able to grasp them*. In this sense too, ***dimensional analysis*** is ***close to*** the ***Kantian concern*** with the *elements* of ***human intuition*** of the ***world***, ... In *dimensional analysis*, then, ***literal symbols represent concepts, not numbers***; the ***things counted rather than*** the ***count itself***." [T. K. Simpson, *Maxwell on the Electromagnetic Field: A Guided Study*, p. **395**, *bold italic underscored emphasis added by* **F.E.D.**].

Motivation. Thus, we may **begin** to fathom *how* an ideographical arithmetic of *qualitative units*; of quantified *ontological* ["kind" of being/monad] ***qualifiers***, and of *metrical* ["unit-of-measure", or 'monad-of-measurement'] ***qualifiers*** -- as distinct from the familiar arithmetic of "pure" ***quantifiers*** -- might be built up.

But ***why*** should we want to construct such an arithmetic?

Our reasons can be stated here, but not fully or quickly demonstrated here. We will merely state some of those reasons for now, leaving their elaboration and demonstration to the sequel.

Our answers include:

(**1**) to break through '*The Nonlinearity Barrier*', which also 'contains' '***The Fusion Barrier***';

(**2**) to help catalyze a 'psycho-historical', cognitive advance within Terran humanity, an advance 'meta-fractally' homologous to that which our ancestors made via their protracted development and gradual species-wide diffusion of pure-quantitative, ideographic arithmetical tools [eventuating in that of the Hindu-Arabic numeration-notation system], and one without which, we hold, Terran *humanity* will be unable to survive *as such* much longer, let alone to move on to the "next levels" in its potential 'evolution' and 'meta-evolution', and;

(**3**) to construct an insight-inciting 'ideographical' or 'symbolical' shorthand for trans-formal thinking; for dialectical, '***ontological-contental***' thinking -- for '***ontodynamical***' thinking -- not stopping short at modeling the fixed-ontology "dynamics" of the "evolution" of "dynamical systems", as does present-day "dynamical systems theory", but constructing further, to encompass the '*meta-modeling*' of '*meta-systems*' and their '*meta-dynamics*' of '*meta-evolution*', i.e., their inherently, immanently, internally '***self-revolutionizing self-change***'. The latter manifests as a ***sequence***, a '*self-propelling, self-propagating* ***progression***' of distinct, "historically-specific" systems/epochs, separated and punctuated by 'revolutions' -- by their ***self-propelled traversals*** of ***self-formed*** 'metafinite conversion singularity' ***boundaries*** -- and characterized by 'ontology change' and 'ontology gain'; a ***cumulative expansion of ontology***; i.e., characterized by «***aufheben***» ***net gains in ontology*** with each successive such singularity. Such a shorthand is a symbolic assistant, an ideographic «***organon***» or '*tool of thought*', for thinking about realities mostly neglected, avoided, or 'unconnected' by current science and its mathematical models. Such realities are describable/modelable via ***model specifications***, via "sets" of *initial condition and boundary condition* ***premises***, such that the calculated *dynamical, temporal consequences* of such a model specification, designed and intended to model/mirror the self-consequences *in reality* for the system they model, bring about, at length, *in time*, the 'suspension'; the 'obsolescence'; the '***self-violation***' -- the dynamical 'negation' or 'refutation' or 'undoing' -- of those dynamical-temporal ***premises***, of that ***model specification***. This process can be comprehended as a new kind, a 'meta-dynamical', '***temporalized***' analogue, of the "atemporal" «***reductio ad absurdum***» proof-procedure within formal logic. But this analogue is one which does ***not*** signify, as in formal/'adynamical'/'Parmenidean' logic, the total, ***absolute***, and timeless untenability and logical non-existence of thus-refuted "atemporal" premises/specifications. It signifies, rather, the ***relative***, temporal, temporary, transitory, transient, transitional, and ***self-suspending*** character of those dynamical, temporal, epochal, ***historical*** and '"***historically-specific***"' premises; of that initial model specification, as also of the ***system/epoch*** that those premises entail. It signifies the "historical specificity", hence the historical-, temporal- & also the internal-, immanent- or self-***limits***; the ***finitude*** of the temporal "length" or duration/'self-durability' of that 'eventity'; of its "fourth" or temporal dimension. It ***implicitly*** signifies the immanent construction of a new, ***successor system*** by that thus ***predecessor system*** via that of a new, qualitatively-changed, ontologically-changed, ***ontologically net-expanded*** system of ***temporal, dynamical premises***, or new ***model-specification***. It signifies a "*raising-by-its-own-bootstraps*", cumulative, «***aufheben***»-negation, and updating-***revision***, with «***aufheben***» *net-expansion*, of those ***temporal premises***; of that ***dynamical*** *model specification*.

It signifies the 'intra-dual' or 'self-dual' *self*-overcoming of the "***system***'s *immanent* ***dynamic***" by its equally-immanent '***meta-system meta-dynamic***'; the self-overflow of that system's epoch into a new, next epoch; that meta-system's *formation* of that new epoch *of* and for itself; of that new epoch of history, with its *new* historical specificity; that meta-system's self-supersession of its own "***historically-specific***", or internal, '*epoch-specific*' "laws"-of-motion, by the *new* "laws" of the *new [kinds of] motion* that it creates *for* and *as* itself -- for and *as* its ontologically ***expanded self*** -- in that very process of [self-induced] 'self-transitioning'. Each "[system/ meta-]***state***" is, in fact, not '***stat***(e)-ic'; it is a '*dyn*ate'.

Such '***epochal transitions***' and "***meta-system transitions***" [cf. Valentin F. Turchin, *The Phenomenon Of Science*, Columbia University Press [NY: **1977**], pp. **viii-xi**, *et passim*.] are often signaled, in the '*purely-quantitative*' ['***unqualified***'] models of analytical dynamics, typically formulated in the form of systems of [typically ***nonlinear***] integro-differential equations, by a ***finite*** time value, a specific 'date' in time, where the [***nonlinear***] differential equation(s) "blow-up", i.e., where the system's metrics "encounter a singularity". This means that the '"solution"' -- the list [vector] of predicted values of some or all of the 'metrical quantifiers' characterizing the dynamical "state" of the system -- '"*explodes*"', '"*becomes infinite*"', '"*becomes meaningless*"', '"*becomes undefined*"', '"*encounters a discontinuity*"', '"*ceases to exist*"', or '"*diverges*"'. That is, '"*the solution is **carried off** to infinity*"', or '"*the solution **leaves** the state-space*"', or '"*the solution...**stops** at some finite instant,* **t***'", or '"*the equations no longer make sense*"' [cf. Florin Diacu, Philip Holmes, ***Celestial Encounters**: The Origins of Chaos and Stability*, Princeton U. Press [Princeton, NJ: **1996**], pp. **82-84**]. All of the above voices and paraphrased phrases are '"breathless"' and 'breath-taken', relatively unguarded and spontaneous voicings of an incipient ***immanent critique*** -- of a ***self-critique*** -- of the standard mathematics of '"dynamical systems"'.

Such "singularities" typically arise via *zero* ***division***. The value of a component 'sub-function', say with a positive function-value, located in the *denominator* of the RHS [Right Hand Side] of the model nonlinear differential *equation itself*, and/or in that of the *solution-function* of that nonlinear differential equation, ***declines*** in absolute value in the positive direction of time-advance, as the **t** parameter-value increases. The presence, and, more specifically, the ***denominator***-presence, of such a 'depletion-function' in a system's 'state-function' or 'solution-function' is part of what makes the system's differential equation -- the result of the 'differentiation' of that solution-function -- ***nonlinear*** in the first place. This 'denominatorized' function's "function" is to measure, as the time parameter-value advances, the degree of '***ontological*** *[**self**-]**conversion***' of a population/«***arithmos aisthetos***» of monads which **(1)** constitute the system's '***fuel ontology***'; which **(2)** are located in the '***driver locus***' of that system's evolution, and **(3)** whose ontological self-conversion is the '***negentropy releasing***' heart-process of that system, the very process that drives that system's evolution, thus constituting its 'essence-ial' activity. The '***ontological self-conversion***' of this population of 'fuel' monads is its self-conversion into a population of higher, 'meta-monads', constituting a new, higher 'onto' -- a 'meta-onto' which is typically omitted of any mention in the model specification. After a ***finite*** duration elapses, during which the system has developed itself or '*consequented*' from its '***temporal premise***', or "initial condition", i.e., its "initial state" -- and, as of a specific, ***finite*** 'date', or value of **t**, call it **t*** -- the value of this denominator-resident 'sub-function' ['***conversion/depletion-function***'] sinks to ***zero***. This ***zero*** value signifies the ***complete*** ontological [self-]***conversion/depletion*** of the '***fuel onto***', though only *within* the system's '***driver-locus***'. Thus, the value of the numerator of the differential equation's RHS differential expression, for that "singular" value of **t**, and/or of the numerator of its solution-expression, undergoes ***division by zero***. If the function-value of this sub-function is the entire denominator, or if it is a "factor" in a "multiplicative" or "product" denominator, then the *entire* denominator-value becomes ***zero*** as of **t = t***. This is a "continuous", gradual transition to a ***zero*** denominator. Yet this "smooth" transition seems to drive an abrupt, "***instantaneously discontinuous***" change in the value of the equation's RHS differential expression, and therefore also in the predicted values of at least some of the system's "***state-variables***" -- i.e., of its '*essence-ial*' "*vital signs*", or '*self-/status-characterizing*' '***stat[e][us]-measurements***' or '***metrics-of-stat[e][us]***'. *The RHS differential expression depicts* an "***infinitely sudden***" jump from ***finite***, albeit 'acceleratedly escalating' values, "to" the "value" of *pure-quantitative* "***infinity***", "at" a *definite*, specific instant of time; at a ***finite*** value of the time-parameter; at a discrete "point in time" -- at the "point" along the **t** axis denoted by **t***. The foregoing account summarizes the '***ideo***-*phenomenology*' of such a '*purely-quantitative*', analytical, dynamical-system model. That's just the *internal*, '*idea-al*', cognitive-psychological, or 'internal-to-the-human-mind model-phenomenology', 'idealization-phenomenology', or '***endo***-*phenomenology*'.

But what actually happens, '***exo***-*phenomenologically*', i.e., in our human sensory-*mental* perceptions of and in our instru-*mental* measurements of, the ***external-objective reality*** that the mental simulation, the '***endo***-*phenomenology*' of such a model putatively models, when that analytical model "goes bonkers" in this zero-division, "singular" way? What typically happens is that the original system specification, including its *premises*, its "*ontological commitments*", i.e., its '***ontology***', has, both "at" and "past" that **t*** point, '*broken down*', or '*become obsolete*'. That model specification and its entailed 'ontology' has been '*surpassed*', or '*superseded*'; '***suspended***', or '***transcended***' as a consequence of the very dynamics mirrored in the model encoding that system and its specifications. Indeed, typically, the 'ontology' of the total system, the system totality or '***conversion-formation***', has 'self-converted'. That is, the system's ontology has both ***contracted locally***, '***convolutely***', within the system's '[***meta-***]***evolutionary engine***', 'negentropy'-generating '***driver-locus***', or '[ontological ***self***-]***conversion-locus***', but has '***evolutely***' net-***expanded globally***, in ***the rest*** of the system, i.e., in the rest of the '[***self-***]***conversion-formation***'.

In summary, how can we further characterize what truly happens "at" **t***? What truly happens near the singularity? What happens near the point at which the primary ***output*** of the solution of the nonlinear differential equation, its "dependent variable", or past-reconstructive/future-predictive 'system-state-function', the measure-able result to which the solution of the 'differential expression' of the nonlinear differential equation equates, ***putatively*** arrives at ***an infinite quantitative change*** at the end of a merely ***finite*** period of [perhaps large but still ***finite***] lead-up escalation? What happens is a '***qualitative***' change. A specifiable and ***finite*** system of ontological change(s) occurs. A '***metafinite***' ***change of*** '***ontology***' occurs. It typically involves a global, 'self-«***aufheben***»' net self-expansion of the ontology of the former, now-predecessor, system, and, thereby, a self-transformation of that former or predecessor system into a qualitatively, ontologically "new", successor-system; into a self-transformed, different 'system-identity'.

'Metrical [Re-]Qualification' of Nonlinear Differential Equations and The 'Meta-Dynamics' of Singularity. Take for instance the classical, founding example -- and therefore also perhaps the crudest example -- of such '***nonlinear singularity***': that of the collision of two [or more] mutually-gravitating bodies, e.g., 'planets', in the Newtonian "many-body problem" of classical celestial mechanics.

In the idealization native to the Newtonian idiom of 'planetary-system dynamics', or 'planetary mechanics', planets are modeled as "mass-points", i.e., as if they had all of their mass concentrated at their '"zero-dimensional"' mass 'center-points' -- that is, at their centers-of-mass, or, given certain additional conditions, their 'centroids'. Each planet is therefore represented solely by the point at its 'centroid', or center of mass, and not as a three-dimensionally-extended body of structured, heterogeneous, complexified matter. Thus, planets are modeled as zero-dimensional, infinitesimal, mathematical-geometrical points, moving in a three-dimensional Euclidean model of physical space -- *moving each other* solely by their gravitational interaction, by the gravitational forces which they exert upon one other per this model.

The magnitude of the [mutually-attracting] gravitational force between two such mass-point "idealized" planets is ***inversely*** related to the "square" or '*self-reflexion*' [i.e., the '*self-multiplication*' or '*self-product*'; the 'first power' [Diophantus' «['***auto-***']***dynamis***»] or "second degree"] of the distance between them. That distance changes continuously as *they move each other*, fundamentally *toward* one another, responding to the gravitational forces which, per this paradigm, they exert upon one another. Indeed, the squaring of this denominator-resident '***function unknown***', [often rendered, generically, e.g., as $\mathbf{r_{j,k}(t)}$ to denote the 'radial' distance between the [idealized] $\mathbf{j}$th planet-point/-centroid and the $\mathbf{k}$th planet-point/-centroid in the planetary system at a[ny] moment, the 'generic moment' denoted by $\mathbf{t}$]; this presence of the *unknown, to-be-solved-for generic time-function-value* or *dynamical-function-value* $\mathbf{r_{j,k}(t)^2}$ in the differential equations of 'Newtonian gravitics', is what renders those equations ***nonlinear***. This presence is also ***part*** of what renders those equations "unsolvable", if they involve the general case of more than two such planets, within the present-day status, form, and content of the theory of mathematical analysis [Recently discovered "closed-form" solutions to the "$\mathbf{n}$-body problem" for $\mathbf{n \geq 3}$ require the '***infinite postponement***' of all collisions to $\mathbf{t = +\infty}$, thus merely transferring the 'infinite unrealism' of ∞ from the gravitic force ***de***pendent variable to the time ***in***dependent variable, thereby rendering these '"solutions"' just as unrealistic as the $\mathbf{f_{j,k}(t^*)}$ "=" ∞ 'solution' already at hand -- **F.*E.D.***].

Now, given the mutual-'attractional' character of this putative 'gravitic force', it is not surprising that mutual collisions of such planets can, and do, occur. Suppose that the timing of such a collision, say between planet $\mathbf{j}$ and planet $\mathbf{k}$, is associated with the time-value $\mathbf{t = t^*}$. Because these two planets have been, in this model, '"idealized"' by being '"***reduced***"' to mass-***points***, collision in this model is signified by the '*co*-incid-ence', the 'super[-im-]***position***', the '***co***[m]-***position***', or the '***co-occupation***' of the same, "***one***", single point in the space of this Euclidean model of physical space by the "***two***" [and therefore "two"-***no-longer***] planets' mass-points. This thus means that the 'centroid'-to-'centroid' distance between the two planets becomes, at the point in time quantitatively named by $\mathbf{t = t^*}$, equal to $\mathbf{0}$, which is the generic 'quantifier' for the distance 'between' a single such "point" and itself. We have $\mathbf{r_{j,k}(t^*)^2 = 0^2 = 0}$. But, since the function-value $\mathbf{r_{j,k}(t)^2}$ resides in the ***denominator*** of the Newtonian gravitic force expression, ***collision means 0-division***. The magnitude of gravitic force between planets $\mathbf{j}$ and $\mathbf{k}$, per the Newtonian differential equation, thus goes to '"*infinity*"' ***at*** $\mathbf{t = t^*}$, ***whatever*** the *pure-quantitative* values of the masses of those two planets, and of the Universal Gravitic "Constant", $\mathbf{G}$, in the ***numerator*** of the RHS [Right Hand Side] of the Newtonian gravitic force model-equation [namely, with $\mathbf{p(t)}$ denoting ***momentum*** as a function of time, $\mathbf{t}$; with $\mathbf{f_{j,k}(t)}$ denoting gravitic ***force*** as a function of time, with $\mathbf{m_j}$ denoting the inertial mass of planet $\mathbf{j}$, and with $\mathbf{m_k}$ denoting the inertial mass of planet $\mathbf{k}$:

$$\mathbf{dp(t)/dt = f_{j,k}(t) = Gm_jm_k/r_{j,k}(t)^2}, \text{ so } \mathbf{f_{j,k}(t^*) = Gm_jm_k/0^2 = Gm_jm_k/0 \text{ "="} \infty}].$$

But this is not what happens in reality at the moment modeled by $\mathbf{t^*}$. As our clock approaches, then reaches $\mathbf{t^*}$, the "force between" planets $\mathbf{j}$ and $\mathbf{k}$ never, in reality, becomes "infinite". At the moment $\mathbf{t^*}$, as well as beyond it, *after* it, the '"idealized"'/'"reduced"' model and reality part ways ***in the extreme***. They diverge from one another '"infinitely"' in a pure quantitative sense. This is because the predicted "infinity" of the model contrasts with an ever-finite actuality. The mathematical '"*term of art*"' for the *difference* between a model's *prediction* of a future moment's measurement of a "time-varying", or "dynamical" variable, and the *actual* value of that variable, as actually, physically measured when that future moment arrives or 'presents', is "***residual***".

The pure-quantitative "***residual***" for this model as of date **t***, at the **t*** moment of '"collision-singularity"', is infinity, ∞. This is because the difference between ∞ and whatever *finite* quantifier-value, call it **f***, is actually measured for the force 'between' the two actual planets as of **t***, is also ∞. The subtraction of the *finite* magnitude of the force actually measured from the "quantity" predicted by the model is the same as the value predicted by the model, because that value is ∞, and the 'magnitude' "infinity" minus ***any*** finite value still equates to "infinity" per the standard theories of transfinite arithmetic: $\infty - f^* = \infty$.

What this "***infinite residual***" means is that this gravitic model becomes ***infinitely wrong*** starting at least from the moment of collision-singularity, **t***.

The actual, physically measured gravitic force between a colliding pair of planets, as **t*** approaches -- as the two planets approach one another -- first rises to a peak but *finite* level. It then ***disappears***, as such. It disappears ***into*** the internal, self-gravit(y)(ies) of the new, coalesced planet(s) and of the multitudinous smaller fragments that may emerge out of the ***mutual coalescence / disintegration*** of those, ***formerly**-two*, planets, as well as *into* the external, 'inter-mutual' gravitational interactions among all of these resulting collision-products, these new bodies, these new singleton ***ontos***, consisting of planet(s), planetoid(s), planetessimal(s), plus debris too small to be called even "planetessimal".

In any account of the actual, ***natural history of gravity, of planets***, and of *planetary systems* or *solar systems* in this cosmos -- as distinguished from formal, "timeless", idealized, 'ahistorical' or 'supra-historical' theories of gravitation -- such collisions are no mere "exceptions", "curiosities", or "rare events". They are the very stuff of the '***self-«bildung»***', the '***self-formation***' or '***self-building***' of planets and planetary systems. Our planets, our solar systems, are '***made of singularities***'; were '***made by singularities***'. They arose as the '***cumula***', the '[ac]*cumulations*' of such '***metafinite conversions***' of smaller bodies into larger, of '"interstellar dust"' into stars and their entourages of planets. Just look at the satellite photos showing the collision-cratered visages of this solar system's "terrestrial" moons and planets [other than Earth and Venus] to see that this is so.

Infinity Residuals and The Paradox of Singularity. We have seen above the sudden, eruptive onset of ***infinity residuals*** at the "***point***-in-time" of singularity; the irruption of ***infinite*** 'homeomorphic defect'; of the becoming ***infinitely wrong*** of the predictions of the mathematical model of the system in question at and past that point. At and after the moment of singularity, the scientific theory embodied in the mathematical model has become "***infinitely***" ***falsified*** empirically. Yet, up until nearly the ***instant*** of **t***, that model had provided a good "fit" to our experienced, measured, empirical actuality, with quite finite and even minimal residuals, ***minute*** or '*minutessimal*' if never infinitesimal. For the case in point -- the Newtonian model of gravitics -- its fitness is sufficient to reliably guide Earth's space-probes to orbits around other planets in its home star/planets-system, and even to manage high precision 'astro-acrobatics', e.g., gravitic "sling-shot" maneuvers.

If we turn away from our ' "infinitely" erroneous' model of the moment **t***, and back to empirical reality, as reality approaches what in reality corresponds to the model's moment of singularity, we find that, indeed, as the planets converge closer and closer to one another, the gravitic attractive force between them intensifies in an acceleratory way. But it ***remains finite***. Before its **t***, it reaches a *finite* maximum, which we denote generically, in such cases as this, by the '***finite-limit***' quantifier-variable $\overset{\circ}{\overline{\wedge}}$, a pictogram/ideogram hybrid, symbolizing an arrow-head, pointing upward in the positive, or $\oplus$, sense of axial direction, colliding with a [***finite***] limit, ceiling, or barrier. Much beyond that finite maximum, ***that*** particular gravitic force component ***ceases to exist***, along with the cessation of existence of the two planetary bodies which exuded it. It is not precisely **0** in magnitude. It no longer has ***any*** magnitude, because it is ***no longer there*** to have ***a*** magnitude. Much past that $\overset{\circ}{\overline{\wedge}}$ point, ***other*** "***laws***" than Newton's gravitic "law" supervene locally. That is, ***other processes*** become dominant -- tidal processes; processes involving material properties of the constitutive matter of the two planets, such as brittle fracture mechanics, visco-thermo-plasticity, and visco-thermo-elasticity. These processes and "laws" are outside of/beyond/not part of the Newtonian model specification/idealization. These processes have the affect of ***dis***-organizing, or of ***dis***-integrating and ***dis***-assembling planets **j** and **k**, as well as of ***re***-organizing, ***re***-integrating, and ***re***-assembling them as something else, something ***qualitatively, ontologically different*** than what was before, relative to the '"***ontological commitments***"' -- the '***ontological presumptions***' or '***pre-assumptions***' -- of our initial model specification. The old pair of planets, in their collision, ***dis***-appears, and new bodies emerge/appear "in their [former] place(s)", or in 'new places'.

Thus, up until the point of this supervention of "laws", the residuals of the Newtonian model are small. But there is a huge "residuals *dis*-continuity" at about $\hat{\Lambda}$, just before $\mathbf{t^*}$. The formerly 'minutessimal' residuals become '''infinitely large''' somewhere in there.

How is it possible to go from minimal, finite residuals to "infinite" residuals so fast, so suddenly? Is this not a ***paradoxical*** mathematical phenomenon? For the model, so convergent with the reality so much of the time, to diverge so markedly at a discrete point/period of time? For the model specification to become so suddenly inadequate; to reveal, so momentaneously, its inadequacy? This phenomenology is what we term '***The Paradox of Singularity***'. It is the paradox -- the unbelievability -- of *an absolutely instantaneous absolute discontinuity* in model fitness; of an instantaneous transition from 'near zero' residuals to 'infinite residuals'. Such a model-phenomenon, we hold, must be a 'presence' of an absence; ***a sign of something amiss***, of something *logically remiss*, of ***something missing***; of something *conceptually, philosophically, cognitively, linguistically, symbolically, culturally, **and psycho-historically*** omissive. It points, we hold, to something crucial that has been left out, elided, neglected, and that ***has gone*** and ***still goes*** unnoticed, at least until this Paradox announces the cost of our unconsciousness. It points to a memetic blind-spot, to the cultural self-conditioning of a specific kind of blindness, to some gaping incompleteness at the foundations of our arithmetic, that only here, at length, at a far end of the 'meta-evolution' of our mathematics, shows itself searingly, and comes back to bite us: '***The Elision of the Qualifiers***'.

Here also obtrudes, in this ***mathematical phenomenon*** of singularity, i.e., in this key component -- one of the two major components -- of '***The Nonlinearity Barrier***', the toxic fruit of another cultural, psycho-historical bias of our civilization: our scientifico-philosophical and mathematical cultures' implicit preference for the Parmenidean. This preference encompasses our bias towards models which exhibit an endless, decelerating, eternal and undeviating taxis toward an asymptotic, one-state, monolithic, single-fixed-point attractor; toward a final, eternal, timeless and changeless equilibrium solution. These misrepresentations of reality are among the propensities of linear and "linearized", i.e., of ***linearly-falsified***, differential equations-models; of '*ir*reflexive' models; of the ***pseudo***-dynamics of linear "dynamical" systems theory. Here also obtrudes our bias in squeamishly and systematically avoiding and under-exploring, for the last **300+** years, the '***self-reflexive***' and '***autokinesic***', '''self-oscillatory''' nonlinear dynamical differential equations-models, with all of their "paradoxical", '''life-like''', and "intractable" propensities. Here obtrudes our avoidance of facing the facts -- of the '''spontaneous''' or self-caused ***self-explosion*** of nonlinear '''equilibria''', and of typical ***nonlinear systems*** in general, not "at" "$\mathbf{t = +\infty}$", as typically with linear "dynamical" systems, but in ***finite*** time!

The Immanent Necessity of a Quanto-Qualitative Mathematics [*if the Language of Mathematics is to Overcome Its Descriptive Inadequacy Regarding "Singularity"*]. Pure quantitative infinity is, apparently, the only way that this pure-quantitative mathematical model can express, or 'translate', the ***qualitative, ontological change in reality*** that occurs as of $\mathbf{t = t^*}$. The "quantity" ∞ functions here as '*the quantitative shadow of the qualitative*', that is, as the quantitative shadow of '***onto-dynamasis***'. On the contrary, the *standard, 'pure-quantitative', "dynamical"* model -- hats still off, even if unknowingly so, to Parmenides -- tacitly assumes '***onto-stasis***'. It seems to resort to ***infinite quantitative change*** as if it were the only available 'pure-quantitative' expression of ***qualitative/ontological change*** in a quantity-only language.

When we '[*re-*]***qualify***' that 'purely-quantitative' differential equation for the force between planets $\mathbf{j}$ and $\mathbf{k}$, via the apparatus of the ${}_{a}\underline{\underline{\mathbf{u}}}$ dialectical arithmetic, even if we use only its sub-ideography for "Dimensional Analysis", we get a very different answer, indeed a ***qualitatively different*** answer. Yet this different answer is one which bears a '*quanto-qualitative*', '*meta-fractal*' similarity to the "*pure-quantitative*" answer, while eliminating its apparently ***infinite quantitative erroneousness***.

We have found it mnemonically useful, in notating the 'qualifier meta-numerals' or 'dialectors' of the dialectical ideographies, to employ three '*ideo-diacritical marks*', '_', '^', and '$^{\circ}$'. The underscore, '_', designates ***dialectical*** '***meta-numerals***', denoting dialectical '***meta-numbers***', which are '***strongly contra-Boolean***' in their self-multiplicative behavior. The "caret" or "hat" mark designates the 'meta-numerals' of 'meta-numbers' which are single "units", single "monads", analytic-geometrically of one unit in "length" or "modulus", as distinct from «***arithmoi***», assemblages of two or more units or monads, ***homogeneous*** in kind, or '***meta-«arithmoi»***', assemblages of multiples of units or monads, ***heterogeneous*** in kind [also termed '***multi-ontic, multi-monadic cumula***']. The raised omicron 'headdress', '$^{\circ}$', carried over from Diophantus' circa 250 C.E. "syncopated" generic '**Mo**nad' symbol, $\overset{\circ}{\mathbf{M}}$, designates 'meta-numbers' which are, like $\overset{\circ}{\mathbf{M}}$, '***quantifiable***', i.e., ***not*** "*additively idempotent*". We employ the symbol '$\leftrightarrow$' to denote the 'modeling', 'association', "'assignment'", "'interpretation'", or 'semantification' of a generic [meta-]number symbol to/via another, "intensional", 'connotative', or 'intuitively meaningful', symbol. Our $\underline{\mathbf{Q}}$ model for the [***meta-***]*systematic-dialectical exposition* of the '*ideo-meta-evolution*' of dialectical arithmetic, from [**1**st step] the [first order] "**N**atural" arithmetic, $\hat{\underline{\mathbf{q}}}_1 \leftrightarrow \underline{\mathbf{N}}$ [of '*pure*' or '*unqualified*' quantification], to [**2**nd step] the $\hat{\underline{\mathbf{q}}}_2 \leftrightarrow \underline{\mathbf{Q}}$ dialectical ideography [of '*unquantifiable*' or ***pure-ontological qualification***] itself, to the [**3**rd step] $\hat{\underline{\mathbf{q}}}_3 \leftrightarrow \underline{\mathbf{U}}$ dialectical ideography [of '*quantifiable* ontological qualification'], 'generates', in its seventh step, the '***idea-onto***' of what we call the 'alpha-mu' or just the 'alpha' arithmetic, denoted ${}_{\alpha}\underline{\mu}$, or $\underline{\alpha}$. In that $\underline{\mathbf{Q}}$ model, that latter system of arithmetic is denoted more explicitly by the symbols $\hat{\underline{\mathbf{q}}}_{\mathbf{MU}}$, or $\hat{\underline{\mathbf{q}}}_{\mathbf{MQN}}$, both $\leftrightarrow \hat{\underline{\mathbf{q}}}_7$. These symbols connote the «***aufheben***» '*assimilation*', '*subordination*', or '*subsumption*' of the $\hat{\underline{\mathbf{q}}}_3 \leftrightarrow \underline{\mathbf{U}}$, $\hat{\underline{\mathbf{q}}}_{\mathbf{U}}$, or $\hat{\underline{\mathbf{q}}}_{\mathbf{QN}}$ ideography of ***Ontological*** '***quanto-qualification***' or '***qualo-quantification***', by the $\hat{\underline{\mathbf{q}}}_4 \leftrightarrow \underline{\mathbf{M}}$ or $\hat{\underline{\mathbf{q}}}_{\mathbf{M}}$ ideography of '***pure-Metrical*** onto-dynamasis'. Syntactically, in the formation of the 'meta-numerals" of the $\hat{\underline{\mathbf{q}}}_7 \leftrightarrow {}_{\alpha}\underline{\mu}$ or $\hat{\underline{\mathbf{q}}}_7 \leftrightarrow \underline{\alpha}$ dialectical ideography's sub-arithmetic for "***Dimensional Analysis***", its '*sub-arithmetic of metrical units*' or '*of metrical monads*' component, denoted $\overset{\hat{\circ}}{\underline{\mu}}_{\underline{\mathbf{U}}}$, 'subsumption' manifests as '***subscriptization***' or '***subscript-ification***'. This '***metrical qualifier***' component of ${}_{\alpha}\underline{\mu}$ or $\underline{\alpha}$ works by transferring the $\underline{\mathbf{U}}$ *meta-numerals* from the '*scriptal*' to the '*sub-scriptal*' level of its metrical units' '*meta-numeral*' formations-of-sub-symbols/-ideograms. The $\underline{\mathbf{U}}$ arithmetic is «***aufheben***»-conserved, and its arithmetical operations ensue, but *only* at the subscript-level for this '***metrical quanto-qualifier***' component of ${}_{\alpha}\underline{\mu}$. That component thus uses two successive subscript levels below the 'scriptal' level. The $\underline{\mathbf{U}}$ *meta-numerals* re-emerge as *subscripts* of the $\overset{\hat{\circ}}{\underline{\mu}}_{\underline{\mathbf{U}}}$ *meta-numerals*. The $\overset{\hat{\circ}}{\underline{\mathbf{u}}}_{\mathbf{j}}$ *meta-numerals* of the $\underline{\mathbf{U}}$ arithmetic themselves already have subscripts drawn from the [higher order] $\underline{\mathbf{N}}$ arithmetic of the "**N**atural" numbers. Thus, in the $\overset{\hat{\circ}}{\underline{\mu}}_{\underline{\mathbf{U}}}$ metrical qualifier *sub*-ideography of the ${}_{\alpha}\underline{\mu}$ ideography, the subscripts have, in turn, subscripts of their own. The "variable" or '***generic***' $\overset{\hat{\circ}}{\underline{\mu}}_{\mathbf{U}}$ unit-metanumber, interpretable as modeling a '*metrical qualifier*', '*metrical unit*[y]', '*metrical monad*', or '*dimensional unit*', may be denoted either as $\overset{\hat{\circ}}{\underline{\mu}}_{\underline{\Sigma}_j \alpha_j \overset{\hat{\circ}}{\underline{\mathbf{u}}}_j}$, or as $\overset{\hat{\circ}}{\underline{\mu}}_{\underline{\partial}\underline{\mathbf{U}}}$. The symbol $\underline{\underline{\Sigma}}$ denotes a non-standard, '***quanto-qualitative***' summation-operation which generalizes the $\mathbf{\Sigma}$ operator of standard, '***pure-quantitative***' summation. The $\underline{\underline{\Sigma}}$ operator encompasses 'syntactically ***anti-reductionist***', '***non-reductive***', 'non-collapsing', '***quanto-qualitative***', "non-amalgamative", and "dimensionally ***in***homogeneous" or "heterogeneous" summation operations, as distinguished from both the [likewise non-standard] operator $\underline{\mathbf{\Sigma}}$ of the "'***purely-qualitative***'"/*heterogeneous* summation operation of the $\underline{\mathbf{Q}}$ arithmetic, and the $\mathbf{\Sigma}$ summation operator native to the standard, "'***purely-quantitative***'"/*homogeneous* arithmetics, such as that of $\underline{\mathbf{N}}$ «***et sequelae***». Per the example above, the summation is over the **N**atural number index denoted **j**, with its implicit minimum or lower limit being **1**, and its implicit ***finite*** Maximum or upper limit being **M**, denoting the largest **N**atural number assigned to a "*fundamental*" metrical unit's subscript in the $\overset{\hat{\circ}}{\underline{\mu}}_{\underline{\mathbf{U}}}$ representation of the "*dimensional system*" in use. The α_j denote **R**eal numbers, which includes both negative **R**eal numbers and the **R**eal number **0**. The latter is applied as $\{\alpha_n = 0\}$ in $\underline{\underline{\Sigma}}\alpha_j \overset{\hat{\circ}}{\underline{\mathbf{u}}}_j$ for all of those $\{\overset{\hat{\circ}}{\underline{\mathbf{u}}}_n\}$ defined for the metrical system in use, but not involved in the given 'metrical qualifier' or ['compound-ed'] metrical unit represented by the given instance of $\overset{\hat{\circ}}{\underline{\mu}}_{\underline{\Sigma}_j \alpha_j \overset{\hat{\circ}}{\underline{\mathbf{u}}}_j}$, and such that $\alpha_n \overset{\hat{\circ}}{\underline{\mathbf{u}}}_n = \mathbf{0}\overset{\hat{\circ}}{\underline{\mathbf{u}}}_n = \mathbf{u}_0$, where $\mathbf{u}_0$ denotes the 'quanto-qualitative', '***ontological zero***' or '***existential zero***' of the $\underline{\mathbf{U}}$ arithmetic. More generally, we use the $\underline{\mathbf{\Sigma}}$ operator for $\underline{\mathbf{Q}}$-based arithmetical and algebraical modeling of 'meta-«***arithmoi***»' or 'multi-«***arithmoi***»' -- '***multi-***[meta-]***ontic***', '***multi-***[meta-]***monadic***' '***consecua-cumula***' -- by means of '***poly-qualinomial***' expressions. We use the $\underline{\underline{\Sigma}}$ operator for $\underline{\mathbf{U}}$-based and ${}_{\alpha}\underline{\mu}$-based representations of richer, '***quanto-qualified***'/'***qualo-quantified***' '***consecuum-cumulum***'-models -- the latter, ${}_{\alpha}\underline{\mu}$ models being '*multi-metrical*' as well as ontologically '*multi-dimensional*' -- by means of [partially] "non-amalgamative", "'***in***homogeneous sum'" or "'heterogeneous sum'" expressions which we characterize as '***poly-quanto-qualinomials***' or, equivalently, as '***poly-qualo-quantinomials***'. In $\overset{\hat{\circ}}{\underline{\mu}}_{\underline{\partial}\underline{\mathbf{U}}}$, the $\underline{\partial}$ denotes a [meta-]***finite*** and '***quanto-qualitative***' '***part-ial****-ization*' operation, generalizing the "'infinitesimalizing'" ∂ operator of standard partial differentiation. The subscript here, $\underline{\partial}\underline{\mathbf{U}}$, thus signifies some/any ***finite*** part or fraction/fragment of the ***space*** [***potentially*** infinite, in Aristotle's sense], ***denoted*** $\underline{\mathbf{U}}$, as part of the arithmetical '***rules-system***', or '***system of arithmetic***', denoted $\underline{\mathbf{U}}$. In $\underline{\partial}\underline{\mathbf{U}}$, the $\underline{\partial}$ operator means, in effect, to draw some/any closed ***finite boundary*** within the space $\underline{\mathbf{U}}$, and to take from $\underline{\mathbf{U}}$ the ***finite*** fraction of $\underline{\mathbf{U}}$ that is within that ***boundary***, leaving out/leave behind the rest of $\underline{\mathbf{U}}$. Thus, the ambiguous result or 'operation-value'/'function-value' $\underline{\partial}\underline{\mathbf{U}}$ generically denotes some/any ***finite part***-ial 'excerpt' of/from $\underline{\mathbf{U}}$. Thus $\underline{\underline{\Sigma}}\alpha_j \overset{\hat{\circ}}{\underline{\mathbf{u}}}_j$ and $\underline{\partial}\underline{\mathbf{U}}$ ***may*** denote, essentially, the same thing.

Suppose we '''*assign*'''/'''*interpret*''' [$\leftrightarrow$] the $\hat{\underline{\mu}}_{\underline{\alpha U}}$ sub-species of *meta-numerals* that results from the $\underline{\underline{\mathbf{M}}}$ subsumption of $\underline{\mathbf{U}}$ as follows:

$\hat{\underline{\mu}}_{[+\hat{\underline{u}}_1]} \leftrightarrow \underline{\mathrm{T}}$, for the Time "*dimension*", meted in "*fundamental*" units of '**sec.**', thus denoted, still to this day, in "*syncopated*" fashion;

$\hat{\underline{\mu}}_{[+\hat{\underline{u}}_2]} \leftrightarrow \underline{\mathrm{M}}$, denoting the inertial Mass "*dimension*", measured in "*fundamental*" units of grams, or **gm.**;

$\hat{\underline{\mu}}_{[+\hat{\underline{u}}_3]} \leftrightarrow \underline{\mathrm{L}}$, denoting the physical-spatial Length "*dimension*", measured in "*fundamental*" units of centimeters, or **cm.**, so that --

$\hat{\underline{\mu}}_{[+\hat{\underline{u}}_3 - \hat{\underline{u}}_1]} \leftrightarrow \underline{\mathrm{V}}$, denoting the physical-spatial Velocity "*dimension*", measured in "*compound*" units of **cm. / sec.**;

$\hat{\underline{\mu}}_{[+\hat{\underline{u}}_3 - 2\hat{\underline{u}}_1]} \leftrightarrow \underline{\mathrm{A}}$, denoting the physical-spatial Acceleration "*dimension*", measured in "*compound*" units of **cm. / sec.**^{2};

$\hat{\underline{\mu}}_{[+\hat{\underline{u}}_2 + \hat{\underline{u}}_3 - \hat{\underline{u}}_1]} \leftrightarrow \underline{\mathrm{P}}$, for the physical-spatial Momentum "*dimension*", measured in "*compound*" units of **gm. × cm. / sec.**^{1};

The 'compounded' dimension of Force, $\underline{\mathrm{F}}$, thus translates to $\underline{\mathrm{ML}} / \underline{\mathrm{T}}^2$, and thus also to $\underline{\mathrm{MV}} / \underline{\mathrm{T}}$, and to $\underline{\mathrm{MA}}$, and to $[\hat{\underline{\mu}}_{\hat{\underline{u}}_2} \times \hat{\underline{\mu}}_{\hat{\underline{u}}_3} / \hat{\underline{\mu}}_{\hat{\underline{u}}_1}{}^2]$. The latter, because multiplying amongst $\hat{\underline{\mu}}_{\underline{\alpha U}}$ meta-numerals equates to *additions* of their subscripts, *dividing to subtracting*, equates to $\hat{\underline{\mu}}_{[+\hat{\underline{u}}_2 + \hat{\underline{u}}_3 - 2\hat{\underline{u}}_1]}$, in **gm.·cm./sec.**^{2}, or **dynes** –

$$(\underline{G})[\underline{G}] = (G)[\underline{G}] = G\underline{G} = (6.67\times10^{-8})[\mathrm{dyne}\cdot[\mathrm{cm.}^2/\mathrm{gm.}^2]] \leftrightarrow ((6.67)(10^{-8}))[[\hat{\underline{\mu}}_{[+\hat{\underline{u}}_2 + \hat{\underline{u}}_3 - 2\hat{\underline{u}}_1]}][\hat{\underline{\mu}}_{[+2\hat{\underline{u}}_3} / \hat{\underline{\mu}}_{[+2\hat{\underline{u}}_2]}]] =$$

$$((6.67)(10^{-8}))[\hat{\underline{\mu}}_{[+\hat{\underline{u}}_2 + \hat{\underline{u}}_3 - 2\hat{\underline{u}}_1 + 2\hat{\underline{u}}_3 - 2\hat{\underline{u}}_2]}] = ((6.67)(10^{-8}))[\hat{\underline{\mu}}_{[-\hat{\underline{u}}_2 + 3\hat{\underline{u}}_3 - 2\hat{\underline{u}}_1]}],$$

-- wherein **G** denotes the Newtonian "Universal" 'Gravitic' "constant" *quantifier*, and $\underline{\mathbf{G}}$ its ***metrical qualifier***.

Thereby, via the operation of its $\hat{\underline{\mu}}_{\underline{\alpha U}}$-'***qualification***' or -'***dimensionalization***', using the $\hat{\underline{\mu}}_{\underline{\alpha U}}$ metrical unit qualifiers or dimensional analysis "dimensions" as assigned above, the "*purely-quantitative*" or '*unqualified*' differential equation --

$$dp(t)/dt = f_{j,k}(t) = Gm_jm_k/r_{j,k}(t)^2$$

-- becomes the '[re-]***qualified***' or '*units-of-measure-**qualified***', 'dimensionally-***qualified***', or 'dimensioned'/'dimensional' [and thus '*quanto-**qualitative***' or '***qualo**-quantitative*'] equation --

$$(dp(t)/dt)[\hat{\underline{\mu}}_{[-\hat{\underline{u}}_1]}][\hat{\underline{\mu}}_{[+\hat{\underline{u}}_2 + \hat{\underline{u}}_3 - \hat{\underline{u}}_1]}] = (f_{j,k}(t))[\hat{\underline{\mu}}_{[+\hat{\underline{u}}_2 + \hat{\underline{u}}_3 - 2\hat{\underline{u}}_1]}] = (f_{j,k}(t))[\underline{\mathrm{F}}] = (Gm_jm_k)[\hat{\underline{\mu}}_{[-\hat{\underline{u}}_2 + 3\hat{\underline{u}}_3 - 2\hat{\underline{u}}_1] + [2\hat{\underline{u}}_2]}]/(r_{j,k}(t)^2)[\hat{\underline{\mu}}_{[+2\hat{\underline{u}}_3]}].$$

When **t** arrives at **t***, this 're-qualified' equation gives a different answer than does the 'unqualified', 'pure-quantitative' equation, namely, the value μ_0, a value *quanto-**qualitatively** different* from ∞ [μ_0 for ${}_{\alpha}\underline{\underline{\mu}}$ is correlative u_0 'for $\underline{\mathbf{U}}$ as denoting the non-standard, '*quanto-qualitative*'/'***ontological zero***' of the ${}_{\alpha}\underline{\underline{\mu}}$ arithmetic.], such that $\mu_0 \;\ne\; \infty$:

$$(dp(t^*)/dt)[\hat{\underline{\mu}}_{[+\hat{\underline{u}}_2 + \hat{\underline{u}}_3 - 2\hat{\underline{u}}_1]}] = (f_{j,k}(t^*))[\,\underline{\mathrm{GM}}^2/\underline{\mathrm{L}}^2\,] \quad = \quad (Gm_jm_k)[\hat{\underline{\mu}}_{[+\hat{\underline{u}}_2 + 3\hat{\underline{u}}_3 - 2\hat{\underline{u}}_1]}] / (0)\cdot[\hat{\underline{\mu}}_{[+2\hat{\underline{u}}_3]}]$$

$$= \quad (G\cdot m_j\cdot m_k)\cdot[\hat{\underline{\mu}}_{[+\hat{\underline{u}}_2 + 3\hat{\underline{u}}_3 - 2\hat{\underline{u}}_1]}] / \left(0\cdot\hat{\underline{\mu}}_{[+2\hat{\underline{u}}_3]}\right) \quad = \quad (G\cdot m_j\cdot m_k)\cdot[\hat{\underline{\mu}}_{[+\hat{\underline{u}}_2 + 3\hat{\underline{u}}_3 - 2\hat{\underline{u}}_1]}] / \mu_0$$

$$= \quad (G\cdot m_j\cdot m_k)\cdot[\hat{\underline{\mu}}_{[+\hat{\underline{u}}_2 + 3\hat{\underline{u}}_3 - 2\hat{\underline{u}}_1]}]\cdot\mu_{-0} \quad = \quad (G\cdot m_j\cdot m_k)\cdot[\hat{\underline{\mu}}_{[+\hat{\underline{u}}_2 + 3\hat{\underline{u}}_3 - 2\hat{\underline{u}}_1]}]\cdot\mu_{+0}$$

$$= \quad (G\cdot m_j\cdot m_k)\cdot[\hat{\underline{\mu}}_{[+\hat{\underline{u}}_2 + 3\hat{\underline{u}}_3 - 2\hat{\underline{u}}_1]}]\cdot\mu_0 \quad = \quad (G\cdot m_j\cdot m_k)\cdot\mu_0 \quad = \mu_0,$$

because, for any $x \in \mathbb{R}$, $\hat{\underline{\underline{X}}} \sqsubset {}_{\alpha}\underline{\underline{\mu}}$:

$$(x)[\hat{\underline{\underline{X}}}] / \mu_0 = (x)\cdot[\hat{\underline{\underline{X}}}]\cdot\mu_{-0} = x\cdot\hat{\underline{\underline{X}}}\cdot\mu_{+0} = x\hat{\underline{\underline{X}}}\mu_0 \quad = \mu_0.$$

What does the sudden, terminal appearance of this '*quanto-qualitative*', or '*qualo-quantitative*', '*existential*' form of *zero, of this 'black hole' of arithmetic, this all-devouring, 'Hridayamic' meta-number*, μ_0, from the ${}_{\alpha}\underline{\underline{\mu}}$ or $\underline{\underline{\alpha}}$ arithmetic, signify?

▮ It signifies that planets j and k, and indeed the entire assumed ontology; the '"ontological commitments"', and '*ontological, existential context*' of the original model, has ceased to be; has been, as it were, '*refuted*' by its own logical/dynamical/temporal/'*historical*' ***consequences***, mirroring the consequences ensuing in the reality that this model models. It signifies that ***reality has***, from time t^* forward, ***revolutionized itself***, and thereby '***exceeded***' and '***superceded***' that initial model specification, its premises and its ontology; has made that model specification and that ontology ***obsolete***; no longer reflective of what is extant; no longer descriptive of what is *existent*, in short, that a 'temporal, dynamical ***auto-«reductio ad absurdum»***' of those model premises has occurred or ensued. Speaking 'ontologically', in the non-reductionist sense of the word '*ontology*', in which the formerly-existent planet j and planet k have '***ontological status***' -- are considered part of "***what is***" according to the model specification -- the old planetary system has ceased to exist, is henceforth '***ontologically, existentially missing/absent***', or, translated ideographically, is henceforth μ_0. A new planetary system, with a new planetary ontology, has supplanted it. ***Reality itself*** has '***negated***', by having ***self-expanded beyond*** -- by having 'quanto-qualitatively' or 'qualo-quantitatively' and 'ontologically' outgrown -- its own past, temporal, temporary, ***historical premises***. Likewise, and correspondingly, our ***model*** of that reality has ***accomplished its own self-obsolescence***, has '*germinated*' the '"***seeds of its own destruction***"' which it contained from its outset. That model ***qua*** its own self-implied, self-predicted historical/temporal/logical ***consequences***, beginning from the moment of its '*meta-finite*', '*self-conversion singularity*', t^*, '*reaches back around*' and ***negates*** itself ***qua*** its own premises, including their ontology, their '"***ontological commitments***"'. We have a ***logical*** but also a ***temporal*** and ***dynamical self-«reductio ad absurdum»*** of the earlier model specification. This means that ***a new ontology***, requiring a revised/expanded model specification, has arisen. This arising reflects the fact that a qualitatively expanded, ontologically ***net***-expanded ***next new, successor system*** in that historical ***sequence of systems*** that we call the '***meta-system***' has arisen from the ***old, predecessor system***. It has arisen, *as a whole*, not as a '*convolute*' but as an '*evolute*' successor, via an operation of '***self-«aufheben»***' enacted by the predecessor system upon itself. The resultant, new planetary system contains and ***«aufheben»*** conserves-in-negating/-superseding the old planetary system and [at least *most of*] its former ontology. We hold that '***The Paradox of Singularity***', is one symptom of an absence, an omission, an 'incompleteness' in our apparently "purely-quantitative" mathematics. It points to, and may eventually call into our consciousness, the need for an arithmetic that can formulate ontological '***dis-exist-entiation***' or '***dis-entiation***', the [e.g., local] ***dis***-appearance of a *formerly appearing* ***ent***ity or ontological category of ***ent***ities/'*event*ities'. As well, the needed arithmetic is one which can formulate the emergence -- the gradual or sudden appearance -- of qualitatively new ***ent***ities; the self-population of new ontological categories. ***Singularity signifies radical 'self-bifurcation'***. The "*state-space*" or "*phase-space*" activity of the mutually-induced motions of planets j and k has, via their collision, reached back around, invaded, and changed the '"control-/parameter-space"' of this "many-body" gravitic system. The "control-/parameter-space" for this specific "state-space" or "phase-space" may be called 'mass-space'. That 'mass-space' has a mass-axis for m_j and, perpendicular to it, another mass-axis, for m_k. As of t^*, those two axes, or dimensions, of this gravitic control-space have, ***in effect*** [i.e., if the dynamical content of that space is to track the ***finite*** actuality of what is occurring in the ***physical*** solar system being modeled], ***disappeared*** from that multi-dimensional space [n-dimensional for an n-body problem] -- ***as if*** collapsing-back or retracting into the origin of that mass-space -- and new axes, new dimensions, reflecting the collision fragments, have ***as if*** irrupted from the origin of that mass-space control-/parameter-space. '***The Paradox of Singularity***' is both a *symptom of*, and a *clue to* a ***missing mathematics***, including to a ***neglected, omitted, elided arithmetic***. That arithmetic must be one which can ideographically and algorithmically symbolize, ***manifestly***, that which has become ***unmanifest***; which can ***symbolically manifest*** such '***de***-manifestation', via a higher form of the zero, that is, a higher, 'meta-form' of the ancient '"placeholder"' for "sand-board" '*missingness*'; the sign for absence, for the assertion of abeyance; for the explicit ***posit***-ing, the ***posit***-ive notation, of the actually ***un-posit***ed or the *no-longer-posited*; for the noting and the notation of *the existentially* ***negated*** and *the vanished*; for the *present*-ing of the *no-longer-present*. Such expression must be, in part, a ***qualitative*** and an ***ontological*** matter in the matter of singularity, for it is the '***de****-presenting*' of a quality or qualities, of an ontological category or 'onto', that needs to be expressed ideographically, arithmetically, mathematically; a '***de***-presenting' that ties to an irruption of new ontology, beyond the ken of the old model specification; beyond its capability to express/formulate that new ontology in any other, less vacant, less vacuous way than via μ_0. It is not just the absence of a 'pure' quantity, or of a quantity ***of something / anything***, that needs to be recorded. It is also ***the absence of a specific something***, e.g., the sudden absence of our hypothetical planets j and k due to their collision-conversion into new bodies; or, e.g., the sudden absence of 'fusion-able' Hydrogen in the now-Helium-converted core of a "main sequence star" that is now leaving the "main sequence", that will next be burning Helium in that core. It is, *in toto*, a '***quanto-qualitative***', or, just as much, a '***qualo-quantitative***' matter, which therefore cannot be adequately reduced, with full meaning intact, to '***un***-qualified', 'pure-quantitative' expression, or adequately formulated in *a* ***language*** *of the purely-quantitative*; of '*quantity-only*', '*quantifier-only*', '***qualifier-less***' quantification-without-'qualification'. The need for and the concept of this distinction between 'pure quantitative' **0** and '[quanto-]qualitative', 'existential', or 'ontological' absence, is encountered in a discussion of Zeno's "Arrow" paradox in a recent book on the history of mathematical concepts of infinity: "The problem is that where the Arrow is metaphysical it is also extremely subtle and abstract. Consider for instance another hidden premise, or maybe a kind of subpremise that's implicit in Zeno...: is it really true that something's got to be either moving or at rest? At first it certainly looks true, provided we take 'at rest' to be a synonym for 'not moving'. Remember LEM [Law of the Excluded Middle -- F.E.D.], after all. Surely, at any given instant *t*, something is either moving or else not moving, meaning that it has at *t* either a Rate [of Speed -- F.E.D.] > 0 or a Rate [of Speed -- F.E.D.] = 0. That in truth this disjunction is *not* valid -- that LEM doesn't really apply here -- can be seen by examining the difference between the number 0 and the abstract word '***nothing***' [or '***no longer any*** of ***just one some***thing', while non-zero quantities of ***other*** things -- constituents of other '***ontos***' -- continue to be present in the context of discourse -- F.E.D.]. It's a tricky difference, but an important one. The [Ancient -- F.E.D.] Greeks' inability to see it was probably what kept them from being able to use 0 in their math, which cost them dearly. But 0 v. nothing is one of those abstract distinctions that's ***almost impossible to talk about directly***; you more ***have to do it with examples*** [or rather, it is difficult to talk about and one needs to talk about it ***inductively***, through ***instances***, ***until*** the concept becomes ***familiar enough*** that (a) name(s) -- (a) word-symbol(s) -- and/or (an) ideographic symbol(s) -- are created for it, so that one can recall it within oneself, and within one's discussants, by such symbolic references -- F.E.D.]. Imagine there's a certain math class, and in this class there's a fiendishly difficult 100-point midterm, and imagine that neither you nor I get even one point out of 100 on this exam. ***Except there's a*** [***qualitative*** -- F.E.D.] ***difference***: you are not in the class and didn't even take the exam, whereas I am and did. The fact that you received 0 points on the exam is thus irrelevant -- your 0 means N/A, nothing -- whereas my zero is an actual zero. Or, if you don't like that one, imagine that you and I are respectively female and male, both healthy and 20-40 years of age, and we're both at the doctor's, and neither of us has had a menstrual period in the past ten weeks, in which case my total number of periods is nothing, whereas yours here is 0 -- and significant. End examples. So it's simply not true that something's always got to be either 0 or not-0; it might instead be nothing, N/A [↔ the '*meta-number*' constant value μ_0 in the ${}_{\alpha}\underline{\mu}$ ideography -- F.E.D.]." [D. Wallace, *Everything and More: A Compact History of* ∞, W. W. Norton, [New York, NY: **2003**], pp. **141-142**, *emphasis* added by F.E.D.]. ▮

'The Nonlinearity Barrier' and *'The Zero Barrier'*: *'The Paradox of Singularity'* and *The Problem of Zero Division* -- **of the Remaining 'Incompleteness' of Standard Arithmetic Regarding the Arithmetic of Zero**. In trying to ***squeeze*** the *'quanto-qualitative'* meaning of μ_0 into that of the *'pure-quantitative'* $\mathrm{Gm_jm_k/0} = \infty$, some of that meaning must escape; some of that meaning is inescapably ***squeezed out***. *In the cases of the 'singularity-prone', 'singularity-pregnant',* ***'autokinesic'***, ***'self-reflexive'***, *or* ***nonlinear*** *differential equations, starting with one as simple-looking as --* $\mathrm{dx(t)/dt} = \mathrm{x(t)(\, x(t)\,)} = \mathrm{x(t)} \cdot \mathrm{x(t)} = \mathrm{x(t)}^2$, *with initial state* $\mathrm{x(0)} = 1$, *whose solution is therefore* $\mathrm{x(t)} = 1/(1 - \mathrm{t})$ *-- which exhibits a "moveable pole" singularity at* $\mathrm{t^*} = 1$, *this extrusion of meaning can appear to become complete; to result in complete "meaninglessness"*. We thus see the mental, ***mathematical phenomenon*** of singularity as a silent immanent critique -- as a *self* critique, standardly unrecognized as such -- of the mathematics which manifests it, and as an immanent manifestation of the necessity for a new level of mathematical analysis, one which can readily 're-*semantify*' such 'self-***de***-semantified', "meaningless" singularities. These are *'meaninglessnesses'* which irrupt suddenly, from a single moment in time, *in the midst of even our most* ***hyper-meaningful***, ***hyper****-accurate models of reality, whose* ***hyper****-accuracy holds true right up to the thresholds of those moments of singularity*. This new level of analysis also calls for -- as we have seen above, and given the ***arithmetical roots*** of nonlinear, dynamical differential-equation singularity in arithmetical 'zero denomination' -- a new, higher level of arithmetic. It calls for a higher arithmetic that can make sense out of, and render tractable, that long ***intractable***, unsolved, and all-but-universally-declared "unsolvable" *trouble, blemish, flaw*, and *'incompleteness'* in our contemporary everyday arithmetic; that *treasure of immanent difficulty, pointing to future conceptual-technological opportunity* -- that relic, and remnant, of an earlier 'conceptual singularity' or "crisis" in our arithmetical 'meta-evolution' -- the still-living fossil of *'The Zero Barrier'*, which, in its still-unconquered part, has been *«aufheben»* inherited by, and carried forward into, as one of the two key ingredients in, our current «***insolubilium***» and '***intractibilium***'; our contemporary, if oft unrecognized, conceptual crisis: *'The Nonlinearity Barrier'*. We mean none other than ***the problem of zero division***. Long before the stage of our dialectical self-argument in which there emerges the $\hat{\underline{\mu}}_{\underline{\mathrm{U}}}$ *sub*-system of arithmetic of the ${}_{\alpha}\underline{\mu}$ system of arithmetic, and the *absence-value*, μ_0, of $\hat{\underline{\mu}}_{\underline{\mathrm{U}}}$, in that 'meta-systematic' dialectical exposition of ***the dialectic of systems of dialectical arithmetic***, i.e., of ***the immanent development of the dialectical ideographies***, modeled by means of $[\, \underline{\mathbf{N}} \,]^{2^{r}}$, this incipient resolution of the ***zero-division*** or '***additive-identity-division***' conundrum has already begun to emerge. It emerges, in a 'pre-vestigial' form, as early as those stages of the ${}_{\underline{\mathbf{W}}}\underline{\mathbf{Q}}$ and ${}_{\underline{\mathbf{W}}}\underline{\mathbf{U}}$ ideographies, which harness the "**W**hole numbers", namely $\mathbf{W} \equiv \{\, 0, 1, 2, 3, 4, 5, 6, \ldots \,\}$, as [re-]sources for the subscripts and superscripts of the *'unquantifiable'* ${}_{\underline{\mathbf{W}}}\underline{\mathbf{Q}}$ 'meta-numerals', and for the subscripts, superscripts, ***and coefficients*** of the *'quantifiable'* ${}_{\underline{\mathbf{W}}}\underline{\mathbf{U}}$ 'meta-numerals'. This incipient resolution becomes even more intriguing among the ${}_{\underline{\mathbf{Z}}}\underline{\mathbf{Q}}$ and the ${}_{\underline{\mathbf{Z}}}\underline{\mathbf{U}}$ dialectical ideographies, which harness the "Integers", namely $\mathbf{Z} \equiv \{\, \ldots, -3, -2, -1, 0, +1, +2, +3, \ldots \,\}$, as parameters -- as sources for the *subscripts* and *superscripts* of the ${}_{\underline{\mathbf{Z}}}\underline{\mathbf{Q}}$ meta-numerals, and for the subscripts, superscripts, ***and coefficients*** of the ${}_{\underline{\mathbf{Z}}}\underline{\mathbf{U}}$ meta-numerals. It helps one to follow calculations in ${}_{\underline{\mathbf{Z}}}\underline{\mathbf{Q}}$ and ${}_{\underline{\mathbf{Z}}}\underline{\mathbf{U}}$ if one visualizes each $\hat{\mathrm{q}}_z$ and $\hat{\mathring{\underline{\mathrm{u}}}}_z$, for all $\mathrm{z} \in \mathbf{Z} - \{0\}$ as *finite*-length, indeed as *unit* length, ray-segments, pointing in a direction perpendicular to those for all other subscripts from $\mathbf{Z} - \{0\}$, and if one visualizes q_0 and u_0, respectively, as denoting ["origin"] "points", the [only] "points" shared in common by all of the unit-rays, corresponding to all of the subscripts from $\mathbf{Z} - \{0\}$. Picture $\hat{\mathrm{q}}_{-z}$ and $\hat{\mathring{\underline{\mathrm{u}}}}_{-z}$ also as unit-rays, collinear with those of $\hat{\mathrm{q}}_{+z}$ or $\hat{\mathrm{q}}_z$ and $\hat{\mathring{\underline{\mathrm{u}}}}_{+z}$ or $\hat{\mathring{\underline{\mathrm{u}}}}_z$, but pointing in the opposite directions. A key difference of the $\hat{\mathrm{q}}_z$ from the $\hat{\mathring{\underline{\mathrm{u}}}}_z$ is that the $\hat{\mathring{\underline{\mathrm{u}}}}_z$ are 'addable' or 'quantifiable' in 'full $\mathbf{Z}$ *multiplicity*', but the $\hat{\mathrm{q}}_z$ are '***unquantifiable***', or *"additively idempotent"* and *'unit-interval confined'*: $\hat{\mathring{\underline{\mathrm{u}}}}_z + \hat{\mathring{\underline{\mathrm{u}}}}_z = 2\hat{\mathring{\underline{\mathrm{u}}}}_z$, $\forall \mathrm{z} \in \mathbf{Z} - \{0\}$, but $\hat{\mathrm{q}}_z + \hat{\mathrm{q}}_z + \ldots + \hat{\mathrm{q}}_z = \hat{\mathrm{q}}_z$. Keep in mind that $\hat{\mathrm{q}}_1, \hat{\mathrm{q}}_2$ & $\hat{\mathrm{q}}_3, \ldots$, and $\hat{\mathring{\underline{\mathrm{u}}}}_1, \hat{\mathring{\underline{\mathrm{u}}}}_2$ & $\hat{\mathring{\underline{\mathrm{u}}}}_3, \ldots$, as well as $\hat{\underline{\mu}}_1, \hat{\underline{\mu}}_2$ & $\hat{\underline{\mu}}_3, \ldots$, etc., herein are 'meta-***numerals***', intending [non-standard] 'meta-***numbers***', i.e., "***constants***", ***not*** "variables"; '***knowns***', ***not*** "unknowns". They do not denote algebraic variables/unknowns, like x, y, & z conventionally do [even though they are "literals", use letters, as do typical algebraic variables], any more than does $\mathrm{i} \in \mathbf{C}$. All the above, plus the principle that $\hat{\mathrm{q}}_{-j} + \hat{\mathrm{q}}_{+j} = \mathrm{q}_0 = \hat{\mathrm{q}}_{-j} + \hat{\mathrm{q}}_{+j}$, plus the '***meta-genealogical evolute*** *product rule*' in ${}_{\underline{\mathbf{Z}}}\underline{\mathbf{Q}}$, namely:

$\hat{\mathrm{q}}_j \times \hat{\mathrm{q}}_k = \hat{\mathrm{q}}_j + \hat{\mathrm{q}}_k + \hat{\mathrm{q}}_{j+k}$, for $\mathrm{j, k} \in \mathbf{Z}$, plus –

$$\mathrm{q}_0 \equiv 0 \times \hat{\mathrm{q}}_j = \hat{\mathrm{q}}_j - \hat{\mathrm{q}}_j = \hat{\mathrm{q}}_{j-j} = \hat{\mathrm{q}}_j \div \hat{\mathrm{q}}_j = \hat{\mathrm{q}}_j / \hat{\mathrm{q}}_j = \hat{\mathrm{q}}_{+j} \times \hat{\mathrm{q}}_{-j} = \hat{\mathrm{q}}_{+j} + \hat{\mathrm{q}}_{-j} + \hat{\mathrm{q}}_{+j+(-j)} =$$

$$\mathrm{q}_0 + \hat{\mathrm{q}}_{j-j} = \mathrm{q}_0 + \mathrm{q}_0 = \mathrm{q}_0 = \hat{\mathrm{q}}_{-j} \times \hat{\mathrm{q}}_{+j} = \hat{\mathrm{q}}_{-j} + \hat{\mathrm{q}}_{+j} + \hat{\mathrm{q}}_{-j+(+j)} = \mathrm{q}_0 + \hat{\mathrm{q}}_{-j+j} = \mathrm{q}_0 + \mathrm{q}_0 = \mathrm{q}_0 =$$

$$\hat{\mathrm{q}}_j^{+1} \times \hat{\mathrm{q}}_j^{-1} = \hat{\mathrm{q}}_j^{+1+(-1)} = \hat{\mathrm{q}}_j^{-1} \times \hat{\mathrm{q}}_j^{+1} = \hat{\mathrm{q}}_j^{-1+(+1)} = \hat{\mathrm{q}}_j^{0} = \mathrm{q}_0 = +\hat{\mathrm{q}}_j \times -\hat{\mathrm{q}}_j = -\hat{\mathrm{q}}_j \times +\hat{\mathrm{q}}_j = \mathrm{q}_0 =$$

$+\hat{\mathrm{q}}_j + [\, -\hat{\mathrm{q}}_j \,] = -\hat{\mathrm{q}}_j + [\, +\hat{\mathrm{q}}_j \,] = \mathrm{q}_0$ [*products of multiplicative inverses* = *sums of additive inverses* = q_0],

plus $\hat{\mathrm{q}}_j + \mathrm{q}_0 = \hat{\mathrm{q}}_j + \mathrm{q}_0 = \hat{\mathrm{q}}_j$ [q_0 as ***additive identity element***], and $\mathrm{q}_0/\mathrm{q}_0 = \mathrm{q}_0^{\pm1+(\mp1)} = \mathrm{q}_0^{0} = \mathrm{q}_0$, *all together imply* -- $q_0 \times q_0 = q_0 + q_0 + \hat{q}_{0+0} = q_0 + \hat{q}_{0+0} = q_0 \div q_0 = q_0$, and

$\hat{q}_j \times q_0 = \hat{q}_j + q_0 + \hat{q}_{j+0} = \hat{q}_j + \hat{q}_j = \hat{q}_j$, and $q_0 \times \hat{q}_j = q_0 + \hat{q}_j + \hat{q}_{0+j} = \hat{q}_j + \hat{q}_j = \hat{q}_j$

[q_0 as ***multiplicative identity element***, *via additive idempotency*], and $\hat{q}_{-j} = +\hat{q}_{-j}^{\,+1} = -\hat{q}_{+j}^{\,+1} = +\hat{q}_{+j}^{\,-1}$,

and $q_0 \div \hat{q}_j = q_0/\hat{q}_{+j} = q_0 \times \hat{q}_{-j} = q_0 + \hat{q}_{-j} + \hat{q}_{+0+(-j)} = \hat{q}_{-j} + \hat{q}_{0-j} = \hat{q}_{-j} + \hat{q}_{-j} = \hat{q}_{-j} = \hat{q}_{+j}^{\,-1} = -\hat{q}_j$;

so also -- $\hat{q}_j \div q_0 = \hat{q}_j/q_0 = \hat{q}_j \times \hat{q}_{-0} = \hat{q}_j + \hat{q}_{-0} + \hat{q}_{+j+(-0)} = \hat{q}_j + \hat{q}_{j-0} = \hat{q}_j + \hat{q}_j = \hat{q}_j =$

$+\hat{q}_{-j}^{\,-1} = -\hat{q}_{-j}^{\,+1} = -\hat{q}_{+j}^{\,-1}$ [note how the '+' & '−' signs can '*make the rounds*', '*rotate together*', or '*circulate*' from *script/coefficient* position to *superscript/exponent* position to *subscript/index* position, and back around again, ***without changing the value/identity of the meta-numeral***.]. ***Division by the additive identity*** in ${}_{\underline{z}}\underline{\Omega}$ yields [*meta-*]*finite, tractable,* ***determinate, well-defined, meaningful results.*** Note that the ***additive identity element*** & ***multiplicative identity element*** of ${}_{\underline{z}}\underline{\Omega}$ are ***identical*** -- identically q_0.

The '***meta-heterosis convolute** product rule*' in ${}_{\underline{z}}\underline{U}$, namely: $\hat{\hat{\underline{u}}}_n \times \hat{\hat{\underline{u}}}_m = \hat{\hat{\underline{u}}}_{n+m}$, for all $n, m \in \mathbf{Z}$,

with $\hat{\hat{\underline{u}}}_n \div \hat{\hat{\underline{u}}}_m = \hat{\hat{\underline{u}}}_n \times \hat{\hat{\underline{u}}}_{-m} = \hat{\hat{\underline{u}}}_{+n} / \hat{\hat{\underline{u}}}_{+m} = \hat{\hat{\underline{u}}}_n / \hat{\hat{\underline{u}}}_m = \hat{\hat{\underline{u}}}_{n+(-m)} = \hat{\hat{\underline{u}}}_{n-m}$, plus

$u_0 = 0 \times \hat{\hat{\underline{u}}}_n = \hat{\hat{\underline{u}}}_n - \hat{\hat{\underline{u}}}_n = \hat{\hat{\underline{u}}}_{n-n} = \hat{\hat{\underline{u}}}_n \div \hat{\hat{\underline{u}}}_n = \hat{\hat{\underline{u}}}_n / \hat{\hat{\underline{u}}}_n = \hat{\hat{\underline{u}}}_{+n} \times \hat{\hat{\underline{u}}}_{-n} = \hat{\hat{\underline{u}}}_{+n+(-n)} =$

$\hat{\hat{\underline{u}}}_{n-n} = u_0 = \hat{\hat{\underline{u}}}_{-n} \times \hat{\hat{\underline{u}}}_{+n} = \hat{\hat{\underline{u}}}_{-n+(+n)} = \hat{\hat{\underline{u}}}_{-n+n} = u_0 = \hat{\hat{\underline{u}}}_n^{\,+1} \times \hat{\hat{\underline{u}}}_n^{\,-1} = \hat{\hat{\underline{u}}}_n^{\,+1+(-1)} =$

$\hat{\hat{\underline{u}}}_n^{\,0} = u_0 = \hat{\hat{\underline{u}}}_n^{\,-1} \times \hat{\hat{\underline{u}}}_n^{\,+1} = \hat{\hat{\underline{u}}}_n^{\,-1+(+1)} = \hat{\hat{\underline{u}}}_n^{\,0} = u_0 = +\hat{\hat{\underline{u}}}_n \times -\hat{\hat{\underline{u}}}_n = -\hat{\hat{\underline{u}}}_n \times +\hat{\hat{\underline{u}}}_n = u_0 =$

$+\hat{\hat{\underline{u}}}_n + [-\hat{\hat{\underline{u}}}_n] = -\hat{\hat{\underline{u}}}_n + [+\hat{\hat{\underline{u}}}_n] = u_0$

[i.e., *in this,* ***convolute*** *product version of* ${}_{\underline{z}}\underline{U}$, *products of mutual* ***multiplicative*** *inverses* = *sums of mutual* ***additive*** *inverses* = u_0],

plus $u_0 + \hat{\hat{\underline{u}}}_n = \hat{\hat{\underline{u}}}_n + u_0 = \hat{\hat{\underline{u}}}_n$ [u_0 as ***additive identity element***], plus $u_0/u_0 = u_0^{\pm 1+(\mp 1)} = u_0^{\,0} = u_0$, ***all together imply*** --

$u_0 \times u_0 = \hat{\hat{\underline{u}}}_{0+0} = u_0$, & $u_0 \div u_0 = u_0/u_0 = u_{+0} \times u_{-0} = u_{+0+(-0)} = u_0$, and

$\hat{\hat{\underline{u}}}_n \times u_0 = \hat{\hat{\underline{u}}}_{n+0} = \hat{\hat{\underline{u}}}_n$, & $u_0 \times \hat{\hat{\underline{u}}}_n = \hat{\hat{\underline{u}}}_{0+n} = \hat{\hat{\underline{u}}}_n$ [u_0 as ***multiplicative identity element***, ***even w/o*** *additive idempotency*];

$u_0 \div \hat{\hat{\underline{u}}}_n = u_0 / \hat{\hat{\underline{u}}}_n = u_0 \times \hat{\hat{\underline{u}}}_{-n} = \hat{\hat{\underline{u}}}_{0+(-n)} = \hat{\hat{\underline{u}}}_{0-n} = \hat{\hat{\underline{u}}}_{-n} = -\hat{\hat{\underline{u}}}_n = \hat{\hat{\underline{u}}}_n^{\,-1}$; *therefore* --

$\hat{\hat{\underline{u}}}_n \div u_0 = \hat{\hat{\underline{u}}}_n / u_0 = \hat{\hat{\underline{u}}}_n \times u_{-0} = \hat{\hat{\underline{u}}}_{+n+(-0)} = \hat{\hat{\underline{u}}}_{n-0} = \hat{\hat{\underline{u}}}_n = +\hat{\hat{\underline{u}}}_{-n}^{\,-1} = -\hat{\hat{\underline{u}}}_{-n}^{\,+1} = -\hat{\hat{\underline{u}}}_{+n}^{\,-1}$

Division by the additive identity in ${}_{\underline{z}}\underline{U}$ yields [*meta-*]*finite, tractable,* ***determinate, well-defined, meaningful results.***

Note that the ***additive identity element*** and the ***multiplicative identity element*** of ${}_{\mathbf{z}}\underline{\mathbf{U}}$ are ***identical*** -- identically u_0, for the product-rules specified above. Note also that ***additive inverses*** and ***multiplicative inverses*** are ***equivalent*** in both ${}_{\mathbf{z}}\underline{\mathbf{Q}}$ & ${}_{\mathbf{z}}\underline{\mathbf{U}}$, given the product rules as specified above, as they are also in $\mathbf{I} = \mathbf{C}/\mathbf{R}$: $-i = i^{-1} = 1/i$. We hold that it is none other than '*The Elision of the Qualifiers*' itself that has led to the zero division conundrum of ordinary arithmetic, later inherited by the '*The Singularity Barrier*' component of the integro-differential equation '*Nonlinearity Barrier*'. We therefore hold that the zero-division conundrum is an adverse consequence, and an artefact, of '*The Elision Of The Qualifiers*', and of the consequent incapability to distinguish "pure-quantitative", scalar zeros, **0**, from quanto-*qualitative, ontological, existential* absences, denoted by $\boldsymbol{\mu}_0$ in the '*metrical qualifier*' sub-arithmetic, as in the '*ontological qualifier*' sub-arithmetic, of the ${}_{\mathbf{o}}\underline{\boldsymbol{\mu}}$ dialectical ideography, and therefore in the ${}_{\mathbf{o}}\underline{\boldsymbol{\mu}}$ ideography as a whole. In the spirit of the foregoing discussion, we therefore endorse the following statement of principles by the founder of **Meta Research**: "Even though this book deals with technical material, I have done my best to make the text readable even to those with little or no background in the field of astronomy. ***The book deals almost entirely with concepts***, with little attention to ways to utilize those concepts in calculations [i.e., of the usual, 'pure-quantitative' kind -- **F.E.D.**]. My college major was mathematics, and my field of specialization in astronomy is Celestial Mechanics, which is itself a field accustomed to descriptions utilizing *the language of mathematics* [this is '*hyper-hypo-bole*': *an understatement truly vast!* -- **F.E.D.**]. But for this book I have invoked a working principle I wish others would use more often: "***Mathematics should be used to describe the operation of models, not to build them.***" In my opinion, equations cannot be made to substitute for the concepts which underlie them. And equations are generally blind to limitations of range and physical constraints. They are ***too general*** and simply ***lack the sort of specificity*** [**F.E.D.**: i.e., are typically too abstract, too '*elided*', too ***lacking in sufficient richness of determinations to encompass the concrete conceptual content***] that true, intuitive understanding demands. Every equation has a domain of applicability -- usually the range of the observations and little, if anything, more. I use the following as a rule of thumb: If an equation can be extrapolated outside its domain and gives a ***singularity*** (basically, a *zero divisor*), that ***singularity does not exist in nature***; instead, *the model needs modification*. Up to now this rule has always proved true." [Tom Van Flandern, ***Dark Matter, Missing Planets, and New Comets***: ***Paradoxes Resolved, Origins Illuminated***, North Atlantic Books [Berkeley: **1993**], p. **xxi**.]. Not content to leave ***unbridged*** such ***gaps*** or ***discontinuities*** -- between successive systems' models' "domains of applicability", and between the 'implied ontologies' or "ontological commitments" of their model-specifications -- we have sought models of '***meta***-systems', i.e., of self-generating, 'qualitatively Peanic' successions of qualitatively, ontologically self-changing, self-expanding, self-revolutionizing systems, and an ideographical language for modeling *them*, which can transcend epochal, ***historical specificities*** and limits of applicability, and which can thereby straddle these *breaks* and *scissures* in the '*meta-continuity*' of the '***meta-dynamical***' '***meta-evolutions***' that are immanently-induced by the '***self-conversion singularities***' or '***ontologically-dynamical self-bifurcations***' within the '***meta-systematical***' systems-progressions of nature, including those of the historical/collective human "mind".

Caveats. There are difficulties, complexities, and subtleties with q_0, u_0 as unified, *additive/multiplicative* "'***identity-elements***'", and with $\boldsymbol{\mu}_0$ as defined above, e.g., regarding "'scalar'" zero values of ***numerator-resident*** quantifier-functions, and of transcendental-function ***argument-resident*** zero divisions, as well as "'order-of-operation'" conventions and 'parity-principle' issues involving the '***meristemal principle***' of [***non-distributive***, 'non-superpositioning', or "'***non-linear***'"] «***aufheben***» 'generalized multiplication', or 'operator operation', which are not addressed in this primer. Standard texts tend to regard the unification of additive identity and multiplicative identity, in the "'algebraic structures'" of "'Modern Abstract Algebra'", such as "'Ring'" structures, as self-contradictory and impossible: "There is one very simple ring that consists only of the additive identity **0**, with addition and multiplication given by $0 + 0 = 0$, $0 \cdot 0 = 0$; this ring is usually called the *trivial ring*. **Corollary** Let R be the ring with identity **1**. If R is not the trivial ring, then the elements **0** and **1** are distinct. *Proof.* Since $R \neq \{0\}$, there exists some non-zero element $a \in R$. If **0** and **1** were equal, it would follow that $a = a1 = a0 = 0$, an obvious contradiction. CONVENTION. Let us assume, once and for all, that any ring with identity contains more than one element. This will rule out the possibility that **0** and **1** coincide." [Burton, D. M., 1972, ***Abstract and Linear Algebra***, Reading, MA: Addison-Wesley, p. 178, **emphasis** *added* by **F.E.D.**]. Yet we have it that $\hat{\mathsf{q}}_j + \mathsf{q}_0 = \hat{\mathsf{q}}_j + \mathsf{q}_0 = \hat{\mathsf{q}}_j$, *as well as that* $\hat{\mathsf{q}}_j \times \mathsf{q}_0 = \hat{\mathsf{q}}_j + \mathsf{q}_0 + \hat{\mathsf{q}}_{j+0} = \hat{\mathsf{q}}_j + \hat{\mathsf{q}}_j = \hat{\mathsf{q}}_j$, *and that* $\mathsf{q}_0 \times \hat{\mathsf{q}}_j = \mathsf{q}_0 + \hat{\mathsf{q}}_j + \hat{\mathsf{q}}_{0+j} = \hat{\mathsf{q}}_j + \hat{\mathsf{q}}_j = \hat{\mathsf{q}}_j$, for ${}_{\mathbf{z}}\underline{\mathbf{Q}}$. The ${}_{\mathbf{z}}\underline{\mathbf{Q}}$ arithmetic, under the '***meta-genealogical evolute product rule***' of 'ontological multiplication', fulfills ***all*** of the rules defining a "[distributive] Ring", except for the classical 'half' of the rule of "'***distributivity***'" [or '***linearity***'], that for "'multiplication over addition'": $\underline{\mathbf{a}} \cdot [\,\underline{\mathbf{b}} + \underline{\mathbf{c}}\,] \neq, \tfrac{>}{<} \ \underline{\mathbf{a}} \cdot \underline{\mathbf{b}} + \underline{\mathbf{a}} \cdot \underline{\mathbf{c}}$, due to the '***meristemal principle***' of our «***aufheben***» '***generalized multiplication***', or '***flexion***', operation. It is the case, for the other 'half' of that rule, that addition "'distributes over'" multiplication in ${}_{\mathbf{z}}\underline{\mathbf{Q}}$: $[\,\underline{\mathbf{b}} + \underline{\mathbf{c}}\,] \cdot \underline{\mathbf{a}} = \underline{\mathbf{b}} \cdot \underline{\mathbf{a}} + \underline{\mathbf{c}} \cdot \underline{\mathbf{a}}$. Thus, the proof quoted above does not, strictly speaking, apply to ${}_{\mathbf{z}}\underline{\mathbf{Q}}$, for it, indeed, depends upon the distributive law for the necessity of its assumption that $a0 = 0$. We plan to address these and other issues of the systems of dialectical arithmetic in a forthcoming treatise, in its third section, '***The Arithmetics of Meta-Evolution***', in a manner concordant with the canons of '***Dialectical Meta-Axiomatics***', and of '***The Gödelian Ideo-Metadynamic***', i.e., '***The Gödelian Dialectic***'. These canons include the axioms-based, rigorous, deductive proof of theorems. They also encompass the evocation of those immanent "'Gödel-formulae'", which locate the '*Gödel-incompletenesses*' of each successive axioms-system of arithmetic. ■ The '***meta-diophantine equations***' to which those "'Gödel-formulae'" map -- ***unsolvable within*** the ***number-ontology*** of the given axioms -- point beyond those axioms, to a 'meta-deductive', dialectical, «***aufheben***» eclosion, adding new axioms, describing new properties of an expanded "*universal set*"/"'*ideo-totality*'", with *added elements*, of higher, '*meta-fractally*' *self-internalizing/power-set-internalizing* "logical type", thus with the ***new, higher kinds*** of '***non-diophantine meta-numbers***' which that thereby expanded universal set 'totality' thereby models. The resulting, qualitatively/'ideo-ontologically' enriched, successor axioms-system of arithmetic, with its expanded '***ideo-ontology of number***' renders solvable, within it, those '***meta-diophantine equations***' that were unsolvable within its predecessor system of arithmetic. But it reveals, in turn, its own '*Gödel-incompletenesses*'; its own "***unsolvable equations***". Thus, this 'Qualo-Peanic' systems-progression ever drives [for] its own self-continuation. ■

The $\underline{\mathbf{Q}}$ arithmetics can model this '*Gödelian Dialectic*', as exemplified -- in terms of the recorded history and 'psycho-archaeology' of arithmetic on this planet -- in the second section of that treatise, entitled '*The Meta-Evolution of Arithmetics*'.

Summary on $_{\alpha}\underline{\underline{\mu}}$. The $_{\mathbf{N},\alpha}\underline{\underline{\mu}}$ or $_{\mathbf{N}}\underline{\underline{\alpha}}$ system of arithmetic involves three component sub-systems of arithmetic:

- The [*ideographical*] arithmetical rules-system for ***quantifiers***, $\underline{\mathbf{N}}$, or $_{\mathbf{N}}\underline{\mu}$, with numerals μ_k, wherein $k \in \mathbf{N}$;
- The [*ideographical*] arithmetic of ***metrical qualifiers***, $\hat{\underline{\mu}}_{\underline{U}}$, with '*meta-numerals*' of the form $\hat{\underline{\mu}}_{\underline{\Sigma}\alpha_j \hat{\underline{u}}_j}$;
- The [*ideographical*] arithmetic of ***ontological qualifiers***, $\hat{\underline{\mu}}_{\underline{\mathbf{N}}}$, with '*meta-numerals*' of the form $\hat{\underline{\mu}}_{k}$, wherein $k \in \mathbf{N}$;
- The latter two '***qualifier arithmetic***' sub-systems can be combined, as $\hat{\underline{\mu}}_{\underline{\mathbf{N}}+\underline{U}}$, and contrasted to the '***quantifier arithmetic***' sub-system, denoted $_{\mathbf{N}}\underline{\mu}$, or $\underline{\mathbf{N}}$, *the* '***quantifier*** half' *of arithmetic thus standing over against the* '***qualifier*** half': $_{\underline{\mathbf{N}},\alpha}\underline{\underline{\mu}} \equiv {}_{\mathbf{N}}\underline{\mu}\hat{\underline{\mu}}_{\underline{\mathbf{N}}+\underline{U}}$, or $\underline{\mathbf{N}}\hat{\underline{\mu}}_{\underline{\mathbf{N}}+\underline{U}}$: $\quad \hat{\underline{\mu}}_{k}\hat{\underline{\mu}}_{\underline{\Sigma}\alpha_j \hat{\underline{u}}_j} = \hat{\underline{\mu}}_{k+\underline{\Sigma}\alpha_j \hat{\underline{u}}_j}$, or, $\hat{\underline{\mu}}_{\partial\mathbf{N}} \times \hat{\underline{\mu}}_{\partial\underline{U}} = \hat{\underline{\mu}}_{\partial\mathbf{N}}\hat{\underline{\mu}}_{\partial\underline{U}} = \hat{\underline{\mu}}_{\partial\mathbf{N}+\partial\underline{U}}$.

Subscript Arithmetics. The $\underline{\mathbf{Q}}$, $\underline{\mathbf{U}}$ and $_{\alpha}\underline{\underline{\mu}}$ systems of arithmetic within dialectical ideography are '*old-arithmetics-**parameterized** new **arithmetics***, and, in particular, '***subscript arithmetics***'. They each advance beyond their predecessor systems of arithmetic by *subsuming* or *assimilating* and «***aufheben***»-conserving them '*parametrically*', at the *coefficient, superscript*, and *subscript* levels. We may thus characterize the initial 'number-spaces' of these successive systems of arithmetic as follows: $_{\mathbf{N}}\underline{\mathbf{Q}} \equiv \{\, \mathbf{N}\hat{q}^{\mathbf{N}}_{\mathbf{N}} \,\} = \{\, \hat{q}^{\mathbf{N}}_{\mathbf{N}} \,\} = \{\, \hat{q}_{\mathbf{N}} \,\}$, *without loss of generality;* $_{\mathbf{N}}\underline{\mathbf{U}} \equiv \{\, \mathbf{N}\hat{\underline{u}}^{\mathbf{N}}_{\mathbf{N}} \,\} = \{\, \mathbf{N}\hat{\underline{u}}_{\mathbf{N}} \,\}$, *without loss of generality;* $_{\underline{\mathbf{N}}}\underline{\underline{\alpha}} \equiv {}_{\mathbf{N},\alpha}\underline{\underline{\mu}} \equiv \{\, \mathbf{N}\hat{\underline{\mu}}^{\mathbf{N}}_{\mathbf{N}+\underline{U}} \,\} = \{\, \mathbf{N}\hat{\underline{\mu}}_{\mathbf{N}+\underline{U}} \,\}$, *without loss of generality, ...*, etc., making explicit, in the $_{\mathbf{N}}\underline{\mathbf{Q}}$ system of arithmetic, the '[***self-***]*subsumption*'/'[***self-***]*subscriptization*' of the $\mathbf{N}$-based $\underline{\mathbf{N}}$ system of arithmetic, and, in the $_{\underline{\mathbf{N}},\alpha}\underline{\underline{\mu}}$ or $_{\underline{\mathbf{N}}}\underline{\underline{\alpha}}$ system of arithmetic, the *subsumption / 'subscriptization'* of the $_{\mathbf{N}}\underline{\mathbf{U}}$ system of arithmetic by the $\underline{\mathbf{M}}$ system of arithmetic. Each '***meta-numeral***' consists of a ***script-level symbol***, which is like a person's "family name", combined with a ***subscript-level symbol***, which is like a person's "individual name". Both 'names' together denote the exact ***identity*** of a given 'meta-numeral's ***meta-number***'. 'Calculations', i.e., transformations of the identities of 'meta-numbers', due to their *interactions* and '*self-interactions*', the latter denoting '*intra-actions*', i.e., to their 'inter-application' or 'mutual-application' and 'self-application', or 'inter- or mutual operation' and 'self-operation', involve algorithmic arithmetical processes especially at the <u>sub</u>script level. These subscript processes proceed in conformity with the rules of the 'subsumed' or 'subscriptized' arithmetic. Those variants of these '***subscript-arithmetics***' that we consider herein are characterized by their somewhat '***sub***script-[as opposed to exponent, power, or ***super***script-]logarithm-like' definition of ***script***-level ***multiplication*** of 'meta-numbers' in terms of their ***subscript***-level ***addition***, of the ***script***-level ***division*** of 'meta-numbers' in terms of their ***subscript***-level ***subtraction***, and of the script-level ***exponentiation*** of 'meta-numbers' in terms of their subscript-level ***multiplication*** [especially for the '***meta-heterosis convolute product rule***']. This «***aufheben***» conservation/subsumption/'subscriptization' of 'earlier' arithmetics -- arithmetics for which computational theorems are already readily available -- has the added advantage that calculations, and proofs of calculation-theorems, for the resulting 'new' arithmetics are facilitated by incorporating aspects of theorems already proven for the '***subsumption-conserved***' 'old' arithmetics. ■

The $_{\alpha}\underline{\underline{\mu}}$ ***Metrical [Sub-]Arithmetic: An Ideography Interpretable for Dimensional Analysis***. The purpose of this sub-section is to briefly show how the $\hat{\underline{\mu}}_{\underline{U}}$ sub-arithmetic of the $_{\alpha}\underline{\underline{\mu}}$ or $\underline{\underline{\alpha}}$ arithmetic can model, in no-longer-"syncopated", but fully-"symbolic" or 'ideographical' and 'algorithmical' fashion, the basic rules of the *arithmetic of **dimensional analysis***, as expressed in the first four theorems of Chapter **5**, on the "***Arithmetic of Dimensions***", pp. **95-96** in the **1998** treatise ***Applied Dimensional Analysis and Modeling***, by T. Szirtes.

That chapter employs that book's standard notational format in which any 'dimensioned' variable $\mathbf{V_k}$ is decomposable into a "<u>m</u>agnitude" 'factor', denoted $\mathbf{m_k}$, and a '<u>d</u>imensional' 'factor', '<u>d</u>imensional unit', or "<u>d</u>imension", denoted $\mathbf{d_k}$, whereby the square brackets 'operator', **[.]**, with the '.' standing for an "'ellipsis dot'" denoting ***any*** [admissible] content/argument, returns just the "<u>d</u>imension" when operating on such variables, $\mathbf{V_k} \equiv \mathbf{m_k}\bullet\mathbf{d_k}$: $[\mathbf{V_k}] = [\mathbf{m_k}\bullet\mathbf{d_k}] \equiv \mathbf{d_k}$. We correspondingly adopt a convention whereby a ***generic*** '***dimensioned quantifier***' variable, i.e., an *explicitly* '***metrically-qualified***' or '***dimensionally-qualified***' variable, is denoted $\mathring{\underline{\mathbf{\Psi}}}_k \equiv \mu_k\bullet\hat{\underline{\mu}}_{\underline{\Sigma}\alpha_j \hat{\underline{u}}_j}$, and whereby the operators **[.]** and **(.)** are defined such that the former 'extracts' only the '*metrical qualifier*', 'metrical unit', or 'metrical monad' component, while the latter 'extracts' only the '*quantifier*' component. We define --

$[\mathring{\underline{\mathbf{\Psi}}}_k] \equiv [\mu_k\bullet\hat{\underline{\mu}}_{\underline{\Sigma}\alpha_j \hat{\underline{u}}_j}] \equiv \hat{\underline{\mu}}_{\underline{\Sigma}\alpha_j \hat{\underline{u}}_j}$, and $(\mathring{\underline{\mathbf{\Psi}}}_k) \equiv (\mu_k\bullet\hat{\underline{\mu}}_{\underline{\Sigma}\alpha_j \hat{\underline{u}}_j}) \equiv \mu_k$. Below, we use the non-standard signs '▛' & '▜' to delimit deductive proofs -- or proposition-sequences that constitute deductive proofs -- as also to enclose proven propositions.

Theorem 1. *Products of Dimensions.* Rule 1 for *the arithmetic of 'dimensioned variables'*, as cited in Szirtes' Chapter, may be stated as follows: ▛ *The product of the dimensions of two variables is the dimension of the product of those two variables* ▟, or, using Szirtes' algebraic symbolism: ▛ $[V_1] \cdot [V_2] = [V_1 \cdot V_2]$ ▟. Using the $_{\alpha}\underline{\mu}$ or $\underline{\alpha}$ sub-notation for the *dimensional analysis* sub-arithmetic, this rule becomes: ▛ $[\overset{\circ}{\underline{\Psi}}_1] \cdot [\overset{\circ}{\underline{\Psi}}_2] = [\overset{\circ}{\underline{\Psi}}_1 \cdot \overset{\circ}{\underline{\Psi}}_2]$ ▟.

Proof of Theorem 1.

Via Szirtes' Notation	#	Via the $_{\alpha}\underline{\mu}$ Sub-Notation of the $\hat{\underline{\mu}}_{\underline{\mu}}$ Sub-Arithmetic for ***Dimensional Analysis***	
Proposition		**Proposition**	**Justification**
▛ $[V_1] = [m_1 \cdot d_1] = d_1$	1	$[\overset{\circ}{\underline{\Psi}}_1] = [\mu_1 \cdot \hat{\underline{\mu}}_{\sum_j \alpha_j \hat{\underline{\mu}}_j}] = \hat{\underline{\mu}}_{\sum_j \alpha_j \hat{\underline{\mu}}_j}$	Definitions: V_k, $\overset{\circ}{\underline{\Psi}}_k$, & [.]
$[V_2] = [m_2 \cdot d_2] = d_2$	2	$[\overset{\circ}{\underline{\Psi}}_2] = [\mu_2 \cdot \hat{\underline{\mu}}_{\sum_j \beta_j \hat{\underline{\mu}}_j}] = \hat{\underline{\mu}}_{\sum_j \beta_j \hat{\underline{\mu}}_j}$	Definitions: V_k, $\overset{\circ}{\underline{\Psi}}_k$, & [.]
$[V_1] \cdot [V_2] = d_1 \cdot d_2 = d_3$	3	$[\overset{\circ}{\underline{\Psi}}_1] \cdot [\overset{\circ}{\underline{\Psi}}_2] = \hat{\underline{\mu}}_{\sum_j \alpha_j \hat{\underline{\mu}}_j} \cdot \hat{\underline{\mu}}_{\sum_j \beta_j \hat{\underline{\mu}}_j} = \hat{\underline{\mu}}_{\sum_j \alpha_j \hat{\underline{\mu}}_j + \sum_j \beta_j \hat{\underline{\mu}}_j}$	Substitution of Equivalents; *Closure of dimensional multiplication*
$V_1 \cdot V_2 = m_1 \cdot d_1 \cdot m_2 \cdot d_2$	4	$\overset{\circ}{\underline{\Psi}}_1 \cdot \overset{\circ}{\underline{\Psi}}_2 = \mu_1 \cdot \hat{\underline{\mu}}_{\sum_j \alpha_j \hat{\underline{\mu}}_j} \cdot \mu_2 \cdot \hat{\underline{\mu}}_{\sum_j \beta_j \hat{\underline{\mu}}_j}$	Substitution of Equivalents
$V_1 \cdot V_2 = (m_1 m_2)[d_1 d_2] = (m_3)[d_3]$	5	$\overset{\circ}{\underline{\Psi}}_1 \cdot \overset{\circ}{\underline{\Psi}}_2 = \mu_1 \cdot \mu_2 \cdot \hat{\underline{\mu}}_{\sum_j \alpha_j \hat{\underline{\mu}}_j} \cdot \hat{\underline{\mu}}_{\sum_j \beta_j \hat{\underline{\mu}}_j} = \mu_1 \cdot \mu_2 \cdot \hat{\underline{\mu}}_{\sum_j \alpha_j \hat{\underline{\mu}}_j + \sum_j \beta_j \hat{\underline{\mu}}_j}$	Rule of Commutation, applied to step 4
$[V_1 \cdot V_2] = d_1 \cdot d_2 = d_3$	6	$[\overset{\circ}{\underline{\Psi}}_1 \cdot \overset{\circ}{\underline{\Psi}}_2] = \hat{\underline{\mu}}_{\sum_j \alpha_j \hat{\underline{\mu}}_j} \cdot \hat{\underline{\mu}}_{\sum_j \beta_j \hat{\underline{\mu}}_j} = \hat{\underline{\mu}}_{\sum_j \alpha_j \hat{\underline{\mu}}_j + \sum_j \beta_j \hat{\underline{\mu}}_j}$	Definition of [.]; *Closure of metrical multiplication*
$[V_1] \cdot [V_2] = [V_1 \cdot V_2]$	7	$[\overset{\circ}{\underline{\Psi}}_1] \cdot [\overset{\circ}{\underline{\Psi}}_2] = [\overset{\circ}{\underline{\Psi}}_1 \cdot \overset{\circ}{\underline{\Psi}}_2]$ ▟	Transitive Rule of Equality, applied to steps 3 & 6; Q.E.D.

Szirtes then generalizes this first Theorem, or "*Theorem of products*", in the form of a corollary, which may be stated as: ▛ *The product of the dimensions of* n *variables is the dimension of the product of those* n *variables* ▟, and, in Szirtes' algebraical ideographic symbolism, as: ▛ $[V_1] \cdot [V_2] \cdot \ldots \cdot [V_n] = [V_1 \cdot V_2 \cdot \ldots \cdot V_n]$ ▟, or, in $_{\alpha}\underline{\mu}$ algebraical ideographic symbolism, as: ▛ $[\overset{\circ}{\underline{\Psi}}_1] \cdot [\overset{\circ}{\underline{\Psi}}_2] \cdot \ldots \cdot [\overset{\circ}{\underline{\Psi}}_n] = [\overset{\circ}{\underline{\Psi}}_1 \cdot \overset{\circ}{\underline{\Psi}}_2 \cdot \ldots \cdot \overset{\circ}{\underline{\Psi}}_n]$ ▟.

Theorem 2. *Quotients of Dimensions.* ▛ *The quotient of the dimensions of two variables is the dimension of the quotient of those two variables* ▟, or: ▛ $[V_1] / [V_2] = [V_1 / V_2]$ ▟, or: ▛ $[\overset{\circ}{\underline{\Psi}}_1] / [\overset{\circ}{\underline{\Psi}}_2] = [\overset{\circ}{\underline{\Psi}}_1 / \overset{\circ}{\underline{\Psi}}_2]$ ▟.

Proof of Theorem 2.

Via Szirtes' Ideography	#	Via the $_{\alpha}\underline{\mu}$ Ideography for ***Dimensional Analysis***	
Proposition		**Proposition**	**Justification**
▛ $[V_1]/[V_2] = d_1/d_2 = d$	1	$[\overset{\circ}{\underline{\Psi}}_1]/[\overset{\circ}{\underline{\Psi}}_2] = \hat{\underline{\mu}}_{\sum_j \alpha_j \hat{\underline{\mu}}_j} / \hat{\underline{\mu}}_{\sum_j \beta_j \hat{\underline{\mu}}_j} = \hat{\underline{\mu}}_{\sum_j \alpha_j \hat{\underline{\mu}}_j - \sum_j \beta_j \hat{\underline{\mu}}_j}$	Definitions: V_k, $\overset{\circ}{\underline{\Psi}}_k$, & [.]; *Closure of dimensional division*
$V_1/V_2 = m_1 \cdot d_1 / m_2 \cdot d_2 =$ $(m_1/1)(d_1/1)(1/m_2)(1/d_2)$	2	$\overset{\circ}{\underline{\Psi}}_1 / \overset{\circ}{\underline{\Psi}}_2 = \mu_1 \cdot \hat{\underline{\mu}}_{\sum_j \alpha_j \hat{\underline{\mu}}_j} / \mu_2 \cdot \hat{\underline{\mu}}_{\sum_j \beta_j \hat{\underline{\mu}}_j}$	Definitions: V_k & $\overset{\circ}{\underline{\Psi}}_k$; Multiplicative inversion of magnitudes & dimensions
$(m_1/1)(1/m_2)(d_1/1)(1/d_2) =$ $(m_1/m_2) \cdot [d_1/d_2] = V_1/V_2$	3	$\mu_1 \cdot \hat{\underline{\mu}}_{\sum_j \alpha_j \hat{\underline{\mu}}_j} / \mu_2 \cdot \hat{\underline{\mu}}_{\sum_j \beta_j \hat{\underline{\mu}}_j} = (\mu_1/\mu_2) \cdot [\hat{\underline{\mu}}_{\sum_j \alpha_j \hat{\underline{\mu}}_j} / \hat{\underline{\mu}}_{\sum_j \beta_j \hat{\underline{\mu}}_j}]$	Rule of Commutation, applied to step 2; Transitive Rule of Equality applied to steps 2 & 3
$[V_1/V_2] = d_1/d_2 = d$	4	$[\overset{\circ}{\underline{\Psi}}_1 / \overset{\circ}{\underline{\Psi}}_2] = \hat{\underline{\mu}}_{\sum_j \alpha_j \hat{\underline{\mu}}_j} / \hat{\underline{\mu}}_{\sum_j \beta_j \hat{\underline{\mu}}_j} = \hat{\underline{\mu}}_{\sum_j \alpha_j \hat{\underline{\mu}}_j - \sum_j \beta_j \hat{\underline{\mu}}_j}$	Definition of [.]; *Closure of dimensional division*
$[V_1]/[V_2] = [V_1/V_2]$	5	$[\overset{\circ}{\underline{\Psi}}_1] / [\overset{\circ}{\underline{\Psi}}_2] = [\overset{\circ}{\underline{\Psi}}_1 / \overset{\circ}{\underline{\Psi}}_2]$ ▟	Transitive Rule of Equality, applied to steps 1 & 4; Q.E.D.

<u>Theorem 3</u>. '''*Theorem of Associativity*''' -- '*Associative Law of Multiplication for Dimensional Meta-Numbers*'.
$\vdash$ *If* d_1, d_2, *and* d_3 *are, respectively, the dimensions of* V_1, V_2, *and* V_3, *then* $\mathsf{d}_1\bullet[\mathsf{d}_2\bullet\mathsf{d}_3] = [\mathsf{d}_1\bullet\mathsf{d}_2]\bullet\mathsf{d}_3$ $\dashv$,

or: $\vdash$ $\mathsf{d}_1\bullet[\mathsf{d}_2\bullet\mathsf{d}_3] = [\mathsf{d}_1\bullet\mathsf{d}_2]\bullet\mathsf{d}_3$ $\dashv$,

or: $\vdash$ $\underline{\hat{\mu}}_{\sum_j \alpha_j \underline{\hat{\mu}}_j} \bullet [\underline{\hat{\mu}}_{\sum_j \beta_j \underline{\hat{\mu}}_j} \bullet \underline{\hat{\mu}}_{\sum_j \gamma_j \underline{\hat{\mu}}_j}] = [\underline{\hat{\mu}}_{\sum_j \alpha_j \underline{\hat{\mu}}_j} \bullet \underline{\hat{\mu}}_{\sum_j \beta_j \underline{\hat{\mu}}_j}] \bullet \underline{\hat{\mu}}_{\sum_j \gamma_j \underline{\hat{\mu}}_j} = \underline{\hat{\mu}}_{\sum_j \alpha_j \underline{\hat{\mu}}_j + \sum_j \beta_j \underline{\hat{\mu}}_j + \sum_j \gamma_j \underline{\hat{\mu}}_j}$ $\dashv$.

<u>Proof of Theorem **3**.</u>

Via Szirtes' Ideography	#	Via the ${}_\alpha\underline{\mu}$ Ideography for ***<u>Dimensional</u> <u>Analysis</u>***	
Proposition		**Proposition**	**Justification**
$\vdash$ $\mathsf{V}_1 \cdot \mathsf{V}_2 \equiv \mathsf{V}_{1,2}$	1	$\underline{\mathring{\Psi}}_1 \cdot \underline{\mathring{\Psi}}_2 \equiv \underline{\mathring{\Psi}}_{1,2}$	New Definition
$\mathsf{V}_2 \cdot \mathsf{V}_3 \equiv \mathsf{V}_{2,3}$	2	$\underline{\mathring{\Psi}}_2 \cdot \underline{\mathring{\Psi}}_3 \equiv \underline{\mathring{\Psi}}_{2,3}$	New Definition
$[\mathsf{V}_{1,2}]\bullet[\mathsf{V}_3] = [\mathsf{d}_1\bullet\mathsf{d}_2]\bullet[\mathsf{d}_3]$	3	$[\underline{\mathring{\Psi}}_{1,2}]\bullet[\underline{\mathring{\Psi}}_3] = [\underline{\hat{\mu}}_{\sum_j \alpha_j \underline{\hat{\mu}}_j} \bullet \underline{\hat{\mu}}_{\sum_j \beta_j \underline{\hat{\mu}}_j}] \bullet \underline{\hat{\mu}}_{\sum_j \gamma_j \underline{\hat{\mu}}_j}$	Definition of **[.]**; Theorem 1, Proposition 6; closure of dimensional multiplication
$[\mathsf{V}_{1,2}]\bullet[\mathsf{V}_3] = [\mathsf{V}_1 \cdot \mathsf{V}_2]\bullet[\mathsf{V}_3]$	4	$[\underline{\mathring{\Psi}}_{1,2}]\bullet[\underline{\mathring{\Psi}}_3] = [\underline{\mathring{\Psi}}_1 \cdot \underline{\mathring{\Psi}}_2]\bullet[\underline{\mathring{\Psi}}_3]$	Definition of $\mathsf{V}_{1,2}$ and of **[.]**
$[\mathsf{V}_1 \cdot \mathsf{V}_2]\bullet[\mathsf{V}_3] = [\mathsf{V}_1]\bullet[\mathsf{V}_2]\bullet[\mathsf{V}_3]$	5	$[\underline{\mathring{\Psi}}_1 \cdot \underline{\mathring{\Psi}}_2]\bullet[\underline{\mathring{\Psi}}_3] = [\underline{\mathring{\Psi}}_1]\bullet[\underline{\mathring{\Psi}}_2]\bullet[\underline{\mathring{\Psi}}_3]$	Theorem 1, reverse direction, applied to LHS of eq. of step **4**
$[\mathsf{V}_1]\bullet[\mathsf{V}_2]\bullet[\mathsf{V}_3] = [\mathsf{V}_{1,2}]\bullet[\mathsf{V}_3]$	6	$[\underline{\mathring{\Psi}}_1]\bullet[\underline{\mathring{\Psi}}_2]\bullet[\underline{\mathring{\Psi}}_3] = [\underline{\mathring{\Psi}}_{1,2}]\bullet[\underline{\mathring{\Psi}}_3]$	Transitive Rule of Equality, applied to steps **5** & **4**
$[\mathsf{V}_1]\bullet[\mathsf{V}_2]\bullet[\mathsf{V}_3] = [\mathsf{d}_1\bullet\mathsf{d}_2]\bullet[\mathsf{d}_3]$	7	$[\underline{\mathring{\Psi}}_1]\bullet[\underline{\mathring{\Psi}}_2]\bullet[\underline{\mathring{\Psi}}_3] = [\underline{\hat{\mu}}_{\sum_j \alpha_j \underline{\hat{\mu}}_j} \bullet \underline{\hat{\mu}}_{\sum_j \beta_j \underline{\hat{\mu}}_j}] \bullet \underline{\hat{\mu}}_{\sum_j \gamma_j \underline{\hat{\mu}}_j}$	Transitive Rule of Equality, applied to steps **3** & **6**
$[\mathsf{V}_1]\bullet[\mathsf{V}_{2,3}] = [\mathsf{d}_1]\bullet[\mathsf{d}_2\bullet\mathsf{d}_3]$	8	$[\underline{\mathring{\Psi}}_1]\bullet[\underline{\mathring{\Psi}}_{2,3}] = [\underline{\hat{\mu}}_{\sum_j \alpha_j \underline{\hat{\mu}}_j}]\bullet[\underline{\hat{\mu}}_{\sum_j \beta_j \underline{\hat{\mu}}_j} \bullet \underline{\hat{\mu}}_{\sum_j \gamma_j \underline{\hat{\mu}}_j}]$	Definition of $\mathsf{V}_{2,3}$ and of **[.]**
$[\mathsf{V}_1]\bullet[\mathsf{V}_{2,3}] = [\mathsf{V}_1]\bullet[\mathsf{V}_2 \cdot \mathsf{V}_3]$	9	$[\underline{\mathring{\Psi}}_1]\bullet[\underline{\mathring{\Psi}}_{2,3}] = [\underline{\mathring{\Psi}}_1] \cdot [\underline{\mathring{\Psi}}_2 \cdot \underline{\mathring{\Psi}}_3]$	Definition of $\mathsf{V}_{2,3}$ and of **[.]**
$[\mathsf{V}_1]\bullet[\mathsf{V}_2 \cdot \mathsf{V}_3] = [\mathsf{V}_1]\bullet[\mathsf{V}_2]\bullet[\mathsf{V}_3]$	10	$[\underline{\mathring{\Psi}}_1] \cdot [\underline{\mathring{\Psi}}_2 \cdot \underline{\mathring{\Psi}}_3] = [\underline{\mathring{\Psi}}_1]\bullet[\underline{\mathring{\Psi}}_2]\bullet[\underline{\mathring{\Psi}}_3]$	Theorem 1, reverse direction, applied to RHS of eq. of step **9**,
$[\mathsf{V}_1]\bullet[\mathsf{V}_2]\bullet[\mathsf{V}_3] = [\mathsf{d}_1]\bullet[\mathsf{d}_2\bullet\mathsf{d}_3]$	11	$[\underline{\mathring{\Psi}}_1]\bullet[\underline{\mathring{\Psi}}_2]\bullet[\underline{\mathring{\Psi}}_3] = [\underline{\hat{\mu}}_{\sum_j \alpha_j \underline{\hat{\mu}}_j}]\bullet[\underline{\hat{\mu}}_{\sum_j \beta_j \underline{\hat{\mu}}_j} \bullet \underline{\hat{\mu}}_{\sum_j \gamma_j \underline{\hat{\mu}}_j}]$	Transitive Rule of Equality, applied to steps **8**, **9**, & **10**
$[\mathsf{d}_1]\bullet[\mathsf{d}_2\bullet\mathsf{d}_3] = [\mathsf{d}_1\bullet\mathsf{d}_2]\bullet[\mathsf{d}_3]$ or $\mathsf{d}_1\bullet[\mathsf{d}_2\bullet\mathsf{d}_3] = [\mathsf{d}_1\bullet\mathsf{d}_2]\bullet\mathsf{d}_3$ or $\mathsf{d}_1[\mathsf{d}_2\mathsf{d}_3] = [\mathsf{d}_1\mathsf{d}_2]\mathsf{d}_3$	12	$\underline{\hat{\mu}}_{\sum_j \alpha_j \underline{\hat{\mu}}_j} [\underline{\hat{\mu}}_{\sum_j \beta_j \underline{\hat{\mu}}_j + \sum_j \gamma_j \underline{\hat{\mu}}_j}] = [\underline{\hat{\mu}}_{\sum_j \alpha_j \underline{\hat{\mu}}_j + \sum_j \beta_j \underline{\hat{\mu}}_j}] \underline{\hat{\mu}}_{\sum_j \gamma_j \underline{\hat{\mu}}_j} \equiv$ $\underline{\hat{\mu}}_{\sum_j \alpha_j \underline{\hat{\mu}}_j + \sum_j \beta_j \underline{\hat{\mu}}_j + \sum_j \gamma_j \underline{\hat{\mu}}_j}$ $\dashv$	Transitive Rule of Equality, applied to steps **11** & **7**; Definition of '$\bullet$', as the *multiplication* operation-sign, in relation to that of **[.]**, as well as of the *juxtaposition* of dimensional symbols; Substitution of 'script-level' juxtapositions [or of 'subscript-level' additions, in the case of the $\underline{\underline{\hat{\mu}}}_{\mathrm{U}}$ arithmetic] for '$\bullet$'; Q.E.D.

Theorem 4. *Powers of Dimensions.* $\vdash$ *The dimension of a power of a variable is that same power of the dimension of that variable* $\dashv$, or: $\vdash [V_k^{\ n}] = [V_k]^n \dashv$, or: $\vdash [\underline{\overset{\circ}{\Psi}}_k^{\ n}] = [\underline{\overset{\circ}{\Psi}}_k]^n \dashv$.

Proof of Theorem **4**.

Via Szirtes' Ideography	#	Via the $_\alpha\underline{\underline{\mu}}$ Ideography for ***Dimensional Analysis***	
Proposition		**Proposition**	**Justification**
$\vdash V_k = m_k \bullet d_k$	1	$\underline{\overset{\circ}{\Psi}}_k = \mu_k \bullet \underline{\overset{\hat{\circ}}{\mu}}_{\underline{\sum_j}\alpha_j\underline{\overset{\hat{\circ}}{u}}_j}$	Definitions: V_k, $\underline{\overset{\circ}{\Psi}}_k$
$[V_k] = d_k$	2	$[\underline{\overset{\circ}{\Psi}}_k] = \underline{\overset{\hat{\circ}}{\mu}}_{\underline{\sum_j}\alpha_j\underline{\overset{\hat{\circ}}{u}}_j}$	Definition of **[.]**
$[V_k]^n = d_k^{\ n} = d$	3	$[\underline{\overset{\circ}{\Psi}}_k]^n = \underline{\overset{\hat{\circ}}{\mu}}_{\underline{\sum_j}\alpha_j\underline{\overset{\hat{\circ}}{u}}_j}^{\ n} \equiv \underline{\overset{\hat{\circ}}{\mu}}_{n\underline{\sum_j}\alpha_j\underline{\overset{\hat{\circ}}{u}}_j}^{\ 1}$	Equality Maintenance Rule applied to **2** ['*meta-heterosis convolute product rule*' applied to the $\underline{\overset{\hat{\circ}}{\mu}}_{\underline{U}}$ sub-arithmetic of $_\alpha\underline{\underline{\mu}}$]; *Closure of metrical* [*self-*]*multiplication(s)*
$V_k^{\ n} = \{m_k \cdot d_k\}^n = m_k^{\ n} \cdot d_k^{\ n}$	4	$\underline{\overset{\circ}{\Psi}}_k^{\ n} = \{\mu_k \underline{\overset{\hat{\circ}}{\mu}}_{n\underline{\sum_j}\alpha_j\underline{\overset{\hat{\circ}}{u}}_j}\}^n = \mu_k^{\ n} \bullet \underline{\overset{\hat{\circ}}{\mu}}_{n\underline{\sum_j}\alpha_j\underline{\overset{\hat{\circ}}{u}}_j}$	Equality Maintenance Rule; Product Rule of Exponentiation, applied both to magnitudes and to dimensions
$[V_k^{\ n}] = [m_k^{\ n} \bullet d_k^{\ n}] = d_k^{\ n}$ $= d$	5	$[\underline{\overset{\circ}{\Psi}}_k^{\ n}] = [\mu_k^{\ n} \bullet \underline{\overset{\hat{\circ}}{\mu}}_{n\underline{\sum_j}\alpha_j\underline{\overset{\hat{\circ}}{u}}_j}] = \underline{\overset{\hat{\circ}}{\mu}}_{n\underline{\sum_j}\alpha_j\underline{\overset{\hat{\circ}}{u}}_j}$	Substitution of Equivalents, or, Equality Maintenance Rule; *Closure of metrical* [*self-*]*multiplication(s)*; Definition of **[.]**
$[V_k^{\ n}] = [V_k]^n$	6	$[\underline{\overset{\circ}{\Psi}}_k^{\ n}] = [\underline{\overset{\circ}{\Psi}}_k]^n \dashv$	Transitive Rule of Equality, steps **3** & **5**; Reflexive Rule of Equality; Q.E.D.

Conceptualization [«begrifflichkeit»] of the Four Theorems. The foregoing **4** theorems, taken in their unity, at least with respect to the arithmetical operations known as **(1)** "***multiplication***", **(2)** its inverse operation, "***division***", and **(3)** that '***meta-multiplication***' operation, *made up out of multiple* ***multiplication*** *operations*, known as "***exponentiation***" [*iterated* ***self****-multiplication* as '***meta-operation***' to multiplication], lead us to conjecture the following '***meta-theorem***', 'made up out of' the four foregoing theorems: "***dimensions***" ***transform like*** "***magnitudes***", and ∴ "***dimensions***" ***and*** "***magnitudes***" ***can be transformed together, in unison,*** as per V_k or $\underline{\overset{\circ}{\Psi}}_k$. That is, '***dimensional meta-numbers***' like d_k or $\underline{\overset{\hat{\circ}}{\mu}}_{\underline{\sum_j}\alpha_j\underline{\overset{\hat{\circ}}{u}}_j}$ *multiply, divide,* and '*exponentiate*' in accord with *the same generic rules* that also apply to ordinary "**R***eal*" *numbers*, "*magnitudes*", or '***pure quantifiers***', like u_k, m_k, μ_k, **2**, **√2**, **e**, **π**, etc.

Theorem **1** suggests that, if you ***multiply*** two '***dimensionally-qualified quantifiers***' or '***quantified dimensional qualifiers***', $V_1 \equiv m_1 \cdot d_1$ and $V_2 \equiv m_2 \cdot d_2$, together, wherein m_1 '***quantifies***' the '***dimensional qualifier***' d_1, and m_2 '***quantifies***' the '***dimensional qualifier***' or '***metrical qualifier***' d_2, or wherein d_1 '***dimensionally qualifies***' the '***quantifier***' m_1, and d_2 '***dimensionally qualifies***' or '***metrically qualifies***' the '***quantifier***' m_2, you get the same, correct, result, i.e., multiplying V_1 by V_2, as you do if you first multiply the '***quantifiers***', m_1 and m_2, then 'multiply' the '***metrical qualifiers***', d_1 & d_2, and *only then 'multiply' those* **2** *'multiplications' together*: $m_1 \bullet m_2 \bullet d_1 \bullet d_2 = V_1 \bullet V_2$.

Theorem **2** suggests that, if you ***divide*** one '***dimensionally-qualified quantifier***' or '***quantified dimensional qualifier***', by another, $V_1 \equiv m_1 \cdot d_1$ by $V_2 \equiv m_2 \cdot d_2$, wherein m_1 '***quantifies***' the '***dimensional qualifier***' d_1, and m_2 '***quantifies***' the '***dimensional qualifier***' or '***metrical qualifier***' d_2, or wherein d_1 '***dimensionally qualifies***' the '***quantifier***' m_1, and d_2 '***dimensionally qualifies***' or '***metrically qualifies***' the '***quantifier***' m_2, you get the same, correct, result, i.e., dividing V_1 by V_2, as you do if you first divide the '***quantifiers***', m_1 by m_2, then 'divide' the '***metrical qualifiers***', d_1 by d_2, and *only then* 'multiply' those two 'divisions' together: $(m_1/m_2) \bullet [d_1/d_2] = V_1/V_2$.

Theorem **3** suggests that '***dimensional qualifiers***' or '***metrical monads***' follow the same '***rule of multiplicative associativity***' as do '***pure quantifiers***', or ordinary, '***unqualified***', "'**R**eal'" numbers. Theorem **4** suggests that, if you self-multiply, **n** times, a '***dimensionally-qualified quantifier***', or '***quantified dimensional qualifier***', $V_k \equiv m_k \cdot d_k$, you get the same, correct, result, as if you first self-multiply the '***quantifier***', m_k, then 'self-multiply' the '***metrical qualifier***', d_k, and *only then* 'multiply' those two 'multiplications' together:

$$(m_k \bullet \ldots \bullet m_k) \bullet [d_k \bullet \ldots \bullet d_k] = m_k^{\ n} \bullet d_k^{\ n} = \{m_k \cdot d_k\}^n = V_k \bullet \ldots \bullet V_k = V_k^{\ n}.$$

The History of '*The Elision of the Qualifier Meta-Numbers*' from Mathematics, and '*The Restoration of the Qualifiers*' in *Dialectical Ideography*. The second supplement to the Introductory letter, ***Supplement B.***, describes eleven illustrative applications of $\underline{\mathbf{Q}}$ and $_{\alpha}\underline{\mu}$ [pronounced *"alpha-mu"*] *'dialectical arithmetics'*, in which we incessantly re-use typically about the first **32** of the *'ontological qualifier* ***meta-numbers***' of those arithmetical systems [e.g., $\hat{q}_1, \hat{q}_2, \hat{q}_3, ..., \hat{q}_{32}$; or $\overset{\delta}{\underline{\mu}}_1, \overset{\delta}{\underline{\mu}}_2, \overset{\delta}{\underline{\mu}}_3, ..., \overset{\delta}{\underline{\mu}}_{32}$], in each case with vastly different *intensions/interpretations/assignments* for those symbols. Similarly, in our daily lives, we incessantly re-use '''the same''' "Natural" numbers over and over and over again in different *meaning-contexts* -- **10, 20, 30** *minutes*; **10, 20, 30** *lbs.*; **10, 20, 30** *miles*; **10, 20, 30** *gallons* of gasoline; **$10, $20, $30**, or '**10$**', '**20$**', '**30$**', etc., etc. -- appropriating again and again the utility of the *"natural"* insight *arrived at* ***so protractedly*** *by our ancient ancestors so long ago* about the ***generic*** principles, patterns, and rules of ***counting units***, *"any"* units; *units of the most diverse kinds*. We are herein asserting, in effect, through our re-use of, e.g., $\{\ \hat{q}_1, \hat{q}_2, \hat{q}_3, ..., \hat{q}_{32}, ...\ \}$ over and again in these applications, that there is a sequencing, an 'abstractable', 'idealizeable', ***generic sequencing*** [perhaps even a '*Peanic*' ***sequencing!***], of ***ontological types/categories***, one at least as 'natural' and as universal as the sequencing of abstract cardinalities, of the abstracted, idealized, generic *'pure quantifiers'* of the 'purely quantitative' "**N**atural" numbers, **1, 2, 3,...**, once a humanity attains to a certain stage in the historical development of knowing.

What we implicitly assert, by reusing the same ***uninterpreted*** *'pure qualifier* meta-numbers' in such different applicational and textual meaning-contexts, is that all of these instances of '***onto-dynamasis***' or of '*non-onto-stasis*' exhibit a common, general, ubiquitous "pattern", or 'meta-pattern' which we term 'quanto-***qualitative***, ***temporal fractal*** structure', or '***meta-fractal*** *structure*', i.e., ***diachronic*** '*quanto-****qualitative*** *scale-progression* ***self-similarity*** *structure*', or '«***aufheben***» ***self-incorporation*** *succession-structure*'. This generic, 'abstractable', 'idealizeable', 'id*e*ograhically-symbolizeable' structure, shared by ***Supplement B.***'s eleven illustrative applications, and by many more, is, of course, accompanied by richly diverse, id*i*ographic uniquenesses and particularities which they do *not* share [but which their ***interpreted***, '***intensional***' ***symbols*** partially capture heuristically, connotatively]. The 'meta-pattern' they do share is captured and codified, in '''purest''', most abstract, form, in the $_{\underline{\mathbf{N}}}\underline{\mathbf{Q}}$ system of arithmetic, in a '''natural''' way comparable to the way different generic patterns of quantification are codified in the 'rules-system' of the $\underline{\mathbf{N}}$ "Natural" arithmetic. Both are '''Natural''' to such an extent that both $\underline{\mathbf{N}}$ and $_{\underline{\mathbf{N}}}\underline{\mathbf{Q}}$ fulfill the first-order Peano Postulates, though the ***unit*** or '*Peanic*' «*arché*» of $\underline{\mathbf{N}}$ is a '*Boolean*' unit[y] or '''monad''' [$\mathbf{x}^2 = \mathbf{x}$; $\mathbf{1}^2 = \mathbf{1}$; also $\mathbf{0}^2 = \mathbf{0}$], whereas those of the '***pure, unquantifiable qualifier***' and of the '*quantifiable*' ***and*** '*qualifiable*', or '***quanto-qualifier***' systems of arithmetic are '***contra-Boolean***' unit[ie]s or '''monads''', e.g., $\hat{q}_1^{\ 2} \neq \hat{q}_1$, and also; $\overset{\delta}{\underline{u}}_1^{\ 2} \neq \overset{\delta}{\underline{u}}_1$; $[\overset{\delta}{\underline{\mu}}_{\overset{\delta}{\underline{u}}_1}]^2 = \overset{\delta}{\underline{\mu}}_{2\overset{\delta}{\underline{u}}_1} \neq \overset{\delta}{\underline{\mu}}_{1\overset{\delta}{\underline{u}}_1}$, for example, $\mathbf{[in.]^2 = [square\ inch] \neq [in.]^1 = [linear\ inch]}$; and $\overset{\delta}{\underline{\mu}}_1^{\ 2} \neq \overset{\delta}{\underline{\mu}}_1$. Consider the instances of the $\underline{\mathbf{N}}$ rules below:

...	**2**[linear inches]	+ **2**[linear inches]	=	**4**[linear inches] ...
...	**2**[loaves of bread]	+ **2**[loaves of bread]	=	**4**[loaves of bread] ...
...	**2**[fish]	+ **2**[fish]	=	**4**[fish] ...
...	**2**[head of cattle]	+ **2**[head of cattle]	=	**4**[head of cattle] ....

'Extracting' only the '***quantifiers***' of the above equations, all are abstracted down to just:

$$\mathbf{2} \quad + \mathbf{2} \quad = \quad \mathbf{4}.$$

As codified in Plato's theory of '*dianoetical*' arithmetic, that of the «***arithmoi monadikoi***», the *"arithmetic of generic* ***monads***", or "***units***" [as opposed to his *"dialectical"* arithmetic, that of the «***arithmoi eidetikoi***», which is modeled by $_{\underline{\mathbf{N}}}\underline{\mathbf{Q}}$], ancient Hellenic humanity, like their Mesopotamian predecessors, enacted this abstraction quite differently from the way in which 'we' have ever since the Renaissance, at least. Along with ***generic*** '***quantifiers***', e.g., **2**, and **4**, their abstraction took up in/with it a '***generic qualifier***'. Denoting that '***qualifier***' per the "syncopated" [abbreviated] fashion of Diophantus' circa 250 C.E. «***Arithmetike***», their abstraction, *in our terms*, looked like:

$$\mathbf{2\mathring{M}} \quad + \mathbf{2\mathring{M}} \quad = \quad \mathbf{4\mathring{M}},$$

or, in the antique, pre-Hindu-Arabic-numerals proto-ideographic idiom of the extant manuscripts of that work, as $\mathring{M}\,\bar{\beta}\,\bar{\beta}\ \iota^{\sigma}\ \mathring{M}\,\bar{\delta}$ [*bar-capped Greek letters denoted numerals via an alphabetic-order-based ordinal correspondence; their juxtaposition denoted addition, not multiplication as it does today, and* 'ι^{σ}' *abbreviated the Roman «****aequalis****»*].

Jacob Klein characterized the *collective-cognitive*, '***psycho-historical***' ***transition*** in mode of thought from that 'representable' as $\mathbf{2\mathring{M} + 2\mathring{M} = 4\mathring{M}}$ to that 'representable' as $\mathbf{2 + 2 = 4}$, as the emergence of "***symbolic***" thinking: "a new way of "understanding", inaccessible to ancient *episteme* . . ." [Jacob Klein, ***Greek Mathematical Thought and the Origin of Algebra***, Dover [NY: 1992], p. 175]. We will attribute this collective-cognitive transition to '***monetic***' thinking, and to a '***monetic abstraction***' or '***monetic elision***', arising from the even greater immersion of Renaissance humanity than of Hellenic humanity in monetized commerce, in money-mediated exchange. Such immersion entails an increasing domination of habitual pre-conscious thought-processes by the '*concrete abstraction*', and the '*concrete elision*' of even *generic* qualifiers, in the physical and mental activities of exchange-value exchange. How utterly commonplace and habitual -- and fundamental to the amazing progress of our species -- has become this abstraction of the "purely quantitative" from its quanto-qualitative context [e.g., from its context of the ***kinds*** of [ev]entities, and of the metrical ***dimensions*** of the various aspects of those [ev]entities, which are being ***quantified***]! And when we habitually, in effect, ***enact equations*** in the commerce of our daily lives, in the "*exchange* [$\rightleftharpoons$] *of equivalents*", such as:

... $\rightleftharpoons$ **20** yards of linen $\rightleftharpoons$ **£2** $\rightleftharpoons$ **10** lbs. of tea $\rightleftharpoons$ **£2** $\rightleftharpoons$ **1** coat $\rightleftharpoons$ **£2** $\rightleftharpoons$ **40** lbs. of coffee $\rightleftharpoons$ ...,

it is hard to grasp what common '***kind***-ness' allows all of these different quantities, ***of such heterogeneous qualities***!, to be equated; what '***onto***', what obscure, mysterious ***kind*** of thing it is that the **£** ***dimension*** measures and finds equal[ly] in all of them; what is the "*substance*" of this "***value***" *which they have in common* or *to which each can be* "***reduced***"! Yet, in this, for its time, *historically progressive praxis* of "*Natural*" arithmetic, of "***pure***" ***quantity-only*** abstraction, we of **F.*E.D*.** have come to diagnose a ***cognitive blind-spot***, an ***eventually crippling elision***, and a ***psycho-historical barrier*** to the next leap of scientific conception and of socio-technological capability that, we hold, our species must make if it is to survive, to prosper, and to grow into its higher destiny. We have come to call this 'blind-spot' '***The Elision of the Qualifiers***', and, via '***psycho-archaeological excavation***', to trace back its articulation, *in explicit form*, at least 414 years, to the 1585 ***Arithmetic*** of Simon Stevin. It has roots and resonances much deeper still, as we have just seen above, in that famed circa 250 C.E. ***proto-algebraic*** work, the ***Arithmetica*** of Diophantus. Therein, Diophantus denoted the "**Mo**nad" or "unit" of arithmetic -- descended, ***with conceptual mutation***, from the Platonic «***Arithmoi* Mo*nadikoi***» -- via his "syncopated" or 'abbreviative' $\mathring{\mathrm{M}}$. He treats it already as an ***abstract, generic***, "*dimensionless*", '***degree zero***', '*dimensionally reductionist*', '***no-longer-geometrical***', and thus as a '***Boolean***', ***qualifier*** [$\mathring{\mathrm{M}}^2 = \mathring{\mathrm{M}}$; *not* **square** $\not\rightleftharpoons$ **line**, *nor* $\mathring{\mathrm{M}}^2 \not\rightleftharpoons \mathring{\mathrm{M}}$]. Ironically, this ***last*** explicit ***quasi-ideographic*** notation for an ***arithmetical qualifier*** concept coincides with the ***first*** known emergence of ***proto-ideographical algebra for 'pure quantifiers'; for "purely-quantitative" "variables" and "constants"***! Although at loci apparently *remote* from the *barriers* to which '***The Elision of the Qualifiers***' contributes *at the advanced frontier of our civilization's mathematico-scientific-technological praxis*, we *of* this civilization do notice, at times, our own civilizational cognitive blind-spot. For example, our children often fleetingly feel, at its point of first inculcation, this '***Elision of the Qualifiers***' in our ideographical arithmetic, this *presence of the absence* of explicit, *symbolical* ***arithmetical*** *qualification. But we normally suppress such noticings*: "... it is instructive to talk to 1st- and 2nd-grade math teachers and find out about how children are actually taught about integers. About ***what***, for example, the number five ***is***. First, they are given, say, five oranges. ***Something they can touch or hold***. Are asked to count them. Then they are given a picture of five oranges. Then a picture which combines the five oranges with the numeral '5' so they associate the two. Then a picture of just the '5' ***with the oranges removed***. The children are then engaged in verbal exercises in which they start talking about the integer 5 *per se*, ***as an object in itself***, apart from five oranges. In other words they are systematically fooled, or awakened, into ***treating numbers as things instead of as symbols for things***. Then they can be taught arithmetic, which comprises elementary relations between numbers. ... ***Sometimes, a kid will have trouble***, the teachers say. Some children ***understand*** that the word 'five' stands for 5, ***but they keep wanting to know*** 5 *what*? 5 ***oranges***, 5 ***pennies***, 5 ***points***? These children, who ***have no problem adding or subtracting oranges or coins***, will nevertheless perform poorly on arithmetic tests. They cannot treat 5 as an object *per se*. They are often then remanded to Special Ed math, where everything is taught in terms of groups or sets of actual objects rather than as numbers "withdrawn from particular examples". The point: ***The basic def. of*** '***abstract***' for our purposes is going to be the somewhat concatenated 'removed from or transcending concrete particularity, sensuous experience'. Used in this way, 'abstract' is a term from metaphysics. ***Implicit in all mathematical theories, in fact, is some sort of metaphysical position***. The ***father of abstraction*** in ***mathematics***: ***Pythagoras***. The ***father of abstraction*** in ***metaphysics***: ***Plato***." [op. cit., David Foster Wallace, ***Everything and More: A Compact History of*** ∞, W. W. Norton, [New York, NY: 2003], pp. 8-10, ***bold italics emphasis*** added by **F.*E.D*.**]. The initiating abstraction that founded our arithmetic, the abstraction of "pure-quantity" into ideography, leaving quality behind, leaving it in rhetorical, phonogramic, phonetic, or "syncopated" symbolization only, was omissive, as is, inherently, all abstraction. All abstraction is a mental '***extraction***' of part of experience, leaving out the rest, leaving the rest behind. All abstraction thus creates, unavoidably, '***homeomorphic defect***'. It creates ***fictions***, however ***useful*** they may be; partial representations which ever only partially grasp the *factual*, the *actual*, the *concrete* truth. But the abstraction that launched arithmetic was a ***progressive*** abstraction. It was, perhaps, the only option for progress within the attained level of knowledge and social-reproductive praxis, and within the biases, of the collective human cognitive capabilities of the time. But have our capabilities not advanced since then, along with our need for more advanced such capabilities?

Have our needs, and our capacity to meet them, not advanced, *especially* as *a very consequence of* the progress made upon the foundation of this antique abstraction, this '*extraction*' of *only* '*pure, unqualified quantity*' into our arithmetical ideography? Perhaps it is time for us to abstract/'extract' again; to return to the roots of the path of abstraction which our civilization has pursued, *primarily* under the impetus of *the universal development of commerce*; of ***universal selling*** and of ***production for** selling* [Marx's "***universal <u>alienation</u>***" and "***production for <u>alienation</u>***"]; of what Marx called "***the development of the exchange-value***": to *re*-reflect upon *the psycho-historical context* of our initial and initiating arithmetico-algebraical abstractions, and then to abstract anew! ■ The cathexis of money-value that emerges with "***the development of the exchange-value***", and, even more so, with ***the cathexis of capital***, or of monetary profit, seems, to its experiencers -- in part, subliminally -- much like '***a love of the purely quantitative***' and it privileges '***the purely quantitative***' with a deeply, sub-consciously-rooted emotional charge -- arousing toward '***the purely quantitative***' an almost worshipful reverence -- as a proxy for the use-value, seemingly the greatest, most universal usefulness of all, of, namely, pure exchange-value, of the very 'opposite' of particular use-value; of the '*purely-monetary*' form of wealth. ■ This time, from the [ad]vantage of our since-developed cognitive, scientific, and technological capabilities and needs, could we not, should we not arrive at a more encompassing representation, a *richer, more complex, more 'concrete', more adequate* and *more serviceable* abstraction, taking more back with us this time, into our mathematical idealizations, and leaving less behind? Might we not arrive, thereby, at *an arithmetical and general mathematical ideography* capable of more aptly describing -- of capturing, even to the point of <u>pre</u>diction -- our *experience* and our *experiments*; more capable of mastering those of their aspects, especially their ***nonlinear*** and ***dynamical*** aspects, that our present mathematical tool kit finds so intractable? Consider the following '*quanto-qualitative' descriptor* rendered entirely in 'phone-etical', 'phonogram-ic' symbols:

five cubic centimeters of Hydrogen [gas] [at room temperature].

'Parse' it, below, to distinguish *<u>quantifiers</u>*, via enclosure by **( )**, from *<u>metrical</u> qualifiers*, via enclosure by **[ ]**, and from *<u>ontological</u> qualifiers*, via enclosure by **⟦ ⟧**, with $\underline{\alpha} \leftrightarrow \underline{\beta}$ denoting '$\underline{\alpha}$ *interprets* $\underline{\beta}$', and with the combination of the 'pre[fix]-***super***-script', **2**, above the 'pre[fix]-***sub***-script', **a**, i.e., '${}^{2}_{a}$', signifying that the $\underline{\mathbf{Q}}$ and $\underline{\mathbf{U}}$ arithmetics are here being ***interpreted*** to describe certain constituents of a 'taxonomy level **2**' universe of discourse, a 'sub-universe' wherein '**a**' is short for '**<u>a</u>toms**', and with the two symbols together denoting the ***interpretation*** or ***assignment*** of the $\underline{\mathbf{Q}}$ and $\underline{\mathbf{U}}$ ideographies to model a universe of discourse that is *contained in*, and that resides *one-level sub-categorical to*, the 'maximal' or 'taxonomy level **1**' universe of discourse, **∀**, and which is denoted by **<u>H</u>**, short for the ontological sub-category/population/'*<u>concrete</u> «<u>arithmos</u>»*' of the <u>H</u>ydrogen atom:

(five) <of> [cubic centimeters] of ⟦Hydrogen⟧

$\mathbf{N}$: 5

The ideographic abstraction due to our classical, 'Peanic' arithmetic of ' "*pure*" /*<u>un</u>qualified <u>quantifiers</u>*'

${}_{\alpha}\underline{\mu}$: $\mu_1 = 5 \quad \leftrightarrow \quad {}^{2}_{a}\mu_H = 5$

or

$\mu_1(\tau) = 5 \leftrightarrow \quad {}^{2}_{a}\mu_H(\tau) = 5$

$\underline{\mathbf{Q}}$: $\quad \hat{\underline{\mathrm{q}}}_1 \leftrightarrow {}^{2}_{a}\hat{\underline{\mathrm{q}}}_H \quad \equiv \quad {}^{2}_{a}\underline{\mathrm{H}}$

'Peanic' Ideographic *counter-abstraction* of ' "*pure*", *<u>un</u>quantifiable <u>ontological</u> qualifiers*'

${}_{\alpha}\underline{\mu}$: $\quad \overset{\hat{o}}{\underline{\mu}}_1 \leftrightarrow {}^{2}_{a}\overset{\hat{o}}{\underline{\mu}}_H \quad \equiv \quad {}^{2}_{a}\underline{\mathrm{H}}$

'Peanic' Ideographic *counter-abstraction* of *<u>quantifiable</u> <u>ontological</u> qualifiers*'

${}_{\alpha}\underline{\mu}$: $\quad \overset{\hat{o}}{\underline{\mu}}_{\overset{\hat{o}}{\underline{u}}_3} \leftrightarrow [\,\underline{\mathrm{L}}\,]$ in cm.; $\overset{\hat{o}}{\underline{\mu}}_{3\overset{\hat{o}}{\underline{u}}_3} \leftrightarrow [\,\underline{\mathrm{C}} = \underline{\mathrm{L}}^3\,]$ in cu. cm.;

$$[\overset{\hat{o}}{\underline{\mu}}_{\overset{\hat{o}}{\underline{u}}_3}]^3 = [\,\overset{\hat{o}}{\underline{\mu}}_{\overset{\hat{o}}{\underline{u}}_3} \times \overset{\hat{o}}{\underline{\mu}}_{\overset{\hat{o}}{\underline{u}}_3} \times \overset{\hat{o}}{\underline{\mu}}_{\overset{\hat{o}}{\underline{u}}_3}\,] = [\overset{\hat{o}}{\underline{\mu}}_{\overset{\hat{o}}{\underline{u}}_3 + \overset{\hat{o}}{\underline{u}}_3 + \overset{\hat{o}}{\underline{u}}_3}] = \overset{\hat{o}}{\underline{\mu}}_{3\overset{\hat{o}}{\underline{u}}_3}.$$

'Peanic' Ideographic *counter-abstraction* of *quantifiable <u>metrical</u> qualifiers*'

'Boolean' units: $\hat{\mathbf{u}}^2 = \hat{\mathbf{u}}$; $1^2 = 1$.

'contra-Boolean' units: $\quad \overset{\hat{o}}{\underline{u}}^2 \not\gtrless \overset{\hat{o}}{\underline{u}}$; $\overset{\hat{o}}{\underline{u}}^2 \neq \overset{\hat{o}}{\underline{u}}$; $\quad \overset{\hat{o}}{\underline{u}}^2 \perp \overset{\hat{o}}{\underline{u}}$;

$$⟦\hat{\underline{\mathrm{q}}}_1⟧^2 \neq ⟦\hat{\underline{\mathrm{q}}}_1⟧;\ ⟦\hat{\underline{\mathrm{q}}}_1⟧^2 \not\gtrless ⟦\hat{\underline{\mathrm{q}}}_1⟧;\ \ \hat{\underline{\mathrm{q}}}_1^{\,2} - \hat{\underline{\mathrm{q}}}_1 \perp \hat{\underline{\mathrm{q}}}_1;$$

$$⟦\overset{\hat{o}}{\underline{\mu}}_1⟧^2 \neq ⟦\overset{\hat{o}}{\underline{\mu}}_1⟧;\ ⟦\overset{\hat{o}}{\underline{\mu}}_1⟧^2 \not\gtrless ⟦\overset{\hat{o}}{\underline{\mu}}_1⟧;\ \ \overset{\hat{o}}{\underline{\mu}}_1^{\,2} = \overset{\hat{o}}{\underline{\mu}}_2 \perp \overset{\hat{o}}{\underline{\mu}}_1;$$

$$[\,\overset{\hat{o}}{\underline{\mu}}_{\overset{\hat{o}}{\underline{u}}_3}]^2 \neq [\,\overset{\hat{o}}{\underline{\mu}}_{\overset{\hat{o}}{\underline{u}}_3}];\ [\,\overset{\hat{o}}{\underline{\mu}}_{\overset{\hat{o}}{\underline{u}}_3}]^2 \not\gtrless [\,\overset{\hat{o}}{\underline{\mu}}_{\overset{\hat{o}}{\underline{u}}_3}];\quad \overset{\hat{o}}{\underline{\mu}}_{\overset{\hat{o}}{\underline{u}}_3}^{\,2} = \overset{\hat{o}}{\underline{\mu}}_{2\overset{\hat{o}}{\underline{u}}_3} \perp \overset{\hat{o}}{\underline{\mu}}_{\overset{\hat{o}}{\underline{u}}_3}.$$

The *'dialectical re-unification'* or *'dialectical synthesis'* of the above ***abstraction*** and ***counter-abstractions*** begins, in our *'meta-systematic'* dialectical, *category-progression/systems-of-arithmetic-progression* exposition, with the ***'quantifiable ontological qualifiers'*** of the addition of the $\underline{\mathbf{U}}$ system of arithmetic. The $\underline{\mathbf{U}}$ language, *unlike* the $\underline{\mathbf{Q}}$ language, has the capacity to translate, ideogramically, a *'phonogramic' description such as* '**one-hundred-sixty-five trillion atoms of Hydrogen**', *viz.*, for $\underline{\hat{\hat{u}}}_1 \leftrightarrow {}^{2}_{a}\underline{\hat{\hat{u}}}_{H}$, as --

$(165 \times 10^{12})[\![{}^{2}_{a}\underline{\hat{\hat{u}}}_{H}]\!]$, given the *'addability'*, or *non*-idempotent addition operation of $\underline{\mathbf{U}}$, versus, for *any* n in $\mathbf{N}$ --

$(165 \times 10^{12})[\![{}^{2}_{a}\hat{q}_{H}]\!] \; = \; (n)[\![{}^{2}_{a}\hat{q}_{H}]\!] \; = \; (1)[\![{}^{2}_{a}\hat{q}_{H}]\!] \; = \; {}^{2}_{a}\hat{q}_{H}$, per the ***idempotent addition*** rule of $\underline{\mathbf{Q}}$.

The $\underline{\mathbf{U}}$ system of rules of arithmetic is *not*, however, capable of expressing, ideogramically, the description '**five cubic centimeters of Hydrogen**', because, though it contains explicit *quantifiers* and *ontological qualifiers*, it lacks explicit ***metrical qualifiers*** with which to express '**cubic centimeters**'. Capacity to translate the latter phonogramic descriptor into ideograms begins, in our exposition of the dialectical ideographies, with the ${}_{\alpha}\underline{\mu}$ system of arithmetic:

$$(5)[\, \underline{\hat{\hat{\mu}}}_{3\underline{\hat{\hat{u}}}_3} \,][\![{}^{2}_{a}\underline{\hat{\hat{\mu}}}_{H}]\!] \; = \; 5 \cdot \underline{\hat{\hat{\mu}}}_{3\underline{\hat{\hat{u}}}_3} \cdot {}^{2}_{a}\underline{\hat{\hat{\mu}}}_{H} \;\; \leftrightarrow \;\; 5 \cdot \underline{\hat{\hat{\mu}}}_{3\underline{\hat{\hat{u}}}_3} \cdot \underline{\hat{\hat{\mu}}}_{1} \; = \; 5\underline{\hat{\hat{\mu}}}_{3\underline{\hat{\hat{u}}}_3 + 1}.$$

Note: The ***subscripts*** of the $\underline{\hat{\hat{\mu}}}_{\underline{U}}$ *'metrical qualifiers'* or ***"dimensional units"*** are themselves *ontological qualifiers*, constituents of the space $\underline{\mathbf{U}}$, i.e., $3\underline{\hat{\hat{u}}}_3 \in \underline{\mathbf{U}}$. This 'meta-fractal' structuring is reflected in our $\underline{\mathbf{Q}}$ ***'ideo-ontodynamic' model*** of the ***'meta-systematic dialectic'*** of the progression of rules-systems of arithmetic, starting from that of $\underline{\mathbf{N}}$, denoting the "first-order" axiomatic rules-system of $\mathbf{N}$, to that of $\underline{\mathbf{Q}}$ to that of $\underline{\mathbf{U}}$ to that of $\underline{\mathbf{M}}$ to that of ${}_{\alpha}\underline{\mu}$. Per that model, ${}_{\alpha}\underline{\mu}$ is modeled as the $\hat{q}_4 \leftrightarrow \underline{\mathbf{M}}$ or *'pure Metrical qualifier'* rules-system ***subsumption*** of the $\hat{q}_3 \leftrightarrow \underline{\mathbf{U}}$ rules-system: $\hat{q}_7 \leftrightarrow {}_{\alpha}\underline{\mu} \; = \; \underline{\mathbf{MU}} - \underline{\mathbf{U}} \; = \; \underline{\mathbf{M}}\hat{q}_{\underline{QN}} - \hat{q}_{\underline{QN}} \; = \; \hat{q}_{\underline{M}}\hat{q}_{\underline{QN}} - \hat{q}_{\underline{QN}} = \hat{q}_{\underline{MQN}} \; = \; \hat{q}_{\underline{MU}}$ $\leftrightarrow \hat{q}_{4+3} = \hat{q}_7$, *i.e.*, as the *'hybrid'* or *'uni-thesis'* or ***'complex re-unity'*** of $\underline{\mathbf{M}}$ and $\underline{\mathbf{U}}$, just as $\underline{\mathbf{U}}$ models as the $\underline{\mathbf{Q}}$ ***subsumption*** of the $\underline{\mathbf{N}}$ arithmetical rules-system, the *'hybrid'*, *'uni-thesis'*, ***'complex re-unity'*** [***'re-unifying complex'***], or ***'complex re-unification'*** of $\hat{q}_2 \leftrightarrow \underline{\mathbf{Q}}$ and $\hat{q}_1 \leftrightarrow \underline{\mathbf{N}}$: $\hat{q}_3 \leftrightarrow \underline{\mathbf{U}} \; = \; \underline{\mathbf{QN}} - \underline{\mathbf{N}} \; = \; \underline{\mathbf{Q}}\hat{q}_{\underline{N}} - \hat{q}_{\underline{N}} \; = \; \hat{q}_{\underline{Q}}\hat{q}_{\underline{N}} - \hat{q}_{\underline{N}} \; = \; \hat{q}_{\underline{QN}} \leftrightarrow \hat{q}_{2+1} \; = \; \hat{q}_3$.

The ideographic representation of '***five cubic centimeters*** **of Hydrogen**' by $(5)[\, \underline{\hat{\hat{\mu}}}_{3\underline{\hat{\hat{u}}}_3} \,][\![{}^{2}_{a}\underline{\hat{\hat{\mu}}}_{H}]\!]$ presupposes certain generalizations of the arithmetical operations of addition and of multiplication. Specifically, it presupposes a concept of *"non-amalgamative addition"* [cf. writings by Charles Muses] and *'non-amalgamative multiplication'*, *viz.* --

Some ***"amalgamative additions"***: **4 apples + 3 apples = 7 apples**; *'homogeneous sums'*;

$$4 + 3 = 7;\; 4i + 3i = 7i;\; 4\hat{\mathbf{x}} + 3\hat{\mathbf{x}} = 7\hat{\mathbf{x}};\; 4\hat{\hat{\mathbf{e}}}_5 + 3\hat{\hat{\mathbf{e}}}_5 = 7\hat{\hat{\mathbf{e}}}_5;\;\; 4\underline{\hat{\hat{u}}}_1 + 3\underline{\hat{\hat{u}}}_1 = 7\underline{\hat{\hat{u}}}_1;\;\; 4\underline{\hat{\hat{\mu}}}_1 + 3\underline{\hat{\hat{\mu}}}_1 = 7\underline{\hat{\hat{\mu}}}_1;$$

Some ***"non-amalgamative additions"***: **4 apples + 3 oranges**; *'heterogeneous sums'* or *'inhomogeneous sums'*;

$$4 + 3i; \quad 4i + 3; \quad 4\hat{\mathbf{x}} + 3\hat{\mathbf{y}}; \quad 4\hat{\hat{\mathbf{e}}}_5 + 3\hat{\hat{\mathbf{e}}}_6; \quad \hat{q}_1 + \hat{q}_2; \quad 4\underline{\hat{\hat{u}}}_1 + 3\underline{\hat{\hat{u}}}_2; \quad 4\underline{\hat{\hat{\mu}}}_1 + 3\underline{\hat{\hat{\mu}}}_2;$$

Some ***'amalgamative multiplications'*** [number of *one* kind × other number of *same* kind = ***single*** number of ***some*** kind]:

$$4 \times 3 = 12;\;\; 4i \times 3i = -12;\;\; 4\hat{\mathbf{x}} \cdot 3\hat{\mathbf{y}} = 0;\; 4\hat{\hat{\mathbf{e}}}_5 \cdot 3\hat{\hat{\mathbf{e}}}_5 = 12;\;\; 4\underline{\hat{\hat{u}}}_1 \cdot 3\underline{\hat{\hat{u}}}_2 \; = \; 12\underline{\hat{\hat{u}}}_3;\;\; 4\underline{\hat{\hat{\mu}}}_2 \cdot 3\underline{\hat{\hat{\mu}}}_2 = 12\underline{\hat{\hat{\mu}}}_4;$$

Some ***'non-amalgamative multiplications'*** [number of *one* kind × other number of *another* kind ≠ ***single*** number of ***either*** kind, and such that both/all '''factors''', or '''multiplicands''' and '''multipliers''', remain distinctly visible in the product expression]:

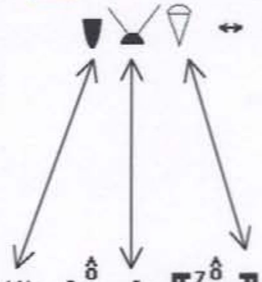
$$4 \times i = 4i; \quad 4 \times \hat{\mathbf{x}} = 4\hat{\mathbf{x}};\;\; 4 \times \hat{\hat{\mathbf{e}}}_5 = 4\hat{\hat{\mathbf{e}}}_5; \quad 4 \times \underline{\hat{\hat{u}}}_1 \; = \; 4\underline{\hat{\hat{u}}}_1;\;\; 4 \times \underline{\hat{\hat{\mu}}}_1 = 4\underline{\hat{\hat{\mu}}}_1; \quad 5 \times \underline{\hat{\hat{\mu}}}_{3\underline{\hat{\hat{u}}}_3} \times \underline{\hat{\hat{\mu}}}_1 \; = \; 5\underline{\hat{\hat{\mu}}}_{3\underline{\hat{\hat{u}}}_3 + 1}.$$

Note: ***'''Non-Amalgamative Addition'''***, or ***'Qualitative Addition'*** -- the ***assertion*** of the ***existence*** and of the ***'''summation'''*** or '''superposition''' or '''aggregation''' of at least two 'terms' denoting '[ev]entities' of different ***kind***, of different ***'''genera'''*** or of different ***'''species'''***; of ***different 'ontological quality'*** -- which gives the '$\substack{+\\ \leftarrow}$' relation its meaning, is the key to the *ever-more-richly, ever-more-concretely* realistic ***'''non-reductionist'''***, ***'contra-atomistic'***, ***'''holistic''' notations***, or ***'ideographies of the totality** and of its **sub-totalities'***, that arise in that 'Qualo-Peanic' dialectical succession of notations, of ideographic languages, of arithmetics, beginning with $\underline{\mathbf{N}} \rightarrow \underline{\mathbf{N}} + {}_{\underline{N}}\underline{\mathbf{Q}} \;\; \rightarrow \;\; \underline{\mathbf{N}} + {}_{\underline{N}}\underline{\mathbf{Q}} + {}_{\underline{N}}\underline{\mathbf{U}} \;\; \rightarrow \;\; \underline{\mathbf{N}} + {}_{\underline{N}}\underline{\mathbf{Q}} + {}_{\underline{N}}\underline{\mathbf{U}} + {}_{\underline{N}}\underline{\mathbf{M}} \;\; \rightarrow \;\; \underline{\mathbf{N}} + {}_{\underline{N}}\underline{\mathbf{Q}} + {}_{\underline{N}}\underline{\mathbf{U}} + {}_{\underline{N}}\underline{\mathbf{M}} + \hat{q}_{\underline{MN}}$, ***«et sequelae»***,

$$\leftrightarrow \quad \hat{q}_1 \;\rightarrow\; \hat{q}_1 + \hat{q}_2 \;\rightarrow\; \hat{q}_1 + \hat{q}_2 + \hat{q}_3 \;\rightarrow\; \hat{q}_1 + \hat{q}_2 + \hat{q}_3 + \hat{q}_4 \;\rightarrow\; \hat{q}_1 + \hat{q}_2 + \hat{q}_3 + \hat{q}_4 + \hat{q}_5,$$ ***«et sequelae»***.

Ideographical Quantifiers, 'Metrical Qualifiers', and 'Ontological Qualifiers', and the Ancient Mesopotamian Origins of Writing. Our *'ideogramic rendering'* of the *'phonogramic'* or "phonetic" phrase '**five cubic centimeters of Hydrogen**' by $\mathbf{(5)}[\,\overset{\hat{o}}{\underline{\mu}}_{\,{}_{3}\hat{\underline{u}}_{3}}\,][\![^{2}_{a}\,\overset{\hat{o}}{\underline{\mu}}_{H}]\!]$, i.e., by a *'non-amalgamative product'* of a ***quantifier***, a ***metrical qualifier***, and an ***ontological qualifier***, respectively, parallels the ***zenith 'meta-state'*** of the early *'meta-evolution'* of human writing praxis in prehistoric Mesopotamia, according to the 'psycho-archaeological' studies, and the 'psycho-historical' theory of the evolution of writing, set forth by Denise Schmandt-Besserat in her ***Before Writing***, and other works, viz., with ↔ "one", ↔ "sila" [an ancient, standard Near-Eastern fluid-volumetric ***unit***-of-***measurement***, or '***metrical «monad»***'], and ↔ "oil" --

↔ "'one sila oil'", or, "'one <of> sila(*s*) <of> oil'"



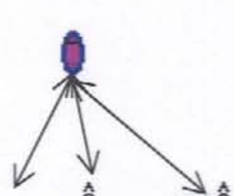
$$\mathbf{(1)}\cdot[\,\overset{\hat{o}}{\underline{\mu}}_{\,{}_{3}\hat{\underline{u}}_{3}}\,]\cdot[\![^{7}_{o}\,\overset{\hat{o}}{\underline{\mu}}_{n}]\!]$$

[Note we have herein asserted, hypothetically, via the '$^{7}_{o}$' prefix, that the *'onto'* of "oil" resides in our taxonomy level **7** universe of discourse, via setting the prefix-superscript '**7**' above the prefix-subscript '**o**', denoting "oils", in the *ontic qualifier*]. ■

■ **'Tokenology' becomes 'Tokenography'**. Per Dr. Schmandt-Besserat's theory, temple-dues goods-accounting, and, eventually, barterable-commodity accounting, progressed through a protracted sequence of qualitatively, '***ideo-ontologically***' distinct rules-systems. Each rules-system in this progression embodied a revolutionary expansion of '***symbols-ontology***', and corresponding '***ideo-ontology***', relative to its predecessor. This sequence of systems of accounting began with the representation of goods via a **3-D** '***tokenology***'. This system of '***token iconology***' was one of *stylized and conventionalized clay-sculpted micro-effigies* of the goods *of the then-extant goods-categories*, deployed as ***manually-'"graspable"'*** ***tangible symbols*** of the qualitatively different ***units*** or ***monads*** for each category of goods. The record of a typical transaction was an assemblage of assemblages of different kinds of tokens, thus a 'multi-«***arithmos***»' or 'meta-«***arithmos***»', eventually delimited by the enclosure of that ensemble of ensembles in a globular, ***opaque***, fired-clay envelope. A growing problem with this system, as the volume/'socio-geographical density' of transactions increased [i.e., as *social* [self-*re*-]*productivity*, reflecting the rising level of *the social self-forces of* [*society self-re-*]*production*, increased], was that, since these envelopes were made of ***opaque*** clay, the record of a given transaction could be '***audited***' only after ***breaking*** its envelope to reveal its token contents, ***a practice which entailed the labor of again producing a new fired-clay envelope after each such 'audit'***. An innovation eventually appeared, in which the fired-clay token contents-*to-be* of an envelope were pressed against the outside of that ***hollowed-out*** clay globe before its firing, while its clay was still moist, thus with sufficient visco-plasticity to receive the ***image*** of a token if one were manually pressed into it -- of an ***impressed*** token. After the emergence of this practice, spheroidal ***hollow*** fired-clay ***envelopes*** began to be superseded by 'rectanguloid' clay-filled or ***solid***, fired-clay ***slabs*** or "***tablets***". There appeared, on the face of the latter, *impressed* and, later, also *stylus-**incised*** token-images. There thereby arose a transition from ***3-D***, *iconic*, to ***2-D***, *inscribed, symbolization*: ***the beginning of writing***. The Mesopotamian branch of humanity thereby achieved the dialectical, 'socio-ontological', and 'psycho-historical' "meta-system transition" [cf. Turchin, *op. cit.*; cf. Logan, ***The Fifth Language***.] --

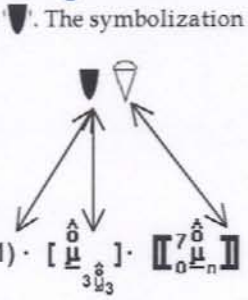
$$\underline{\textbf{tokenology}} \rightarrow \underline{\textbf{tokenology}} + \underline{\Delta}[\,\underline{\textbf{tokenology}}\,] = \underline{\textbf{tokenology}} + \underline{\textbf{tokenography}} \;\not\supset\; \underline{\textbf{tokenology}}.$$

The earlier praxis of **token iconology** became a compound one, of **token iconology** + **token iconography**. Then, later, the social praxis of **token iconology** died out. The **token iconography** praxis then 'meta-evolved' further, into 'picto-ideo-gramic' writing, and, later still, into that plus 'phono-gramic' writing. The earliest tokens-system presented a *'primitive undifferentiated unity'* of ***quantifier***, '***metrical qualifier***', and '***ontological qualifier***'. For example, the "tangible", "palpable", **3-D** '***micro-iconic***' token, which we depict via '', denoted '***one unit***', ***and*** '***one sila***' [to which we have, on this page, *re-assigned* $\overset{\hat{o}}{\underline{\mu}}_{\,{}_{3}\hat{\underline{u}}_{3}}$ from its former denotation, **cm.**^{3}], ***and*** '***oil***' all at once, all in one:



$$\mathbf{(1)}\cdot[\,\overset{\hat{o}}{\underline{\mu}}_{\,{}_{3}\hat{\underline{u}}_{3}}\,]\cdot[\![^{7}_{o}\,\overset{\hat{o}}{\underline{\mu}}_{n}]\!]$$

Later still, within the epoch of inscribed tokenography, the symbol-ontology bifurcated into a separate '*ontological qualifier*' ' ', the *impressed*, **2-D** successor to **3-D** ' ', versus an *incised* '*primitive undifferentiated unity*' [our conjecture] of *quantifier* and '*metrical qualifier*', ' '. The symbolization for '**one** *unit* of *oil*' became:



Finally, ' ' bifurcated into ' ' and/versus ' '. ***Incised***, ideogramic -- only vestigially pictogramic -- and full-fledged/'pure-quantitative' ***numerals*** emerged from formerly ***impressed*** symbols for the hybrid, 'ontological-metrical qualifiers' for grain, which was ***the most important use-value, and also the proto-money commodity*** of the economy of that ancient time and place. The incised sign ' ', which earlier denoted, "univocally", '***one ban*** *of* ***grain***, and which later '*chameleonically*' denoted '***one of the implicit standard/customary metrical units***' of the given kind -- ***any*** given kind -- of commodity to whose symbol it was juxtaposed, finally became the universal '*natural numbers*' ***numeral/'quantifier*** for '***one explicit abstract*** or ***generic unit***', presaging, in a more concrete thinking/writing praxis, Plato's later, more abstract theory of the «***arithmos monadikos***». The incised sign ' ', which earlier denoted, "univocally", '***one bariga***' of *grain*, finally became the universal '*natural numbers*' ***numeral/'quantifier*** for '***ten explicit abstract*** or ***generic units***'. Thus we arrived at ' ' as the expression for '***one sila*** *of* ***oil***', paralleled by our $_{\alpha}\underline{\underline{\mu}}$ ideographic expression **(1)**[ ][ ].

The *Rebus* Origins of Phonogramic Characters and of Phonetic Writing. After this degree of explicitude was reached regarding the differentiation of 'proto-written' ***quantifier***, '***metrical qualifier***', and '***ontological qualifier***' symbols, the intensifying need to express the specific personal ***names*** of the individual human transactors, as a key part of the transaction audit record, presented a problem. Individuals' names typically lacked direct ideographic/pictographic representability. Indirect, ***rebus*** strategies, applying the available ideo-pictographic symbols to represent the ***sounds*** of the ***words*** that ***named*** those ***symbols*** themselves, or that named the ***referents*** of those symbols, emerged. For example, ' L&' is a '*quasi*-phonetical' ***rebus*** encoding for the word also encoded -- ***purely***-phonetically or 'phonogramically' -- by the string of English 'phonogamic' symbols '**heartland**'. These strategies then generalized, from the symbolization of the personal names of the rulers' '*tributary*' or '*taxes-subject*' subjects, to that of the goods- or commodities-objects of the former heartland of the predecessor tokenographic ideo-pictographic symbolic system. This generalization led to ***syllabaries*** and ***logographies***, and, eventually, to full-fledged '***phonogramic***' ***symbolization*** of word-elements, and to ***phonetic alphabets***. [See D. Schmandt-Besserat, ***Before Writing***: ***From Counting to Cuneiform***, vol. I, University of Texas Press, [Austin, TX: 1992], pp. 184-192]. In this process, ***ideography*** survived mainly in the '***graphy***' [writing] of numerals. The ' ' and ' ' '*sub-rules-systems*' components of the '***zenith meta-state***' of the old rules-system of proto-writing passed over into '***phonogramy***'. After that, mostly just the ' ' or, equivalently, just the ' ' '*sub-rules-system*' components remained within 'ideogramy'. And so the '***metrical qualifier***' and '***ontological qualifier***' components were ***elided*** from *ideographical* representation, and therefore from the ***syntax*** of ***arithmetic***, if not, for a long time, from its ***conceptual semantics***. Heretofore, the ideography of '***metrical qualifiers***' has languished in retardation at the "syncopated" level of symbolism ever since, and the ideography of '***ontological qualifiers***' has hardly developed at all. The semantic, conceptual *inclusion of* [at least generic, abstract] ***qualifiers*** in arithmetic *was* maintained in the longstanding *philosophy of arithmetic* eventually distilled into Plato's theory of the «***arithmos monadikos***» [*op. cit.*, Jacob Klein, ***Greek Mathematical Thought and the Origin of Algebra***, pp. 88-99.]. According to that theory, for example, as also according to Aristotle's and other ancient accounts, a single unit did not belong to number; there was no "[*natural*] ***number one***". Number began with the smallest *group* or *assemblage* [«***arithmos***»] of multiple ***qualitative*** units or «***Monads***»: [natural] "number" began with **(two)[units]**. One unit by itself was, in their conception, '***purely qualitative***', ***not quantitative in the least***. One unit by itself was *a* ***qualitative*** *entity*, representable by a ***qualifier***, *not* by a ***quantifier*** *in any way*. It was a *monad*, a *unit*, an *instance*, a *specimen*, an *exemplar* of *some* ***kind*** of *thing*: one ***drachma***, one ***stade***, one ***papyrus***, etc. The «***arché***» of "**N**atural" number, denoted generically by M per Diophantus, was conceived as something ***qualitative***, ***not*** as something ***quantitative***, *at all*, *in any way*, let alone as '***purely***' so. Today, on the contrary, in the conceptions native to the current 'psycho-historical' state, or 'dynate', of Terran humanity, the unit of arithmetic, almost universally denoted **1**, is considered to be a purely quantitative entity. A vast -- if still mostly 'unremarked' -- 'psycho-historical'/'"ideological"' self-transformation of humanity intervenes between these two views of '"Number"', as it does between the mentalities behind these views.

One unit, by itself, was grasped as being ***ontological***, or ***metrical***, like '$\bigtriangledown$' or '$\checkmark$', respectively, not like the final meaning of '●'. According to Plato's theory, as recounted by Aristotle, the realm of the Platonic "Forms" or «***Ιδεα***»*ς*, of the «***arithmoi eidetikoi***», also contained a lower order of ***generic*** '«***Μοναδ***»'*ς*. Each such «***Μονας***» was an *abstract*, generic ***qualitative unit*** or *unity*, but one qualitatively identical to all of the other such units or unities, and thus capable of being mentally marshaled to the service of representing, indifferently, the sensible assemblages, or «***arithmoi aisthetoi***»; the ensembles of units of any material entity in the world of our outer experience, thereby making arithmetical enumerations and calculations possible. These abstract, idealized, generalized Platonic '***qualifiers***' are akin to what Diophantus denoted with his $\mathring{\mathrm{M}}$. However, Diophantus' unit-symbol or monad-symbol represents a conceptually more advanced form, e.g., no longer '***a-tom-ic***' or '***un-cutt-able***', but '***tom-ic***'; conceptually capable of being '***cut***-up', '***frac***tured', '***frac***ted', or '***frac***tionated' into proto-rational-number "***fractions***" of unity. ■

The European Renaissance of Commerce and the Historical Completion of 'The Elision of the Quantifiers'. The re-awakening of widespread, frequent, and regularized commerce brightened, slightly, the European "***Dark Ages***", into the global '***Dim Ages***', wherein we remain to this day. This re-awakening deepened and widened *the money-mediated exchange experience*, ultimately beyond even the zenith level of the ancient Hellenistic / Roman world. That experience involves a seeming *homogenization* of *qualitatively, ontologically* ***in****homogeneous commodities* based upon the *seeming "pure quantity"* of price -- i.e., based upon *an increasingly qualitatively obscure "common-denominator" ontological qualifier of* ***economic*** *value*, measured in *likewise increasingly sensuously remote units of* ***currency***. Under this deepened, intensified, and accelerated influence of "the development of the exchange-value", even that generic, idealized, abstract, 'Platonic-Diophantine', $\mathring{\mathrm{M}}$ form of qualifier-ideography was elided in the early stages of the European rebirth of ideographic arithmetic and proto-algebra. A near-total eclipse of ontological and metrical ***qualifier ideograms*** from arithmetico-algebraic and general mathematical ideography was now complete. That elision was progressive for its time. A comprehensive and coherent ideographic ***arithmetic of qualifiers*** was not within reach for the knowledge/science and for the cognitive-psycho-historical 'meta-state' of humanity at that time. A comprehensive and coherent ideographic ***arithmetic of quantifiers*** *was* within its reach, in the form of the advanced ***numerical ideography*** of the Hindu numerals-system. The separation of *quantitative ideography* from *qualitative phonogramy*, and this near-total eclipse of *qualitative ideography*, freed arithmetic, algebra, and mathematics as a whole for an enormous and accelerated, albeit one-sided, development of its quantitative side, one that a premature struggle to develop a comprehensive and coherent ***quanto-qualitative ideography*** would have hindered and retarded. ***Ancient arithmetic*** had had to separate completely from ***ancient geometry***; the concept of "***discrete***" «***arithmos***» from the concept of "***continuous***" and dimensional ***magnitude***. This separation began as early as the Pythagorean discovery of ***incommensurable magnitudes*** and "***irrational ratios***", starting with $1{:}\sqrt{2}$. Arithmetic and geometry thereafter developed in relative mutual insularity for a long period, until their ***synthesis*** or ***re-unification*** could be fruitful, eventuating in Cartesian 'algebraical geometry' and, later, in the axiomatization of the "**R**eal Numbers". This split abetted an ***implicit*** conception of '*degree zero*' or "***dimensionless***" [∴ '***Boolean***'] units in arithmetic. Such ***implicitly*** *dimensionless* ***unit-qualifiers*** enabled, e.g., $x^2 = 5^2$ to no longer ***implicitly*** connote, for example, **25** "***square***" *units of recti-****linear*** *length*, thus ***inhomogeneous*** with, and ∴ ***not*** "***amalgamatively***" '***addable***' with, $x^1 = x = 5^1 = 5$ "***linear***" length units. Such units allow us to *logically* equate, e.g., $x^2 + x$ to $5^2 + 5 = 25 + 5 = 30$ [***implicitly***, *thirty abstract, generic,* ***dimensionless*** *units*] in our seemingly "*pure-quantitative*" arithmetic-algebra:

- Early Ancient "*geometric*" view: $x^2 + x = 25 \cdot \square + 5 \cdot | \neq 30 \cdot \square$, or $30 \cdot |$, because $\square \neq |$;
- Late Ancient 'diophantine' view: $x^2 + x = (5^2 + 5) \cdot \mathring{\mathrm{M}} = (25) \cdot \mathring{\mathrm{M}} + (5) \cdot \mathring{\mathrm{M}} = 30 \cdot \mathring{\mathrm{M}} = (5^2 \cdot \mathring{\mathrm{M}}^2) + 5 \cdot \mathring{\mathrm{M}}$;
- Late Medieval+, '"*symbolical*"' view: $x^2 + x = 5^2 + 5 = 25 + 5 = 30$;
- *Dialectical-Ideographic* view: for $1 \leq n \in \mathbf{N} \ni M$ = Max. *number of "fundamental" dimensions in the metrical system;*

$$(x\hat{\mathring{\mu}}_{u_0})^2 + (x\hat{\mathring{\mu}}_{u_0}) = (5\hat{\mathring{\mu}}_{u_0})^2 + (5\hat{\mathring{\mu}}_{u_0}) = 25\hat{\mathring{\mu}}_{u_0+u_0} + 5\hat{\mathring{\mu}}_{u_0} = 25\hat{\mathring{\mu}}_{u_0} + 5\hat{\mathring{\mu}}_{u_0} = 30\hat{\mathring{\mu}}_{u_0};$$

$$(x\underline{\hat{\mathring{\mu}}}_{\hat{\mathring{\underline{u}}}_n})^2 + (x\underline{\hat{\mathring{\mu}}}_{\hat{\mathring{\underline{u}}}_n}) = (5^2)\underline{\hat{\mathring{\mu}}}_{2\hat{\mathring{\underline{u}}}_n} + (5)\underline{\hat{\mathring{\mu}}}_{\hat{\mathring{\underline{u}}}_n} = 25\underline{\hat{\mathring{\mu}}}_{2\hat{\mathring{\underline{u}}}_n} + 5\underline{\hat{\mathring{\mu}}}_{\hat{\mathring{\underline{u}}}_n} \neq 30\underline{\hat{\mathring{\mu}}}_{2\hat{\mathring{\underline{u}}}_n}; \quad \neq 30\underline{\hat{\mathring{\mu}}}_{\hat{\mathring{\underline{u}}}_n},$$

where, using the double-underscored 'Pi' symbol, $\underline{\underline{\prod}}$, to denote the standard iterated product operator, $\prod$, but generalized to encompass non-standard, '***quanto-qualitative***', '***non-amalgamative***' ***multiplication*** --

$$\hat{\mu}_{u_0} = [\ \coprod_{k=1,M} \hat{\mu}_{-\hat{\underline{u}}_k}\ ]^0 = [[\ \coprod_{k=1,M} \hat{\mu}_{-\hat{\underline{u}}_k}\ ]/[\ \coprod_{k=1,M} \hat{\mu}_{-\hat{\underline{u}}_k}\ ]] = \hat{\mu}_{+\hat{\underline{u}}_1+\hat{\underline{u}}_2+\hat{\underline{u}}_3+\ldots+\hat{\underline{u}}_M-\hat{\underline{u}}_1-\hat{\underline{u}}_2-\hat{\underline{u}}_3-\ldots-\hat{\underline{u}}_M} = \hat{\mu}_{+u_0+u_0+\ldots+u_0} = \hat{\mu}_{u_0},$$

so that, $[\hat{\mu}_{+\hat{\underline{u}}_1+\ldots+\hat{\underline{u}}_n+\ldots+\hat{\underline{u}}_M-\hat{\underline{u}}_1-\ldots-\hat{\underline{u}}_n-\ldots-\hat{\underline{u}}_M}] \times \hat{\mu}_{+\hat{\underline{u}}_n} = \hat{\mu}_{u_0+2\hat{\underline{u}}_n-1\hat{\underline{u}}_n} = \hat{\mu}_{+\hat{\underline{u}}_n}$, given $1 < n < M$;

where also $u_0 = 0 \cdot \hat{\underline{u}}_n = \hat{\underline{u}}_n - \hat{\underline{u}}_n = [\![\hat{\underline{u}}_n]\!] \div [\![\hat{\underline{u}}_n]\!] = [\![\hat{\underline{u}}_n]\!]^{+1}/[\![\hat{\underline{u}}_n]\!]^{+1} = [\![\hat{\underline{u}}_n]\!]^{+1} \times [\![\hat{\underline{u}}_n]\!]^{-1} = [\![\hat{\underline{u}}_n]\!]^{+1+(-1)} = [\![\hat{\underline{u}}_n]\!]^{0} = \hat{\underline{u}}_{n-n}$;

and where also $u_0 + u_0 = u_0$, and $u_0 \times \hat{\underline{u}}_n = \hat{\underline{u}}_n \times u_0 = \hat{\underline{u}}_{0+n} = \hat{\underline{u}}_{n+0} = \hat{\underline{u}}_n$ [***multiplicative** **identity***],

and $\hat{\underline{u}}_n/u_0 = \hat{\underline{u}}_n \div u_0 = \hat{\underline{u}}_n \times u_0^{-1} = \hat{\underline{u}}_n \times u_{-0} = \hat{\underline{u}}_{+n+(-0)} = \hat{\underline{u}}_{n-0} = \hat{\underline{u}}_n$, and $u_0 + \hat{\underline{u}}_n = \hat{\underline{u}}_n + u_0 = \hat{\underline{u}}_n$ [***additive** **identity***],

and $u_0/\hat{\underline{u}}_n = u_0 \div \hat{\underline{u}}_n = u_0 \times \hat{\underline{u}}_n^{-1} = u_0 \times \hat{\underline{u}}_{-n} = \hat{\underline{u}}_{+0+(-n)} = \hat{\underline{u}}_{0-n} = +\hat{\underline{u}}_{-n}^{+1} = -\hat{\underline{u}}_{+n}^{+1} = +\hat{\underline{u}}_{+n}^{-1}$;

$\therefore [\hat{\mu}_{u_0}] \times [\hat{\mu}_{u_0}] = [\hat{\mu}_{u_0+u_0}] = [\hat{\mu}_{u_0}]$, for the '*meta-heterosis convolute product rule*' of $_{\mathbf{Z}}\underline{\underline{\mathbf{U}}}$: $\hat{\underline{u}}_j \times \hat{\underline{u}}_k = \hat{\underline{u}}_{j+k}$, for $\mathbf{j, k} \in \mathbf{Z}$. Thus, the "***dimensionless***" or '*degree zero*' ***metrical*** ***unit-qualifier*** within $_{\alpha}\underline{\underline{\mu}}$ or $\underline{\underline{\alpha}}$, denoted $\hat{\mu}_{u_h}$, is '***Boolean***', or ***multiplicatively*** "***idempotent***" in its self-multiplication behavior, like $\mathbf{\mathring{M}}$. It is also *quantifiable*, $\hat{\mu}_{u_0} + \hat{\mu}_{u_0} = 2\hat{\mu}_{u_0}$, wherein $+3\hat{\mu}_{u_0} > +2\hat{\mu}_{u_0}$, as is $\mathbf{\mathring{M}}$, for which also $\mathbf{\mathring{M}} + \mathbf{\mathring{M}} = 2\mathbf{\mathring{M}}$, and $3\mathbf{\mathring{M}} > 2\mathbf{\mathring{M}}$, *not unquantifiable*, i.e., "***additively idempotent***", as is Boolean addition, e.g., $0_B + 0_B = 0_B$, and even 'contra-Boolean' $_{\mathbf{N}}\underline{\underline{\mathbf{Q}}}$ when it comes to addition, $\hat{q}_n + \hat{q}_n = \hat{q}_n$. Indeed $\hat{\mu}_{u_0}$ matches Diophantus' $\mathbf{\mathring{M}}$ in many ways [Note: The 'meta-number' $\hat{\mu}_{u_0}$ denotes *neither* the "***origin***" nor the «***arché***» of the '***metrical qualifier sub-space***' of the $_{\alpha}\underline{\underline{\mu}}$ or $\underline{\underline{\alpha}}$ space. It, denotes, rather, the generic unit-vector-like ***unit-length segment*** of a '*degenerate*' ***axis***, ***dimension***, or "***qualified number-line***" within that space, *perpendicular to all of the others*, one which also has its own "*positively-signed*" vs. "*negatively-signed*" '***number-rays***', '***number-wings***', or '***number-branches***', such that also $-\hat{\mu}_{u_0} < +\hat{\mu}_{u_0}$. The '"point"'-like "***origin***" of *both* the '***metrical qualifier space***', $\hat{\mu}_{\underline{U}}$, a 'meta-number space' which is a ***sub***-space to the $_{\alpha}\underline{\underline{\mu}}$ or $\underline{\underline{\alpha}}$ space, as well as of the '***ontological qualifier space***', $\hat{\mu}_{\underline{N}}$, also an $_{\alpha}\underline{\underline{\mu}}$ or $\underline{\underline{\alpha}}$ sub-space, and of that $_{\alpha}\underline{\underline{\mu}}$ or $\underline{\underline{\alpha}}$ or $\hat{\mu}_{\mathbf{N+\underline{U}}}$ space as a whole, is denoted μ_0. But ***neither*** $\hat{\mu}_{u_0}$ ***nor*** μ_0 are '***contra-Boolean***'. They are both '***Boolean***' *in their* ***self-multiplication behavior***, *unlike* the units of the ***typical*** *mutually-perpendicular* ***axes***, ***dimensions***, or '***qualified number-lines***' within that space, and within the space $_{\alpha}\underline{\underline{\mu}}$ as a whole, which are, on the contrary, and overwhelmingly, '***strongly contra-Boolean***'. Here we encounter the «***aufheben***» geometrical structure of the '"*analytical-geometrical*"' *space* of the $_{\mathbf{R}\,\alpha}\underline{\underline{\mu}}$ arithmetic and of the spaces of its predecessor dialectical arithmetics, with respect to the space of the $\mathbf{R}$ arithmetic. The 'Boolean' value mapped to the apparent '"infinitesimal"' "***point***" denoted μ_0, located at the ***central core*** of $_{\alpha}\underline{\underline{\mu}}$ space, can be grasped as and graphed as an analytical-geometrical ***image*** of a 'receded' ***vestige*** of a '***meta-finite***' ***version*** of the '*pure-quantitative*' ***space*** of the self-multiplicatively '*Boolean*' unit or monad, $\mathbf{1}$, namely, the space $\mathbf{R}$ of the arithmetic $\underline{\mathbf{R}}$, self-«***aufheben***»-conserved within the $_{\mathbf{R}\,\alpha}\underline{\underline{\mu}}$ or $_{\mathbf{R}}\underline{\underline{\alpha}}$ space. This 'receded' "***point***" is 'meta-fractally', and vastly, scale-diminished in that 'recedence', as if a ***potentially*** infinite, multi-dimensional ***space*** of $\mathbf{R}$ axes, of $\mathbf{R}^n$, were reduced to a scale that appears "***point***-sized" -- vanishingly small -- relative to the new scale of the $_{\mathbf{R}\,\alpha}\underline{\underline{\mu}}$ successor arithmetic's successor-space; a "***point***" located at the "origin"/source/center of the likewise multi-dimensional, potentially-infinite, '***meta-finite***' $_{\mathbf{R}\,\alpha}\underline{\underline{\mu}}$ or $_{\mathbf{R}}\underline{\underline{\alpha}}$ space. That $\mathbf{R}$ arithmetic is the ultimate predecessor, the «***arché***», of all of the '***pure-qualifier***' and also '***contra***-Boolean', '***quanto-qualifier***', '*dialectical*' 'meta-arithmetics', from $_{\mathbf{R}}\underline{\underline{\mathbf{Q}}}$ to $_{\mathbf{R}}\underline{\underline{\mathbf{U}}}$ to $_{\mathbf{R}\,\alpha}\underline{\underline{\mu}}$ or $_{\mathbf{R}}\underline{\underline{\alpha}}$, and, we hold, to $_{\mathbf{R}\,\beta}\underline{\underline{\mu}}$ or $_{\mathbf{R}}\underline{\underline{\beta}}$ and beyond, in our '***meta***-systematic-dialectical, ***systems***-progression/***categorial***-progression $\underline{\mathbf{Q}}$-based model of the 'meta-evolution' of that '***meta***-system' of dialectical arithmetics]. By now, we hold, Terran human civilization has arrived -- in great part by means of these separative developments, and under the *incentive* and the *compulsion* of the pursuit of "***the exchange-value***" -- at a level and scale of scientific knowledge and of conceptual penetration and appropriation of nature [both that of pre-human/extra-human nature, and of human, mental nature, including of ideographic written language, even to the degree of axiomatic, mathematical, 'ideographic' or "symbolic" formal logic], such that we can fruitfully carry out the development of an arithmetical, algebraical, and analytical ***ideography of ontological and metrical qualifiers***, as well as of ***other kinds of qualifiers*** that [we hypothesize] are emergent in the successor systems of $_{\alpha}\underline{\underline{\mu}}$ or $\underline{\underline{\alpha}}$, namely $_{\beta}\underline{\underline{\mu}}$ or $\underline{\underline{\beta}}$, $_{\gamma}\underline{\underline{\mu}}$ or $\underline{\underline{\gamma}}$, $_{\delta}\underline{\underline{\mu}}$ or $\underline{\underline{\delta}}$, etc. Moreover, we hold that these ideographies will be found, *in order to be successful as such*, to be, in general, '***strongly contra-Boolean***', '*meta-axiomatic*', '*meta-deductive*', '*meta-formal*' -- indeed, '***dialectical***' -- in their logics. We are moving, in our move from $\underline{\mathbf{N}}$ to $\underline{\mathbf{N}} + \underline{\mathbf{Q}}$, i.e., from the '"sphere"' of the ' "***pure***", ***unqualified quantifiers***' of $\underline{\mathbf{N}}$ arithmetic to embrace also their opposite, *the* ' "***pure***", ***unquantifiable qualifiers***' *of* $\underline{\mathbf{Q}}$ *arithmetic*; from the '*amalgamative additivity*' of **(1)·[atom] + (1)·[atom] = (2)·[atom]**s, to a special kind of ***categorial***, '***super***-*amalgamative addition*' operation, denoted by '⊕' below, viz. --

$$\langle\ \underline{\text{atoms}}\ \rangle \oplus \langle\ \underline{\text{atoms}}\ \rangle \oplus \langle\ \underline{\text{atoms}}\ \rangle \oplus \ldots \oplus \langle\ \underline{\text{atoms}}\ \rangle \quad = \quad \langle\ \underline{\text{atoms}}\ \rangle$$

-- i.e., to "***idempotent addition***" or "***additive idempotency***" *à la* the arithmetic of later/contemporary Boolean algebra, in which not only $0_B + 0_B = 0_B$, but also $1_B + 1_B = 1_B$.

In the *'ontological arithmetic'* or *'arithmetic of ontology'* of interpreted **Q**, positing or writing down '$\langle \underline{\text{atoms}} \rangle$' is tantamount to asserting that the existence/*finite manifestation* of the *'onto'* [*onto*logical *category*] of *"chemical elements"* is ***possible*** in the current *'context-of-discourse'* -- i.e., in the current *'meta-state'* of the *universe of discourse*.

Writing out an expression such as '$\langle \underline{\text{atoms}} \rangle \oplus \langle \underline{\text{atoms}} \rangle$' only says the same thing *redundantly*, as in a pleonasm.

But in **Q**, *un*like in Boolean arithmetic, we have too a kind of "non-idempotent", *'super-potent' multiplication*, which adds (a) new "class(es)" of ontology to a universe[-of-discourse] as a result of the also qualitatively self-reproductive/conserving [and, in actuality, ***quantitatively** self-**expansive***] '***self***-multiplication' of the pre-existing part of the ontology of that universe --

$$\langle \underline{\text{atoms}} \rangle \otimes \langle \underline{\text{atoms}} \rangle = \langle\, \underline{\text{atoms}} \oplus \underline{\Delta\text{atoms}} \,\rangle = \langle\, \underline{\text{atoms}} \oplus \underline{\text{molecules}} \,\rangle \;\gtrless\; \langle \underline{\text{atoms}} \rangle,$$ i.e.;

$$\langle \underline{\text{atoms}} \rangle^2 \neq \langle \underline{\text{atoms}} \rangle,$$ ***not*** in the sense of $\gtrless$, but of $\langle \underline{\text{atoms}} \rangle^2 \gtrless \langle \underline{\text{atoms}} \rangle$, and of $\underline{\text{atoms}} \sqsubset \underline{\text{molecules}}$

-- which is quintessentially *'contra-Boolean'* [". . . *we cannot conceive of the addition of any class . . . to the universe* . . ." [George Boole, *An Investigation of the Laws Of Thought. . .* [NY: Dover, 1958], p. 50*n*.]. Thus, we have, in the ontological interpretation of the **Q** arithmetic, the *'contra-Boolean'* principle of ***the ontological self-expansion of the universe of discourse***, in which the *multiplication-modeled* '***self-interaction***' of a given *onto* -- e.g., that of "chemical elements" or ***atoms*** -- signifies both the *«aufheben»* *'conservation-moment' reproduction* of the representative population of the ontological category of '***atoms***', and the *«aufheben»* *'elevation-moment'* ***generation***, by '***self-internalization***', and ***population***, of a new, ***qualitatively** different* ***meta***-*onto*[logical category], in this case, that of ***molecules***, brought into being by the '***metafractal**-ogenic*', '***self-«aufheben» self-incorporation***', '***self-subsumption***', or '***self-involution***' of the manifold of ***atoms***, in the sense that molecules are '***meta***-atoms' *made up out of multiple* atoms. This outer/*'objective-extensive'* or *'exo-empirical'* process of '***self-internalization***' of physical [ev]entities to *create new* '*exo-ontology*' -- of sub-nuclear "particles" to form sub-atomic "particles"; of sub-atomic "particles" to form atoms; of atoms to form molecules; of molecules to form prokaryotic cells; of prokaryotic cells to form eukaryotic cells; of eukaryotic cells to form multicellular *'meta-biota'* [*"meta-zoa"* & *"meta-phyta"*]; of meta-zoa to form animal societies -- forms '***mixed***', '***multi-dimensional***' realities: '***qualo-fractally***' or '***meta-fractally***' *structured* '***multi-meta-ontic***', '***multi-meta-monadic***' '***cumula***', or '***meta-«arithmoi»***'. This outer, external, physical-objective '***meta**-dynamic*', or '***dialectic***', of successive *'self-internalizations'* has 'inner' analogs in *the internal-to-the-mind*, conceptual, *'subjective-intensional'*, and *'intro-empirical'* process of '***The Gödelian Ideo-Metadynamic***', or 'Gödelian Dialectic', which forms *ideative* '***meta-fractals***'; *progressively-up-scaling qualitative self-similarity structures of sets/ideas*; '***multi-meta-ontic ideo-cumula***', or '***ideo-meta-«arithmoi»***'. That '***ideo-metadynamic***' of the '***meta-evolution***' of ***systems of ideas*** is a '***meta-formal***', *'meta-deductive'*, '***meta-axiomatic***' *process of creating new 'ideo-ontology'*. It is wrought, in *set-theoretic models* of arithmetics and of the '***meta-evolving***' species of number, by the '***self-incorporation***' of the sets which represent the predecessor kinds of numbers/arithmetics. This process yields '***sets of sets***' [*of sets . . .*] -- '***meta-sets***', *'made up out of'* or *"containing"*, among their elements, their predecessor '***sets*** [*of sets . . .*]', as well as the [proper] subsets of those predecessor sets. It thereby forms ***qualitatively new*** 'species' of sets, of higher Russellian-Gödelian "logical type", which, via their appropriate axiomatizations -- i.e., via accretions of new axioms -- may model the next higher type of number/arithmetic. That next-higher arithmetic is necessary to overcome the Gödel-incompleteness of the predecessor arithmetic or system of numbers. It is necessary to render the "Gödel formulae", the ***undecidable propositions*** -- asserting the "diophantine"-***unsolvability*** of certain "diophantine" equations in the predecessor arithmetic -- decidable, provable as true propositions, relative to the new, expanded axioms-system. And that next higher arithmetic will perhaps also render the formerly "diophantine"-unsolvable equations solvable, in a 'non-diophantine' sense, using its new kinds of '***non***-diophantine numbers' as solutions. But each such successor system of set-species and their corresponding number-species has its own new *"insolubilia"* and *'undecidabilia'*, its own new Gödel-incompleteness(es). These '***strongly contra-Boolean***' *principles* of ***dialectical***, *«aufheben» logic*, may be abstracted-out as follows for **(1)** the generic, ***minimally-interpreted*** **Q** arithmetic, **(2)** that arithmetic *interpreted for* ***historical dialectic***, or '"dialectic of [both pre-human and human] nature"', and **(3)** that arithmetic *interpreted* for ***meta-systematic dialectical***, *categorial-progression*, ***idea-systems**-progression* ***idea**-exposition*:

(1) $[\![\forall\, \hat{\underline{X}} \in {}_{N}\underline{Q}]\!] [\![\hat{\underline{X}} \longrightarrow\!\exists\; \lnot [\![\hat{\underline{X}}]\!] = \text{«aufheben»}[\![\hat{\underline{X}}]\!] = \underline{\text{auf}}[\![\hat{\underline{X}}]\!] = \hat{\underline{X}}[\![\hat{\underline{X}}]\!] = \hat{\underline{X}}^2 = [\![\hat{\underline{X}} \boxplus \Box[\![\hat{\underline{X}}]\!]\,]\!] \;\gtrless\; \hat{\underline{X}}\,]\!]$;

(2) $\langle \forall\, \hat{\underline{X}} \in {}_{N}\underline{Q} \rangle \langle \hat{\underline{X}} \longrightarrow \lnot\langle \hat{\underline{X}} \rangle = \text{«aufheben»}\langle \hat{\underline{X}} \rangle = \underline{\text{auf}}\langle \hat{\underline{X}} \rangle = \hat{\underline{X}}\langle \hat{\underline{X}} \rangle = \hat{\underline{X}}^2 = \langle \hat{\underline{X}} \oplus \underline{\Delta}\langle \hat{\underline{X}} \rangle\rangle \;\gtrless\; \hat{\underline{X}}\,\rangle$;

(3) $(\!| \forall\, \hat{\underline{X}} \in {}_{N}\underline{Q} |\!)(\!| \hat{\underline{X}} \longrightarrow\!\exists\; \lnot(\!| \hat{\underline{X}} |\!) = \text{«aufheben»}(\!| \hat{\underline{X}} |\!) = \underline{\text{auf}}(\!| \hat{\underline{X}} |\!) = \hat{\underline{X}}(\!| \hat{\underline{X}} |\!) = \hat{\underline{X}}^2 = (\!| \hat{\underline{X}} \oplus \underline{\Delta}(\!| \hat{\underline{X}} |\!)|\!) \;\gtrless\; \hat{\underline{X}}\,|\!)$,

which is the foundation for all of the example-models which follow, in ***Supplement B***. The above expressions each assert, among other principles, one which holds that each $\hat{\underline{X}}$ is its own '**auf**' -- its own self-*«aufheben» operator*; its own 'meta-evolutionary', 'meta-dynamical', ***dialectical*** *self*-negation/*self*-transformation operation.

In the ***Supplement B*** examples, we may, for typographical convenience, drop the '$\hat{}$' *symbol-element*, or '***ideographical diacritical mark***', [which signifies the unit-status [e.g., modulus equal to *unity*, etc.] of these '***dialectical meta-numbers***'], where the presence of other, contextual cues so allows. Also for typographical convenience, we may use standard **[ ]** parentheses, or "brackets", instead of, e.g., $[\![\ldots]\!]$, $\langle\ldots\rangle$, or $(\!|\ldots|\!)$, to enclose *un*interpreted **Q** "pure" '***ontological qualifiers***' as well as $\hat{\mu}_{\underline{u}}$ '***metrical qualifiers***', where context permits this without confusion. In addition, we may use the standard '+' sign in place of the '$\boxplus$' sign of the *un*interpreted **Q** arithmetic, the '$\oplus$' sign of the **Q** arithmetic interpreted for ***historical exo-dialectic***, and the '$\oplus$' sign of that arithmetic interpreted for ***categories-progression*** /***systems**-progression* '[***meta-**]**system-atic ideo-dialectic***', in contexts where the generalization of '+' to encompass these operations of superposition, aggregation, or "addition"', including their '***non**-amalgamative*', as well as their *idempotent*, or '***super**-amalgamative*' aspects, is clear.

Our introductory letter and its two supplements, *Supplement A.* and *Supplement B.*, are designed to provide background preparatory to our planned transmission to you of the three *Briefings* from Part I. of *Dialectical Ideography: A Contribution to the Immanent Critique of Arithmetic*, under separate cover. The contents of this mailing are also designed to introduce you to Part II. through Part V., as well as to Part I. In summary, the explorations recounted therein surround some new, "Non-Standard" arithmetics whose operations provide ideographic images, and a '*quanto-qualitative*' computational mimesis, for some deep and general patterns of '*meta*-evolutionary' process at all known levels of universe '*meta*-evolution'. They emerge by way of an *immanent* critique of the "Standard Arithmetic", '***The Pure-Quantifier Arithmetic***', **N**, with '**N**' denoting just the ***first-order*** part of the Peano axiomatic rules-system of the "Natural Numbers". They do so by '*explicitizing*' or '*outering*' the Standard, first-order, Peano rules-*system*'s '***intra-duality***', its implicit harboring of "Non-Standard" models of itself [as implied by the Löwenheim-Skolem theorem, as well as by the first-order conjunction of the Gödel completeness and incompleteness theorems]. This **N**-rooted critique of **N** leads to an initial, *«arché»* dialectical ideography which is an '***explicitization***' of a '*Peanic*' but "*Non*-Standard" "*Natural*" arithmetic. The latter can be described as '***The Pure-Qualifier Arithmetic***', herein denoted by the symbol **Q**. It is *an arithmetic interpretable as one of "pure", unquantified* and '*unquantifiable*' ["*additively idempotent*"] ***ontological*** '***qualifiers***', whose units also form a '***strongly contra-Boolean*** [*arithmetic and*] ***algebra***', i.e., which follow a "law" of '*intra*-duality', a strong contrary to Boole's "*Fundamental Law of Thought*" or "*law of* [*exo*-]*duality*". This **Q** arithmetic is counterpoint to that '*ideography* of "pure", *unqualified* '***quantifiers***', all with '***Boolean***' unities, which the "Standard" arithmetics of, e.g., the "**N**atural" through the "**R**eal" numbers, present. Dialectical synthesis of the latter, '*pure-quantitative*' arithmetics with '*pure-qualitative*' **Q** leads to the 'addition' of **U**, of a new, 'ideo-ontologically-expanded' system of arithmetic interpretable as a '***quanto-qualitative***' or '***qualo-quantitative***' ***ideography*** of '***quantifiable ontological qualifiers***', or of '***ontologically-qualifiable quantifiers***', for mathematical '***meta-modeling***' of objective and subjective **U**niverse[-*of-discourse*] '*multi-population meta-distribution*' '***meta-evolution***'. The resulting **U** system of dialectical arithmetic is capable of modeling -- i.e., can be interpreted for -- the '***meta-fractal***' multi-population dynamics and '***meta-dynamics***' of the '***self-[meta-]evolutions***' of '***multi-meta-ontic***', '***multi-meta-monadic***' ***cumula***, i.e., of concrete, empirical '***multi-«arithmoi aisthetoi»***' or '***meta-«arithmoi aisthetoi»***'. Further dialectical/immanent self-critique and dialectical synthesis, encompassing *quantifiable* '***metrical qualifiers***' as well as *quantifiable* '***ontological qualifiers***', leads to $_{\alpha}\underline{\mu}$ or $\underline{\alpha}$, whose '***metrical qualifier***' as well as '***ontological qualifier***' aspects both resurrect and advance the explicit, proto-ideographic "units" or "monads" last seen in Diophantus' circa 250 C.E. proto-algebraic work. As noted above, Diophantus denoted those '**Mo**nads', in his syncopated fashion, by $\overset{\circ}{M}$. The $_{\alpha}\underline{\mu}$ system of arithmetic includes, in addition to a [sub-]arithmetic interpretable for ***ontological qualifiers*** *á la* **Q** and **U**, plus a [sub-]arithmetic of ***quantifiers*** *á la* **N** and **U**, a new sub-arithmetic of ***Metrical qualifiers***. The latter arithmetical sub-system concretizes, 'explicitizes', and renders fully operational, as a 'non-syncopated', thoroughly ideographical-symbolic, 'operatorial' and algorithmic arithmetic, the arithmetic implicit in classical "dimensional analysis", as we have seen above. Moreover, it leads "naturally" to a 'semantification' of the "meaningless" singularities, or zero-division finite-time infinite values, and the resulting infinity residuals -- *infinitely erroneous* and quantitatively *infinitely falsified* empirically -- that plague the solution of especially the ***nonlinear*** and "*partial*" integro-differential equation-models of mathematical analysis, including those that codify most of the presently-known "laws" of nature. It does so by means of the metrical, ontological, etc., '[*re-*]*qualification*' of those '*pure-quantitative*' or '***unqualified***' equations. All of these ***immanent developments***, all of these "conceptual leaps", or 'ideo-ontological revolutions', from each predecessor ideas-/language-system to its successor ideas-/language-system; from **N** to **Q** to **U** to $_{\alpha}\underline{\mu}$ and beyond, ***including the partial hybrids/uni-theses in-between***, are derived and presented and even 'calculated' in the form of a '***meta-systematic*** *dialectical*' argument, guided by an '***ideo-onto-logical***'/'***ideo-onto-dynamical***' model of this '***meta-system***' and its '***meta-systemasis***'; this ***systems-as-categories***, ***systems-progression-as-categories-progression exposition***, all cast in the ideographical language of the **Q** arithmetic itself. That **Q** arithmetic serves, thus, as both **(1)** the language in which the whole progression of such categories/systems of arithmetic, grasped as cognitive '***meta-states***', is modeled, and also **(2)** just one particular system among those **Q**-modeled '***meta-states***' of that very '***cognitive-***' or '***ideo-meta-evolution***'.

Each "pure" '***qualifier***' of the **Q** ideography, generically denoted $\hat{\mathrm{q}}_x$, is interpreted as denoting a specific *ontological category*, '***ontological monad***', onto-"*type*", onto-"*predication*", or onto-"*quality*", one *non-quantitatively unequal*, i.e. *qualitatively unequal* to all of the others [$\neq$ not as $\gtrless$, but as $\not\gtrless$] $n \gtrless m \Rightarrow \hat{\mathrm{q}}_n \not\gtrless \hat{\mathrm{q}}_m$.

The **Q** arithmetics' conceptual '''*summation*''' or '''*addition*''' *operation* can algorithmically, syntactically model, via the ***inhomogeneous***, ***non-amalgamative***, *and* '***non-reductionist***' ***superposition***, ***aggregation***, or [*ac*]***cumulation*** of distinct ***ontological qualifiers***, the internal, mental, subjective-objective, or even the external-objective, actual, physical '''addition''', or 'geometrical', 'physical-spatial' co-existence and inter-mixing, of '***ontological monads***', of the local populations of the individual units of a given 'ontological category'. It does so by means of a geometrically/dimensionally self-expanding, '***meta-evolving Possibility-Space***', '*Ontology-State Space*', '*multi-meta-ontic cumulum*', or '***meta-«arithmos»***' interpreted for an *ontologically self-expanding universe of discourse*. This is precisely the kind of 'process-object' or 'eventity' that **Q** '*meta-models*' and the **Q** language were designed to model.

Addition in $_{\mathbf{N}}\mathbf{Q}$'s '***strongly contra-Boolean*** *arithmetic*', as in '*Boolean arithmetic*', is "*idempotent*": $\forall n \in \mathbf{N}, \quad \hat{\mathrm{q}}_n + \hat{\mathrm{q}}_n = \hat{\mathrm{q}}_n$.

The Q *Dialectical Ideography* as a "'Non-Standard Model'" of First-Order 'Peanic' "*Natural Numbers*" Arithmetic, with 'contra-Boolean' Algebraic Logic. The 'contra-Boolean' logic of the **Q** arithmetics is relevant to modeling wherever the application of an operation *to itself* produces a new, qualitatively/ontologically different, 'meta-finitely' higher, 'meta-fractally scaled-up' operation -- for example, wherever the ***self-inclusion*** of a [sub-]totality-*as-operation*, the incarnation of that [sub-]totality as a ***whole as*** a new ***element*** among its own previous elements, "inside" that [sub-]totality, yields a new, qualitatively different, ontologically expanded [sub-]totality 'eventity' or [sub-]totality-*as-operation*, one thereby self-escalated in 'degree', in "logical type", or in 'dimensionality'. In our interpretation of these arithmetics, their "*multiplication*" models '***meta-evolution***' -- *qualitative*, ontology-[*self-*]*change*; *ontological net expansion*. This «***aufheben***» [*self-*]*innovation* **+** [*self-*]'***multiplication***' or [*self-*]'***proliferation***' of new ontological quali*ties*/***activities*** is interpreted as arising via '[***self-induced***] ***bifurcation***' or '***meta-finite conversion singularity***'. This emergence of new, unprecedented ontological qualities from either the *self/other, mutual **inter**-action*, or the '***intra****-action*' or "***self-interaction***", of predecessor onto-qualities is modeled via a proliferation of appropriately-interpreted '***ontological qualifiers***', e.g., of **Q** '***meta-numerals***' or '***dialectors***'. These ideographical symbols and symbolic expressions thus provide ***mathematical metaphors***, *ideographical **mimeses***, as well as '***memeses***' or '***semantifications***', for the '***meta-dynamics***' of the ***self-expanding ontologies*** of such '***meta-evolving***', or [***aperiodically** and **acceleratively***] '***self-revolutionizing***' ***universes***[***-of-discourse***]. As mentioned above, the co-possibility of "*Non-Standard* Models" along with "*Standard* Models" of **N** arithmetic is a first-order co-implication ***of the Gödel Completeness and Incompleteness theorems***, when they are co-applied to the *first-order* Peano Postulates, which were *intended* and *expected* to cover *only* the "**N**atural" Numbers: "Most discussions of Godel's proof. . .focus on its quasi-paradoxical nature. It is illuminating, however, to ignore the proof and ponder the implications of the theorems themselves. It is particularly enlightening to consider together both the ***completeness*** and ***incompleteness theorems*** and to clarify the terminology, since the names of the two theorems might wrongly be taken to imply their incompatibility. The confusion arises from the two different senses in which the term "***complete***" is used within logic. In the ***semantic*** sense, "***complete***" means "capable of proving whatever is ***valid***", whereas in the ***syntactic*** sense, it means "capable of ***proving*** or ***refuting*** [i.e., of "***deciding***" -- **F.E.D.**] ***each sentence*** of the theory". Gödel's ***completeness theorem*** states that every (countable) ***first-order*** theory, whatever its non-logical axioms may be, is complete in the former sense: Its theorems coincide with the statements ***true*** in ***all models*** of its axioms. The ***incompleteness theorems***, on the other hand, show that if formal number theory is consistent, it fails to be ***complete*** in the second sense. The ***incompleteness theorems*** hold also for ***higher-order*** formalizations of number theory. If only ***first-order*** formalizations are considered, ***then the completeness theorem applies as well***, and ***together they yield not a contradiction, but an interesting conclusion***. Any sentence of arithmetic that is ***undecidable*** must be ***true in some models*** of ***Peano's axioms*** (lest it be ***formally refutable*** [as it would be were it true in ***no*** models of the Peano axioms -- **F.E.D.**]) and ***false in others*** (lest it be ***formally provable*** [as it would be were it true in ***all*** models of the Peano axioms -- **F.E.D.**]). ***In particular, there must be models of first-order Peano arithmetic whose elements do not "behave" the same as the natural numbers***. Such ***nonstandard models*** were ***unforeseen*** and ***unintended*** but they cannot be ignored, for their existence implies that *no **first-order** axiomatization of number theory can be adequate to the task of deriving as theorems* **exactly** *those statements that are true of the* ["standard" -- **F.E.D.**] ***natural numbers***." [John W. Dawson, Jr. , ***Logical Dilemmas*: *The Life and Work of Kurt Gödel***, A. K. Peters [Wellesley, MA: 1997], pp. 67-68, ***blue bold italic emphasis*** added by **F.E.D.**]. For example, in terms of the standard geometric interpretations of the space **N**, and of our usual 'analytical-geometrical' interpretation of the space $_{\mathbf{N}}\mathbf{Q}$, the two following assertions are ***true*** in one "***interpretation***" or "***model***" of what we here denote by '**§**', standing for a '*genericized*' ***first***-order Peano-Postulates-based, axiomatic ***rules-system*** for the "**N**atural Numbers", and ***not*** in another such "***model***", with § denoting the "'space'" or "'number-set'" for **§**:

(1) True in **N**, *not in* $_{\mathbf{N}}\mathbf{Q}$: **x**, **y** in §, & **x** ≠ **y** implies "point" **x** *is the to the left of* **y** [**x** < **y**], **OR** "point" **x** *is to the right of* **y** [**x** > **y**];

(2) True in $_{\mathbf{N}}\mathbf{Q}$, *not in* **N**: $\hat{x}$, $\hat{y}$ in §, & $\hat{x} \neq \hat{y}$ implies $\hat{x}$ *and* $\hat{y}$ *denote **mutually-perpendicular unit-length line-segments*** [$\hat{x} \perp \hat{y}$].

The ***Löwenheim-Skolem theorem*** has similar implications: "The research begun in 1915 by Leopold Löwenheim (1878-c. 1940), and simplified and completed by Thoralf Skolem (1887-1963) in a series of papers from 1920 to 1933, disclosed new flaws in the structure of mathematics. The substance of what is now known as the Löwenheim-Skolem theory is this. Suppose one sets up axioms, logical and mathematical, for a branch of mathematics or for set theory as a foundation for all of mathematics. The most pertinent example is the set of axioms for the whole numbers. One intends that these axioms should completely describe the ***positive whole numbers*** [i.e., the "**N**atural numbers, **N** -- **F.E.D.**] and only the whole numbers. But, ***surprisingly, one discovers that one can find interpretations -- models -- that are drastically different and yet satisfy the axioms***. Thus, whereas the set of whole numbers is countable, or, in Cantor's notation, there are only $\aleph_0$ of them, there are ***interpretations*** that contain as many elements as the real numbers, and even sets larger in the transfinite sense. The converse phenomenon also occurs. That is, suppose one adopts a system of axioms for a theory of sets and one intends that these axioms should permit and indeed characterize non-denumerable collections of sets. One can, nevertheless, find a countable (denumerable) collection of sets that satisfies the system of axioms and other transfinite ***interpretations*** quite apart from the one intended. In fact, every consistent set of axioms has a countable ***model***.. . .In other words, ***axiom systems that are designed to characterize a unique class of mathematical objects do not do so***. Whereas Gödel's incompleteness theorem tells us that a set of axioms is not adequate to prove all the theorems belonging to the branch of mathematics that the axioms are intended to cover, ***the Löwenheim-Skolem theorem tells us that a set of axioms permits many more essentially different*** ['***qualitatively different***', '***ideo-ontologically different***', unequal in the '$\frac{\ne}{}$' sense -- **F.E.D.**] ***interpretations than the one intended***. The axioms do not limit the interpretations or models. ***Hence mathematical reality cannot be unambiguously incorporated in axiomatic systems***.* *Older texts did "prove" that the basic systems were ***categorical***: that is, all the ***interpretations*** of any basic axiom system are isomorphic -- they are essentially the same but differ in terminology. But the "proofs" were loose in that logical principles were used that are not allowed in Hilbert's metamathematics and the axiomatic bases were not as carefully formulated then as now. ***No set of axioms is categorical, despite "proofs" by Hilbert and others***.. . .One reason that unintended ***interpretations*** are possible is that each axiomatic system contains ***undefined terms***. Formerly, it was thought that the axioms "defined" these terms ***implicitly***. But the axioms do not suffice. Hence the concept of ***undefined terms*** must be altered in some as yet unforeseeable way. The Löwenheim-Skolem theorem is as startling as Gödel's incompleteness theorem. It is another blow to the axiomatic method which from 1900 even to recent times seemed to be the only sound approach, and is still the one employed by logicists, formalists, and set-theorists." [Morris Kline, ***Mathematics*: *The Loss of Certainty***, Oxford U. Press [NY: 1980], pp. 271-272, ***blue bold italic emphasis*** added by **F.E.D.**].

<u>The 'Qualitative Peanicity' of $_{\mathbf{N}}\underline{\mathbf{Q}}$ as Non-Standard "Natural" Arithmetic</u>. An axiomatic basis for geometry, via Euclid's five postulates, was already extant circa 300 B.C.E. Though Plato is reported to have advanced a philosophy of arithmetic -- of both the *«arithmos monadikos»* and the *«arithmos eidetikos»* -- already circa 380 B.C.E., the discovery [or re-discovery?] of an axiomatic basis for arithmetic was 'delayed' by ~2,400 years, until the publication of five postulates for "**N**atural" arithmetic by Giuseppe Peano in 1889 C.E. Even without invoking directly the intricate logical-ideographical machinery behind the two Gödel theorems invoked in the quote above, we can see, ***by inspection***, that the four ***first-order*** Peano Postulates encompass more than the arithmetic of **N** alone. *They are termed "**first-order**" because they address only "**logical individuals**", individual **numbers** in this case*, but ***not*** *qualities-in-common* or "predicates" of sets [*of sets...*] of such "logical individuals"/numbers. Those first four axioms of Peano's five axioms for the "Natural Numbers" define part of what we denote by $\underline{\mathbf{N}}$, and are classically stated as follows --

The *first-order* ***Peano Postulates*** [*earlier version*] for the Standard **N**atural Numbers, denoted **N**:

P1: $\mathbf{1}$ is a [**N**atural] Number, or: $\mathbf{1} \in \mathbf{N}$. <u>Define a 'successor function'</u>: $\mathbf{s} \mid [\ \forall \mathbf{n} \in \mathbf{N},\ \mathbf{s(n) = n+1}\ ]$.

P2: The successor of any [**N**atural] Number is a[lso a] [**N**atural] Number, or: $\mathbf{n} \in \mathbf{N} \Rightarrow \mathbf{s(n)} \in \mathbf{N}$.

P3: No two [**N**atural] Numbers have the same successor, or: $\mathbf{n, m} \in \mathbf{N}\ \&\ \mathbf{n \neq m} \Rightarrow \mathbf{s(n) \neq s(m)}$.

P4: $\mathbf{1}$ is not the successor of any [**N**atural] number, or: $\neg \exists \mathbf{x} \in \mathbf{N} \mid \mathbf{s(x) = 1}$ [i.e., the "diophantine equation" $\mathbf{x + 1 = 1}$ is an ***unsolvable equation*** within $\underline{\mathbf{N}}$]. [cf.: Reese, W., *<u>Dictionary of Philosophy & Religion</u>: Eastern & Western Thought*, Humanities Pr. [Atlantic Heights, NJ: 1987], pp. 418-419]. The following re-rendition may help to bring forward their ***generic*** content, not limited to **N**:

g_1: *The '<u>a</u>rcheic monad'* [or *«<u>a</u>rché»*] $\underset{*}{\mathbf{a}}$ *is a constituent of the <u>S</u>pace* $\underset{*}{\mathbf{S}}$ *of the 'Rules-<u>S</u>ystem'* $\underline{\underset{*}{\mathbf{S}}}$: $\underset{*}{\mathbf{a}} \sqsubset \underset{*}{\mathbf{S}}$;

This postulate, together with g_4, means that this "space" has $\underset{*}{\mathbf{a}}$ as its definite *'origin'* or *'beginning'*;

g_2: ***The successor of any constituent of*** $\underset{*}{\mathbf{S}}$ ***is also a constituent of*** $\underset{*}{\mathbf{S}}$: If $\underset{*}{\mathbf{b}} \sqsubset \underset{*}{\mathbf{S}}$, then $\underset{*}{\mathbf{s}}[\underset{*}{\mathbf{b}}] \sqsubset \underset{*}{\mathbf{S}}$:

<u>Note</u>: *all of the successors of* $\underset{*}{\mathbf{a}}$ *are therefore part of the Discrete <u>S</u>equence-'<u>S</u>pace'*, or *'**Consecuum**'*, $\underset{*}{\mathbf{S}}$;

g_3: ***Distinct constituents of the <u>S</u>equence*** $\underset{*}{\mathbf{S}}$ ***have distinct successors***; for every $\underset{*}{\mathbf{b}}, \underset{*}{\mathbf{c}} \sqsubset \underset{*}{\mathbf{S}}$, if $\underset{*}{\mathbf{b}} \neq \underset{*}{\mathbf{c}}$,

then $\underset{*}{\mathbf{s}}[\underset{*}{\mathbf{b}}] \neq \underset{*}{\mathbf{s}}[\underset{*}{\mathbf{c}}]$; This postulate is necessary to secure the 'singularity'/uniqueness of each term in any such '***«arché»-onic sequence***';

g_4: ***The 'archeic monad'*** $\underset{*}{\mathbf{a}}$ ***has no predecessor in the '<u>S</u>equence-<u>S</u>pace'*** $\underset{*}{\mathbf{S}}$ ***of the 'Rules-<u>S</u>ystem'*** $\underline{\underset{*}{\mathbf{S}}}$:

There does not exist $\underset{*}{\mathbf{x}} \sqsubset \underset{*}{\mathbf{S}}$ such that $\underset{*}{\mathbf{s}}[\underset{*}{\mathbf{x}}] = \underset{*}{\mathbf{a}}$ -- in short, $\underset{*}{\mathbf{a}}$ *is the **«arché»** of* $\underset{*}{\mathbf{S}}$.

Inspection of these first four, first-order Peano Postulates thus reveals that they can describe ***anything that qualifies*** as what we term an *'**archeonic**' **sequence**, '**archeonic <u>consecuum</u>**'*, or *'**archeonic <u>cumulum</u>**'*, by which we mean a '***beginningful***' [vs. "beginning***<u>less</u>***", like **Z**, **Q**, **R**, and **C**] but ***<u>potentially</u> endless*** sequence of *unique, discrete, <u>consecutive</u>* '[meta-]numbers', *with no '[meta-]numbers'* of the space in question situated *in-between* any pair of '[meta-]numbers' in this sequence of 'successorship'. At least for the classical Peano **s**uccessor function, each successor 'contains' and thus '"conserves"', while ***<u>thereby</u>*** also '"surpassing"', its predecessor, in a vestigial ***«aufheben»*** fashion: $\mathbf{s(n) = n+1}$, so $\mathbf{n} \sqsubset \mathbf{s(n)}$. Since each $\mathbf{s(n)}$ is an ***«arithmos»***, made up of some multiplicity of the '***archeic <u>monad</u>***', or ***<u>unit</u>***, **1**, we also have that the ***«arché»*** too is '"contained/conserved/surpassed"' in all of its ***«sequelae»***: $\mathbf{1} \sqsubset \mathbf{s(n)}$. Suppose humanity should discover, via protracted practical and theoretical exploration, that their maximal [cosmological] universe-of-discourse regarding natural history exhibits the following historical sequence of '***physical cumula***' or '***physical sums***' [for which we use a '***physical addition' sign***, '⊕'] of ***multiplicities of each kind*** of ***multiple kinds*** of [ev-]entities:

⟨...<u>atoms</u>...⟩ → ⟨...<u>atoms</u>... ⊕ <u>molecules</u> ⟩ → ⟨...<u>atoms</u>...⊕ <u>molecules</u>...⊕ <u>atomic/molecular hybrid formations</u> ⟩ →

⟨...<u>atoms</u>...⊕ <u>molecules</u>...⊕ <u>atomic/molecular hybrid formations</u>...⊕ <u>cells</u> ⟩ →

⟨...<u>atoms</u>...⊕ <u>molecules</u>...⊕ <u>atomic/molecular hybrid formations</u>...⊕ <u>cells</u>...⊕ <u>hybrids</u> ⊕ <u>multicellular organisms</u> ⟩ →...,

or, seeing that "molecules" are 'meta-atoms' [made up out of multiple "atoms"], and, in turn, that prokaryotic "living cells" are 'meta-molecules' [made up out of multiple "molecules"], and so on, that --

⟨...<u>atoms</u>...] → ⟨...<u>atoms</u>...⊕ <u>meta-atoms</u>] → ⟨...<u>atoms</u>...⊕ <u>meta-atoms</u>...⊕ <u>atom/meta-atom hybrid formations</u> ⟩ →

⟨...<u>atoms</u>...⊕ <u>meta^1-atoms</u>...⊕ <u>atom/meta^1-atom hybrid formations</u>...⊕ <u>meta-meta-atoms</u> ⟩ →

⟨...<u>atoms</u>...⊕ <u>meta^1-atoms</u>...⊕ <u>atom/meta^1-atom hybrids</u>...⊕ <u>meta^2-atoms</u> ⊕ <u>atom/meta^2-atom hybrids</u>...⊕ <u>meta^3-atoms</u> ⟩ →....

Suppose further that such 'meta-fractal' and 'meta***finite*** meta-regress' historical sequences, or ['*time-marking*' and '*time-defining*'] *<u>temporal</u>* '***orders-of-appearance***' of ***<u>kinds</u>*** of [ev]entities, or '***ontos***', are found also to be ubiquitous among the taxonomic 'sub-levels'; the ***non***-maximal ***<u>sub</u>**-universes-of-discourse* of their ***sub***-classifications of the classes of kinds of entities appearing in the historical sequence of that maximal universe-of-discourse. Thus we might have, for example, "within" the 'onto' of **<u>atoms</u>**, the 'sub-onto' of **<u>Hydrogen atoms</u>**, herein denoted **<u>H</u>**, and the 'sub-onto' of **<u>Helium atoms</u>**, herein denoted **<u>He</u>**, such that --

⟨ <u>H</u> ⟩ → $^{-2}$⟨ <u>H</u> ⟩ = <u>H</u> "of" <u>H</u> = ⟨ <u>H</u> ⊕ <u>ΔH</u> ⟩ = ⟨ <u>H</u> ⊕ <u>meta-H</u> ⟩ = ⟨ <u>H</u> ⊕ <u>He</u> ⟩ $\frac{\rightarrow}{\leftarrow}$ <u>H</u>; <u>He</u>, and, moreover, **⟨ <u>H</u> ⊕ <u>He</u> ⟩ →...,**

whereby **<u>He</u>** atoms are grasped as 'meta-**<u>H</u>**' atoms, made up out of multiple **<u>H</u>** atoms, e.g., either via ***stellar*** *nucleosynthesis* ['***reproductive*** accumulation' process for **<u>He</u>**], or via '***cosmological*** *nucleosynthesis*' ['***original*** or ***primitive*** accumulation' of **<u>He</u>**].

If so, might they find it useful to construct a <u>*generic*</u> '*archeonic consecuum*', ${}_{\underline{N}}\underline{Q} \equiv \{ \hat{q}_1, \hat{q}_2, \hat{q}_3, \ldots\}$, for the Rules-System of a new kind of, by-then, to-them, "natural", arithmetic, call it ${}_{\underline{N}}\underline{Q}$, in which --

Q1: $\hat{q}_1 \in {}_{N}\underline{Q} \sqsubset {}_{\underline{N}}\underline{Q}$; <u>Define a 'successor function'</u>: $\underline{s} \mid [\![\forall n \in N, \underline{s}[\![\hat{q}_n]\!] = \hat{q}_{s(n)} = \hat{q}_{n+1}]\!]$.

Q2: $[\![\forall n \in \mathbf{N}, \hat{q}_n \in {}_{N}\underline{Q}]\!] \Rightarrow [\![\underline{s}[\![\hat{q}_n]\!] = \hat{q}_{s(n)} = \hat{q}_{n+1} \in {}_{N}\underline{Q}]\!]$;

Q3: $[\![\forall \hat{q}_m, \hat{q}_n \in {}_{N}\underline{Q}]\!] \; [m \neq n] \Rightarrow [\![\underline{s}[\![\hat{q}_m]\!] \neq \underline{s}[\![\hat{q}_n]\!]]\!]$; indeed, it $\Rightarrow \underline{s}[\![\hat{q}_m]\!] \gtrless \underline{s}[\![\hat{q}_n]\!]$;

Q4: $[\![\nexists \hat{q}_x \in {}_{N}\underline{Q}]\!] \mid [\![\underline{s}[\![\hat{q}_x]\!] = \hat{q}_{s(x)} = \hat{q}_1]\!]$

-- and for which ${}_{N}\underline{Q}$'s '*meta-numerals*' can be *interpreted* as denoting the '*ontos*' of any given, '*fitting*' historical sequence of *ontic* self-expansion to which they are [$\therefore$ *rightly*] applied, for purposes of 'meta-modeling' of the dialectical processes generically denoted by $[\![\hat{q}_1]\!]^{2^{\tau}}$, as follows [wherein we sometimes use '$-\!\boxplus\!-$', instead of '$\boxplus$', to stress the *mutual oppositeness* of two '*ontic terms*']:

$\hat{q}_1$ = ['***first*** <u>***thesis***</u>', '***«arché»*** <u>***thesis***</u>', or '***initiating*** <u>***thesis***</u>']: *Stipulated initial/inaugurating* '*onto*' for given historical order of appearance of '*ontos*'; '*seed*' *onto of universe*[-of-discourse], $= [\![\hat{q}_1]\!]^{2^0} = [\![\hat{q}_1]\!]^1$;

$\hat{q}_2$ = ['***first*** <u>***contra-thesis***</u>']: The **2**nd-arising '*onto*' for the given order of '*ontos*', *qualitatively* '*opposing*' the first, $= \hat{q}_{1+1}$; together with the first, forming the first '***antithesis-sum***', $= [\![\hat{q}_1]\!]^{2^1} = \hat{q}_1 -\!\boxplus\!- \hat{q}_2$;

$\hat{q}_3$ = ['***first full*** <u>***uni-thesis***</u>']: The **3**rd-arising '*onto*', hybridizing/reconciling/<u>**uni**</u>fying the **1**st & **2**nd, $= \hat{q}_{2+1}$, and ***opposing their*** mutual ***opposition***, i.e., the sub-whole or '***antithesis-sum***' formed by the "'***non-amalgamative sum***'" of the first two 'ontos';

$\hat{q}_4$ = ['***second*** <u>***contra-thesis***</u>']: The **4**th-appearing '*onto*', $= \hat{q}_{2+2}$, '*opposing*' the [sub-]whole, '***synthesis-sum***', or '***meta-thesis***' formed of the '***qualitative sum***' of the first **3** '*ontos*', forming the '***antithesis-sum***' $= [\![\hat{q}_1]\!]^{2^2} = [\![\hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3]\!] -\!\boxplus\!- \hat{q}_4$;

$\hat{q}_5$ = ['***first partial*** <u>***uni-thesis***</u>']: The **5**th-appearing '*onto*', hybridizing/*mediating*/unifying the **4**th & **1**st '*ontos*', $= \hat{q}_{4+1}$;

$\hat{q}_6$ = ['***second partial*** <u>***uni-thesis***</u>']: The **6**th-appearing '*onto*', hybridizing/reconciling/unifying the **4**th & **2**nd '*ontos*', $= \hat{q}_{4+2}$;

$\hat{q}_7$ = ['***second full*** <u>***uni-thesis***</u>']: The **7**th-appearing '*onto*', hybridizing/reconciling/unifying the **4**th and **3**rd '*ontos*', and, thereby, unifying the **2**nd 'contra-thesis *onto*' with the **1**st 'contra-thesis *onto*' & the '***«arché» onto***', $= \hat{q}_{4+3} = \hat{q}_{4+2+1} = \hat{q}_{2^2+2^1+2^0}$;

$\hat{q}_8$ = ['***third*** <u>***contra-thesis***</u>']: The **8**th-appearing '*onto*', $= \hat{q}_{4+4}$, '*opposing*' the [sub-]whole, '***synthesis-sum***', or '***meta-thesis***' formed of the '***qualitative sum***' or '<u>meta</u>-***«arithmos»***' formed by the first **7** '*ontos*', forming the '***antithesis-sum***' $= [\![\hat{q}_1]\!]^{2^3}$;

$\hat{q}_9$ = ['***third partial*** <u>***uni-thesis***</u>']: The **9**th-appearing '*onto*', hybridizing/reconciling/unifying the **8**th & **1**st '*ontos*', $= \hat{q}_{8+1}$;

$\hat{q}_{10}$ = ['***fourth, partial,*** <u>***uni-thesis***</u>']: The **10**th-appearing '*onto*', hybridizing/reconciling/unifying the **8**th & **2**nd '*ontos*', $= \hat{q}_{8+2}$;

$\hat{q}_{11}$ = ['***fifth, partial,*** <u>***uni-thesis***</u>']: The **11**th-appearing '*onto*', hybridizing/reconciling/unifying the **8**th & **3**rd '*ontos*', $= \hat{q}_{8+3}$;

$\hat{q}_{12}$ = ['***sixth, partial,*** <u>***uni-thesis***</u>']: The **12**th-appearing '*onto*', hybridizing/reconciling/unifying the **8**th & **4**th '*ontos*', $= \hat{q}_{8+4}$;

$\hat{q}_{13}$ = ['***seventh, partial,*** <u>***uni-thesis***</u>']: The **13**th-arising '*onto*', hybridizing/reconciling/unifying the **8**th & **5**th '*ontos*', $= \hat{q}_{8+5}$;

$\hat{q}_{14}$ = ['***eighth, partial,*** <u>***uni-thesis***</u>']: The **14**th-appearing '*onto*', hybridizing/reconciling/unifying the **8**th & **6**th '*ontos*', $= \hat{q}_{8+6}$;

$\hat{q}_{15}$ = ['***third full*** <u>***uni-thesis***</u>']: The **15**th-appearing '*onto*', hybridizing/reconciling/unifying the **8**th and **7**th '*ontos*', thereby unifying the **3**rd, **2**nd, and **1**st 'contra-thesis ***ontos***' and the '***«arché» onto***', $= \hat{q}_{8+7} = \hat{q}_{8+4+2+1} = \hat{q}_{2^3+2^2+2^1+2^0}$;

$\hat{q}_{16}$ = ['***fourth*** <u>***contra-thesis***</u>']: The **16**th-emerging '*onto*', $= \hat{q}_{8+8}$, '*opposing*' the [sub-]whole, '***synthesis-sum***', or '***meta-thesis***' formed of the '***qualitative* sum**' of the first **15** '*ontos*', forming the '***antithesis-sum***' $= [\![\hat{q}_1]\!]^{2^4} = [\![\hat{q}_1]\!]^{16} =$

$$[\![\hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \boxplus \hat{q}_4 \boxplus \hat{q}_5 \boxplus \hat{q}_6 \boxplus \hat{q}_7 \boxplus \hat{q}_8 \boxplus \hat{q}_9 \boxplus \hat{q}_{10} \boxplus \hat{q}_{11} \boxplus \hat{q}_{12} \boxplus \hat{q}_{13} \boxplus \hat{q}_{14} \boxplus \hat{q}_{15}]\!] -\!\boxplus\!- \hat{q}_{16}; \ldots$$

. . . ?

But such would be a sequence, not of '*generic Peanic* <u>*quantifiers*</u>', as is the **N** sequence **{1, 2, 3, ...}**, but, rather, one of '*generic Peanic* <u>*qualifiers*</u>', $\{ \hat{q}_1, \hat{q}_2, \hat{q}_3, \ldots\}$. And it would not be '*mono-ontic*' and '*mono-monadic*' at best [as is **N**, via its '***archeic monad***', **1**, if we assume an ***implicit*** single metrical or ontological "*dimension*" or '*onto*' that each **n** in **N**, *in any given application*, is always counting, or, a *generic* onto, ***«à la»*** Plato's and Diophantus' ***«arithmos monadikos»***, with its '***archeic monad***' denoted by Diophantus' '$\overset{\circ}{\mathrm{M}}$']. It would depict, instead, on the contrary, a '<u>multi</u>-***«arithmos»***', or '<u>meta</u>-***«arithmos»***'. It describes something both '***multi-***[*meta*]***-ontic***', and '***multi-***[*meta*-]***monadic***'. It would depict a '***multi-meta-ontic***', '***multi-meta-monadic***', '***meta-fractal***', 'meta-<u>***finite***</u>' '***cumulum***'.

The 'Vestigial' «*Aufheben*»-'Dialecticality' Of [Even] The "Natural" Counting Process. Notice, in the above, how even the 'Peanic' '''perfection''' of the '''purely-quantitative''' abstraction does not escape being itself a vestigial model of the dialectical process -- does not posit a true exception to the universality of dialectic. The 'Peanically' abstracted, genericized, idealized ***process of counting*** itself -- the process of '***Peano-Succession***' of the **N** abstraction, which is so oft presumed to be the extreme opposite of, the opposing pole to, all dialectical process, is in fact none other than a '***shadow-form***', a '***spectral-form***' of the generic, 'qualitative-Peanic', ***dialectical succession***; of the 'purely-qualitative', 'purely-ontological' '***Dialectical Progression***' of the $_{\mathbf{N}}\underline{\mathbf{Q}}$ abstraction/idealization of dialectical process. The ***counting process*** is what is left of the ***dialectical process*** when abstraction leaves behind only those '''*ghosts of departed quality*''', those '''*specters of departed ontology*''', which is what the standard "**N**atural" numbers, -- **1, 2, 3, ...** -- really ***are***! '*Departed*' here means precisely '*abstracted*', or '*abstraction-extracted*'.

Thus, given the classical Peano '''*succession operation*''', '''*succession operator*''', or '''*succession-function*''', **s**, defined such that, for all of the "**N**atural" numbers, denoted generically by **n**, that are contained in **N**, which denotes the '''set''' or '''space''' **{1, 2, 3, ...}**, or in other symbols, given a successor-function $\mathbf{s} \mid [\forall \mathbf{n} \in \mathbf{N}][\mathbf{s}(\mathbf{n}) = \mathbf{n}+1]$, we have the Peano ***model of the counting process***, as follows, following from the assertion [denoted $\vdash$] of the existence [denoted $\exists$] of the abstract, '***pure-quantifier***' unit, or 'monad', **1**, plus the definition of the Peano **s** function, with the stipulation that the '*increment*' or '*finite difference*' which each application of **s** to any "**N**atural" Number, **n**, adds to that **n**, is constant, is always the same: $[\forall \mathbf{n} \in \mathbf{N}][\ \mathbf{s}(\mathbf{n}) = \mathbf{n} + \Delta\mathbf{n} = \mathbf{n} + \Delta \mid \Delta\mathbf{n} = 1 = \Delta\ ]$:

$\vdash \exists 1;$

$s(1) = 1 + \Delta 1 = 1 + \Delta = 1 + 1 = 2 \neq 1;$

$s(2) = 2 + \Delta 2 = 2 + \Delta = 2 + 1 = 1 + 1 + 1 = 3 \neq 2, 1;$

$s(3) = 3 + \Delta 3 = 3 + \Delta = 3 + 1 = 1 + 1 + 1 + 1 = 4 \neq 3, 2, 1;$

. .   .   .

. .   .   .

. .   .   . , ...,

wherein '$\neq$', of course, here denotes '$\gtrless$', and, in particular, '>', ***not*** '$\lesseqgtr$'.

So, we have the '''*pure-quantitative*''' ***counting process***, idealized as --

$$1 \ni\!\!\rightarrow s(1) = 1+\Delta = 1+1 = 2 \ni\!\!\rightarrow s(2) = 2+\Delta = 2+1 = 3 \ni\!\!\rightarrow s(3) = 3+\Delta = 3+1 = 4 \ni\!\!\rightarrow s(4) = 4+\Delta = 4+1 = 5 \ni\!\!\rightarrow \ldots$$

-- as also a '***spectral-form***' of the ***dialectical*** [or] '***«aufheben» process***', such that, as we saw above, each **n** is '***conserved***' in **s(n)**, while it is also '***changed***', '***made other***' than/to its [former] '***self***', or '***negated***' by being '***elevated***' -- '***elevated***' in quantitative magnitude -- *by exactly one unit in each application of the* **s** *operation.* Thus, as we saw above, $\forall \mathbf{n} \in \mathbf{N}$, *both* $\mathbf{n} \sqsubset \mathbf{s}(\mathbf{n})$ *and* $1 \sqsubset \mathbf{s}(\mathbf{n})$. That is, making explicit the repeated application of the **s** function, we have '***the counting paradigm***' modeled as:

$$1 \ni\!\!\rightarrow s(1) = 2 \ni\!\!\rightarrow ss(1) = s(2) = 3 \ni\!\!\rightarrow sss(1) = ss(2) = s(3) = 4 \ni\!\!\rightarrow ssss(1) = sss(2) = ss(3) = s(4) = 5 \ni\!\!\rightarrow \ldots,$$

or, using the standard '''superscript'''/'''exponent'''/'''power''' ***operation-repetition/iteration-notation***, modeled by $\mathbf{s}^{\tau}(1)$ as $\tau \in \mathbb{T}$, or:

$$s^0(1) = 1 \ni\!\!\rightarrow s^1(1) = s(1) = 2 \ni\!\!\rightarrow s^2(1) = s^1(2) = 3 \ni\!\!\rightarrow s^3(1) = s^2(2) = s^1(3) = 4 \ni\!\!\rightarrow s^4(1) = s^3(2) = s^2(3) = s^1(4) = 5 \ldots.$$

This '***pure quantifier***', '***vestigial-dialectic of counting***' ***parallels*** and ***mirrors***, in 'spectral' form, the '***pure-qualifier***', '***pure-ontological***', or '***pure-categorial***' dialectic modeled by the '''*un*-interpreted''' -- actually, '***minimally***-interpreted' -- $_{\mathbf{N}}\underline{\mathbf{Q}}$ arithmetic. The $\underline{\Sigma}\hat{q}_n$ succession of '***meta-«arithmoi»***', denoted by $\llbracket\, \hat{q}_1 \,\rrbracket^{\tau}$ as $\tau \in \mathbb{T}$, in that generic, abstracted, idealized and 'universalized', $_{\mathbf{N}}\underline{\mathbf{Q}}$ version of the dialectic, also features a '***succession-operator***' which is none other than $\hat{q}_1$ itself, the '***«arché»***' or '***archeic monad***' of its '***archeonic consecuum***'. Moreover, the increment that transforms that predecessor '***meta-«arithmos»***', $\llbracket\, \hat{q}_1 \,\rrbracket^{\tau} = \llbracket\, \hat{q}_1 \boxplus\ldots\boxplus \hat{q}_{\tau} \,\rrbracket$, into its incremented or successor '***meta-«arithmos»***', $\llbracket\, \hat{q}_1 \,\rrbracket^{\tau+1} = \llbracket\, \hat{q}_1 \boxplus\ldots\boxplus \hat{q}_{\tau+1} \,\rrbracket$, is supplied, each time, precisely via 'multiplication' of the predecessor '***meta-«arithmos»***' by its 'archeic monad' -- that is, via $\hat{q}_1$ '''***times***''' $\llbracket\, \hat{q}_1 \,\rrbracket^{\tau}$, i.e., $\hat{q}_1^{\,+1} \boxtimes \llbracket\, \hat{q}_1 \,\rrbracket^{\tau}$ yields $\llbracket\, \hat{q}_1 \,\rrbracket^{\tau+1}$:

$$\underline{\sigma}\llbracket\, \hat{q}_1 \,\rrbracket^{\tau} = \llbracket\, \hat{q}_1 \,\rrbracket^{s(\tau)} = \llbracket\, \hat{q}_1 \,\rrbracket^{\tau+1} = \hat{q}_1 \boxtimes \llbracket\, \hat{q}_1 \boxplus\ldots\boxplus \hat{q}_{\tau} \,\rrbracket = \llbracket\llbracket\, \hat{q}_1 \boxplus\ldots\boxplus \hat{q}_{\tau} \,\rrbracket \boxplus \hat{q}_1 \boxtimes \llbracket\, \hat{q}_{\tau} \,\rrbracket\rrbracket = \llbracket\llbracket\, \hat{q}_1 \boxplus\ldots\boxplus \hat{q}_{\tau} \,\rrbracket \boxplus \llbracket\, \hat{q}_{\tau} \boxplus \hat{q}_{\tau+1} \,\rrbracket\rrbracket =$$

$$\llbracket\, \hat{q}_1 \boxplus\ldots\boxplus \hat{q}_{\tau} \boxplus \hat{q}_{\tau} \boxplus \hat{q}_{\tau+1} \,\rrbracket = \llbracket\, \hat{q}_1 \boxplus\ldots\boxplus \hat{q}_{\tau} \boxplus \hat{q}_{\tau+1} \,\rrbracket = \llbracket\, \hat{q}_1 \boxplus\ldots\boxplus \hat{q}_{\tau+1} \,\rrbracket = \llbracket\, \hat{q}_1 \boxplus\ldots\boxplus \hat{q}_{s(\tau)} \,\rrbracket = \llbracket\, \hat{q}_1 \,\rrbracket^{s(\tau)} = \llbracket\, \hat{q}_1 \,\rrbracket^{\tau+1}, \text{ i.e.,}$$

$$\llbracket\, \hat{q}_1 \,\rrbracket^{1} = \hat{q}_1;\ \ \underline{\sigma}\llbracket\, \hat{q}_1 \,\rrbracket^{1} = \hat{q}_1\llbracket\, \hat{q}_1 \,\rrbracket^{1} = \llbracket\, \hat{q}_1 \,\rrbracket^{1+1} = \llbracket\, \hat{q}_1 \,\rrbracket^{2} = \llbracket\, \hat{q}_1 \boxplus \hat{q}_{1+1} \,\rrbracket = \llbracket\, \hat{q}_1 \boxplus \hat{q}_2 \,\rrbracket;$$

$$\underline{\sigma}\llbracket\, \hat{q}_1 \,\rrbracket^{2} = \hat{q}_1\llbracket\, \hat{q}_1 \,\rrbracket^{2} = \llbracket\, \hat{q}_1 \,\rrbracket^{2+1} = \llbracket\, \hat{q}_1 \,\rrbracket^{3} = \hat{q}_1\llbracket\, \hat{q}_1 \boxplus \hat{q}_2 \,\rrbracket = \llbracket\llbracket\, \hat{q}_1 \boxplus \hat{q}_2 \,\rrbracket \boxplus \hat{q}_1\llbracket\, \hat{q}_2 \,\rrbracket\rrbracket = \llbracket\llbracket\, \hat{q}_1 \boxplus \hat{q}_2 \,\rrbracket \boxplus \llbracket\, \hat{q}_2 \boxplus \hat{q}_{2+1} \,\rrbracket\rrbracket =$$

$$\llbracket\, \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \,\rrbracket = \llbracket\, \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \,\rrbracket;\ \ \underline{\sigma}\llbracket\, \hat{q}_1 \,\rrbracket^{3} = \hat{q}_1\llbracket\, \hat{q}_1 \,\rrbracket^{3} = \llbracket\, \hat{q}_1 \,\rrbracket^{3+1} = \llbracket\, \hat{q}_1 \,\rrbracket^{4} = \hat{q}_1\llbracket\, \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \,\rrbracket =$$

$$\llbracket\llbracket\, \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \,\rrbracket \boxplus \hat{q}_1\llbracket\, \hat{q}_3 \,\rrbracket\rrbracket = \llbracket\llbracket\, \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \,\rrbracket \boxplus \llbracket\, \hat{q}_3 \boxplus \hat{q}_{3+1} \,\rrbracket\rrbracket = \llbracket\, \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \boxplus \hat{q}_3 \boxplus \hat{q}_4 \,\rrbracket = \llbracket\, \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \boxplus \hat{q}_4 \,\rrbracket; \ldots.$$

The ${}_{\underline{N}}\underline{\mathbf{Q}}$ *'Arithmetic of Dialectics'* as a 'contra-Boolean' **Arithmetic, with a** 'contra-Boolean' Algebra. We also call the reader's attention to something perhaps not as readily discernible as the conceptual co-possibility of '*contra-Standard*' or 'Non-Standard" "Natural" arithmetics, like ${}_{\underline{N}}\underline{\mathbf{Q}}$ within $\underline{\mathbf{N}}$, together with "Standard" "Natural" arithmetics like $\underline{\mathbf{N}}$, also within $\underline{\mathbf{N}}$: namely, the *conceptual co-possibility* of '*contra-Boolean*' together with '*Boolean*' arithmetics and algebras of '*logic*'. That co-possibility resides in the '*logical controvertibility*' of what Boole termed "*the fundamental law of thought*", or "*law of duality*", which he expressed via the equation $\mathbf{x}^2 = \mathbf{x}$, whereas the equation characteristic of ${}_{\underline{N}}\underline{\mathbf{Q}}$ is its '*strong* negation', not just $\mathbf{x}^2 \neq \mathbf{x}$ in the sense of $\mathbf{x}^2 \gtrless \mathbf{x}$, but in the deeper sense of inequality expressed by $\underline{\mathbf{x}}^2 \not\gtrless \underline{\mathbf{x}}$:

"**Proposition IV**. *The axiom of metaphysicians which is termed* ***the principle of contradiction****, and which affirms that it is impossible for any being to possess a* ***quality****, and* ***at the same time*** *not to possess it, is a consequence of* ***the fundamental law of thought****, whose expression is* $x^2 = x$. Let us write this equation in the form

$$x - x^2 = 0,$$

whence we have

$$x(1 - x) = 0; \tag{1}$$

both these transformations being justified by the axiomatic laws of combination and transposition (**II**.**13**). Let us, for simplicity of conception, give to the symbol **x** the particular ***interpretation*** of *men*, then $1 - x$ will represent the class of "not-men" (**Prop**. **III**.) Now the formal product of the expressions of two classes represents that class of individuals which is common to both of them (**II**. **6**). Hence $x(1 - x)$ will represent the class whose members are ***at once*** "men," and "not men", and the equation (1) thus express the principle, *that a class whose members are* ***at the same time*** *men and not men does not exist* [i.e., is equal to the class **"Nothing"**, denoted '**0**' -- **F**.*E*.*D*.]. In other words, that *it is* ***impossible*** *for the same individual to be* ***at the same time*** *a man and not a man*. Now let the meaning of the symbol **x** be extended from the representing of "men," to that of any class of beings characterized by the possession of any ***quality*** whatever; and equation (1) will then express that it is impossible for a being to possess a ***quality*** and not to possess that ***quality at the same time***. But this is identically that "***principle of contradiction***" which Aristotle has described as the fundamental axiom of all philosophy. "It is impossible that the same ***quality*** should belong and not belong to the same thing. . . . This is the most certain of all principles. . . Wherefore they who demonstrate refer to this as an ultimate opinion. For it is by nature the source of all other axioms." ... The above ***interpretation*** has been introduced not on account of its immediate value in the present system, but as an illustration of a significant fact in ***the philosophy of the intellectual powers***, viz., that ***what has commonly been regarded as the fundamental axiom of metaphysics is but the consequence of a law of thought, mathematical in its form***. I desire to direct attention also to the circumstance that ***the equation*** (1) ***in which that fundamental law of thought is expressed is an equation of the second degree*** [i.e., is an ***algebraically nonlinear*** equation if **X** is taken as denoting a "variable" or "unknown" to be solved-for – **F**.*E*.*D*.].*

Without speculating at all in this chapter upon the question, whether that circumstance is necessary in its own nature, we may venture to assert that if it had not existed, the whole procedure of the understanding would have been different from what it is. Thus it is a consequence of the fact that ***the fundamental law of thought is of the second degree***, that we perform the ***operation*** of ***analysis*** and ***classification***, by ***division into pairs of opposites***, or, as is technically said, by ***dichotomy***. Now if the equation in question had been of the ***third degree***, still admitting of interpretation as such, the ***mental division*** must have been ***threefold in character***, and we must have proceeded by a species of ***trichotomy***, the real nature of which it is impossible for us, with our existing faculties, adequately to conceive, but the laws of which we might well investigate as an object of ***intellectual speculation***. ... The ***law of thought*** expressed by equation (1) will, for reasons which are made apparent by the above discussion, be occasionally referred to as the "***law of duality***". *Should it here be said that the existence of the equation $x^2 = x$ necessitates also the existence of the equation $x^3 = x$, which is of the ***third degree***, and then inquired whether that equation does not indicate a process of ***trichotomy***; the answer is, that the equation $x^3 = x$ is ***not interpretable*** in the system of logic. For writing it in either of the forms:

$$x(1 - x)(1 + x) = 0, \tag{2}$$

$$x(1 - x)(-1 - x) = 0, \tag{3}$$

we see that its ***interpretation***, if possible at all, must involve that of the factor $1 + x$, or of the factor $-1 - x$. ***The former is not interpretable, because we cannot conceive of the addition of any class*** x ***to the universe*** 1 ***[whereas, per*** $\{\ \underline{\mathbf{Q}}_t\ \}$ ***models, the universe of discourse is continually adding new qualities/'ontos'/classes, e.g.,*** $\{\ \underline{\Delta \mathbf{q}}_x\ \}$***, to itself, and is thus constantly self-expanding, qualitatively or ontologically, as the result of the interaction and self-interaction of the previously-posited 'ontos',*** $\{\ \underline{\mathbf{q}}_x\ \}$ -- **F**.*E*.*D*.]; the latter is ***not interpretable***, because the symbol -1 is not subject to the law $x(1 - x) = 0$, to which all class symbols are subject. Hence the equation $x^3 = x$ admits of ***no interpretation*** analogous to that of the equation $x^2 = x$. Were the former equation, however, true independently of the latter, i.e., were that ***act of mind*** *which is denoted by the* ***symbol*** **x**, such that its ***second repetition*** should ***reproduce*** the result of a ***single operation***, but not its ***first or mere repetition***, it is presumable that we should be able to ***interpret*** one of the forms (2), (3), which under the actual conditions of thought we cannot do. There exist operations, known to the mathematician, the law of which may be adequately expressed by the equation $x^3 = x$ [e.g., the Musean hypernumber $\boldsymbol{\varepsilon}$, the unity of the "'*counter-complex numbers*'", and a "'*proper square-root of positive* ***R****eal unity*'", *mimed by the matrix operator* $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ -- **F**.*E*.*D*.]. But they are of a nature altogether foreign to the province of general reasoning." [G. Boole, ***An Investigation of the Laws Of Thought on Which are Founded the Mathematical Theories of Logic and Probabilities***, [New York: Dover, 1958], pp. ***49-51***; ***50n***.; originally published in 1854; ***bold*** *italics* **emphasis** ***added*** by **F**.*E*.*D*.].

Boole, in one of his manuscripts on the philosophy of logic recently published for the first time, reiterates his 'quadripartite' partitioning of logical definition-equations in the example of his "development" of the term $\mathbf{y}$ = "*rational beings*", via an "'abstraction'" of "animals" from "'human beings'" defined as "rational animals", modeled 'logical-arithmetically' by means of the 'Boolean division' **rational_beings*animal_beings/animal_beings**, and using the four possible '*Boolean fractions*' of the form **a/b** for Boolean **a** and **b**: "Suppose that from the proposition "Men = rational animals" it were required to find explicitly a definition of "rational beings" in terms of "men" and "animals". If we represent the concept "men" by **x** and "rational beings" by **y** and "animals" by **z** we have the equation,

$$x = yz \tag{1}$$

Hence

$$y = x/z \tag{2}$$

and developing the second member

$$x = (1)zx + (0)z(1 - x) + (1/0)(1 - z)x + (0/0)(1 - z)(1 - x) \tag{3}$$

The *interpretation* of which is the following:

1st Rational beings consist of ***all*** [$1 = 1/1$ -- F.*E*.*D*.] animals that are men, ***no*** [$0 = 0/1$ -- F.*E*.*D*.] animals that are not men [porpoise, for example, if they too are "rational animals", are either already excluded by the premise/definition that $x = yz$, if x is defined so as to exclude them, or already included if x is defined so as to already include them as "rational animals", i.e., as "human(s)" -- F.*E*.*D*.], and ***an indefinite remainder*** (***some***, none, ***or*** all) [$0/0$, i.e., an *'ambiguous'*, "'undefined'", or "'*indeterminate*'" portion/magnitude -- F.*E*.*D*.] ***of beings that are neither animals nor men***. [What might such beings possibly be? -- angels?; sentient robots?; extra-terrestrial ***'planimals'***? -- F.*E*.*D*.]

2ndly Men that are not animals ***do not exist*** [$1/0$; division of "'the Universe'", 1, by "'Nothing'", zero, 0 -- "Infinity" or ***'singularity'***, signifying "'***impossibility***'" -- F.*E*.*D*.]."

[G. Boole, ***Selected Manuscripts on Logic and Its Philosophy***, I. Grattan-Guinness, G. Bornet, eds., Birkhauser [Boston, MA: 1997], p. 98, ***bold italic emphasis*** added by F.*E*.*D*.].

Aside: Let us engage an "*intellectual speculation*" of our own involving Boole's contentions above. Suppose, "in our future", a new class, a new *'onto'*, of ***android robots*** emerges, part of which we denote by r, which we posit to be a *part* of the 'not-human' class, denoted $(1 - h)$: $r \sqsubset (1 - h)$. Suppose further that, with the emergence of r, there also emerges a praxis of *'cyborg prosthetics'* or of "'*cyborg bionics*'", whereby some, e.g., aging or ailing, human individuals, given the inelasticity of the demand for the avoidance of [premature] incapacity or even death, cause themselves to be outfitted with "artificial" devices to replace some of their damaged/worn biological body parts, *devices also shared by some of the android robots* [e.g., robotic legs], specifically and only among the robots denoted, collectively, by r. Such a praxis of "'cyborg bionics'" would thereby create a further new, 'hybrid' class, with both "human", biological, and "robotic", "artificial", qualities, a new class which we here denote by q_{rh}. Should, then, the class of 'humans', h, be conceived of as having [somehow] already always, in advance, included this ***new***, hybrid group, q_{rh}, a later outgrowth of both h and of r; *as a part of* h: $q_{rh} \sqsubset h$? Or, should the q_{rh} class be accounted, «*au contraire*», as part of the android robot class, i.e., $q_{rh} \sqsubset r \sqsubset (1 - h)$? Or, should q_{rh} be accounted as a part of ***both*** h and $r \sqsubset (1 - h)$, *thus as part of a therefore newly non-empty* ***intersection/overlap*** of h and $(1 - h)$: $h(1 - h) \sqsupset hr = q_{rh}$?

Should q_{rh} *instead* be accounted as part of ***neither*** h ***nor*** r ***nor*** the ***old*** $(1 - h)$ ***nor*** the ***old*** $(1 - r)$? If so, would q_{rh} constitute a ***new, unprecedented***, *never-before-extant*, "***third***" category of being or of ontology, a category of "cyborgs", c; a «***tertium quid***» vis-à-vis ***both*** h & r and the ***old*** $(1 - h)$ and the ***old*** $(1 - r)$? Part of the ***intersection*** of the ***new, expanded*** $(1 - h)$ & $(1 - r)$: $(1 - h)(1 - r) \sqsupset q_{rh} = c$? Or, should ***both*** r & q_{rh}, if both are later outgrowths of h, be accounted as [***later-in-time***] but newer parts of a "time-varying", *'dynamical class'* or *'dynamical category'* h itself; as parts of the ***'anticipatable'***, ***'expectable' self-extension, self-elaboration***, and ***self-development*** within the h ontological category of this thus ***changing and self-changing*** ontology of the human species?

Boole's Ideography as 'Symbolical' Simulation of Mental Operations?: Ideographic-Algorithmic Models of Mental Actions. Boole is more explicit as to the significance of the "*law of thought*" expressed in $x^2 = x$ in his earlier book on the subject -- on his *'operatorial* ***ideography of*** *[formal] logic'* -- *in terms of* the ***'mental actions'*** or "*operations of mind*" which it is designed to model, or *simulate symbolically*: "Let us employ the symbol 1, or unity, to represent the Universe, and let us understand it as comprehending every ***conceivable class*** of ***objects whether actually existing or not***, it being premised that ***the same individual may be found in more than one class***, inasmuch ***as it may possess more than one quality in common with other individuals***. ... The symbol x ***operating upon*** any ***subject*** comprehending ***individuals or classes***, shall be supposed to ***select from*** that ***subject*** all the Xs which it contains. ... When no ***subject*** is expressed, we shall suppose 1 (the Universe) to be the ***subject*** understood, so that we shall have $x = x(1)$, the meaning of either term being the ***selection from the Universe*** of all of the Xs which it contains, and the ***result*** of the ***operation*** being in common language, the ***class*** X, *i.e.* the ***class*** of which each member is an X. From these premises it will follow, that the ***product*** xy [i.e., $x(y(1))$ -- F.*E*.*D*.] will represent, in ***succession***, the ***selection*** of the ***class*** Y [from the Universe, 1: $y(1)$ -- F.*E*.*D*.], and the ***selection*** from the ***class*** Y of such ***individuals*** of the ***class*** X as are contained in it, the result being the ***class*** whose members are both Xs and Ys. ... From the ***nature*** of the ***operation*** which the symbols x, y, z, are conceived to represent, we shall designate them as ***elective symbols***. An expression in which they are involved will be called an ***elective function***, and an equation of which the members are ***elective functions***, will be termed an ***elective equation*** [***whence, we denote Boolean Arithmetic by*** $\mathbb{E}$, ***connoting the Arithmetical Rules-System for the 'space' of 'Elector Operators', denoted*** $\mathbb{E}$ -- F.*E*.*D*.]. It will not be necessary that we should here enter into the analysis of that ***mental operation*** which we have represented by the ***elective symbol***. It is ***not*** an ***act of Abstraction*** according to the common acceptation of that term, because ***we never lose sight of the concrete***, but it may probably be referred to an exercise of the faculties of ***Comparison*** and ***Attention***. Our present concern is rather the ***laws of combination and of succession***, by which its ***results*** are ***governed***, and of these it will suffice to ***notice*** the following. ... **3rd**. The result of a given ***act*** of ***election*** performed twice, or any number of times ***in succession***, is the result of the same ***act*** performed once. If from a ***group*** of objects we ***select*** the Xs, we obtain a ***class*** of which all the members are Xs. ***If we repeat the operation on this class no further change will ensue***: in ***selecting*** the Xs we take the ***whole***. Thus we have $xx = x$, or $x^2 = x$; and supposing the same ***operation*** to be n times performed, we have $x^n = x$, which is the mathematical expression of the law above stated [Note that the embrace here of the latter expression of this "law", the form $x^n = x$, which encompasses n such that $n > 2$, contradicts what Boole will say later, in ***The Laws Of Thought***, as quoted above -- F.*E*.*D*.].* *The office of the ***elective symbol*** x, is to ***select*** individuals comprehended in the ***class*** X. Let the ***class*** X be supposed to embrace the Universe; then, whatever the ***class*** Y may be, we have $xy = y$ [i.e., $x = 1$, so $xy = x(y) = 1(y) = y$ -- F.*E*.*D*.]. The office which x performs is now equivalent to the symbol +, in one at least of its ***interpretations***, and ***the index law*** (3) gives $+^n = +$, which is the known property of that symbol. ... The third law (3) we shall denominate the ***index law***. It is peculiar to ***elective symbols***, and will be found of great importance in enabling us to ***reduce*** our results to forms meet for ***interpretation***." [G. Boole, ***The Mathematical Analysis Of Logic***, Thoemmes Press [Sterling, VA: 1998], pp. 15-18, 17n.; originally published in 1847; ***bold italics and underscore emphasis added*** by F.*E*.*D*.].

Boole's logic thus tacitly assumes an absolutely completed and *forevermore fixed universe* of ideas, or of human knowledge, with all concepts and categories "*cut-and-dried*", and in *final intellectual equilibrium*. Consider, for example, the following series of Boolean equations ['depicting' or expressing *symbolically* [i.e., *ideographically*] *a series of mental actions*]: $\mathbf{x(x(1)) = x(x) = xx = x^2 = x^1 = x}$. This series asserts that the "*[s]election*", ***by*** the '*[s]elector*' *operator* **x**, ***of*** *all individuals '''belonging''' to* the concept **X**, ***from*** out of the universal concept, which is also the '*identity [s]elector*' or 'identity operator', denoted **1**, ***followed by*** the "*[s]election*" of the '''belongings''' of the concept **X**, again, but this time ***by*** the '*[s]elector*' **x** itself, and this time also ***from*** out of the class corresponding to its own concept, **X**, denoted again by the '*[s]elector*' operator **x**, yields nothing but the class corresponding to the concept **X** itself, again, denoted at last by the '*[s]elector*' or *mental-operation-symbol* **x** *standing **alone***. The latter *implicitly* also again denotes **x(1)** or **x·1** or **x1**; the "*[s]election*", *by* **x**, of all *logical individuals '''belonging''' to* the concept **X** *from out of* the '*universe operator*', which also denotes the class of all individuals '''belonging''' to the *universe concept*, **1**.

According to the formulation $\mathbf{x(x(1)) = x(x) = x}$, and its Boolean interpretation, the human mind, as ***subject***, i.e., as controlling and initiating ***agent*** of mental ***action***, '*holding in thought*' or '*mentally embodying*' inwardly and '***im-person-ating***' ***presently***, and '***semantically***', a given ***action of conception***, symbolized ideographically by '**x(_)**', ***confronts***, as ***object***, the results or product of its own '***pastly***' thought-activity regarding the "same" content/topic, e.g., as remembered inwardly and/or as ***presently*** recorded outwardly, e.g., in written form -- '***syntactically***' or in '***syntactical representation***' of its inward semantics -- this 'objective' form being symbolized ideographically by '**_(x)**'. This confrontation of the '*subjective-semantical*' and '*objective-syntactical*' forms of **x** thus posits a '*time-offset **self**-confrontation*' of the '***im-person-ated***' concept in question, which we here therefore symbolize ideographically by **x(x)**. The Boolean "***fundamental law of thought***" holds that no change, no cognitive gain, no conceptual improvement, no intellectual progress arises thereby, from this ***self-confrontation*** of an idea, because the concept or conceptual activity denoted by **x** is always already totally completed, absolutely finished, mentally reproducible ***without error***, and ***without improvement*** or correction of any past error, since no such past error or inadequacy of conception is assumed to [have ever existed, or to any longer] exist. All improvement of ideas, were it ever to have been needed, must have always already occurred in the past, and is always already *all over with* by the time we get to the operations of mind codified, symbolized, and simulated in Boole's ***logical ideography***. Boolean logic is thus a logic of '***the simple reproduction of ideas***'. What if we step back from this idealization of conceptual perfection in our *internal* world, reflecting implicitly also a presupposed eternal stasis in the ontology of the "states of affairs" in our *external* world? What if we seek to describe more adequately, more accurately, more concretely, and more richly the actual processes of ideation within the actually-observed 'dynamicity' and 'meta-dynamicity' of our internal and external worlds? Do we not find that '***reflection***' -- indeed, such '***self**-re-flex-ion*' and '***self**-re-flux-ion*' or '***self**-critique*' *of ideas* -- with a '***contra***-Boolean' $\underline{\mathbf{x}}$ as *subject* or ***agent*** of *present* thought applied to itself, i.e. to $\underline{\mathbf{x}}$ again as *object / material* of present thought / *result* of *past* thought, oft may yield an increment of cognitive gain, of cumulative theoretical progress, and of *universe-of-discourse*, '*ideo-taxonomical*', "***categorial***", '***ideo-ontological***' *expansion*, here denoted $\underline{\Box \mathbf{x}}$ [using the generic ${}_{\underline{\mathbf{N}}}\underline{\mathbf{\Omega}}$ notation]? Per the '***meta-genealogical evolute product rule***' for ${}_{\underline{\mathbf{N}}}\underline{\mathbf{\Omega}}$, we have --

$$\underline{x}\llbracket\, \underline{x}\,\rrbracket = \underline{x} \boxtimes \underline{x} = \underline{xx} = \underline{x}^2 = \llbracket \underline{x} \boxplus \underline{x} \boxplus \underline{\Box}\llbracket\, \underline{x};\, \underline{x}\,\rrbracket\,\rrbracket = \llbracket \underline{x} \boxplus \underline{x} \boxplus \underline{\Box}\underline{x}\,\rrbracket = \llbracket\, \underline{x} \boxplus \underline{\Box}\underline{x}\,\rrbracket \gtrless\ \underline{x}^1 = \underline{x} \qquad \leftrightarrow$$

$$\hat{q}_n\llbracket\hat{q}_n\rrbracket = \hat{q}_n\hat{q}_n = \hat{q}_n^{\ 2} = \hat{q}_n \boxplus \hat{q}_n \boxplus \underline{\Box}\llbracket\, \hat{q}_n;\, \hat{q}_n\,\rrbracket = \hat{q}_n \boxplus \hat{q}_n \boxplus \underline{\Box}\hat{q}_n = \hat{q}_n \boxplus \underline{\Delta}\hat{q}_n = \hat{q}_n \boxplus \hat{q}_{n+n} = \hat{q}_n \boxplus \hat{q}_{2n} \gtrless \hat{q}_n\ [n \in N],$$

with $\hat{q}_{2n} \perp \hat{q}_n$, which is just the special case, for $n = m$ rather than $n \gtrless m$, of $\hat{q}_{n+m} \perp \hat{q}_n, \hat{q}_m$, in [given $\underline{x} \gtrless \underline{y}$] --

$$\underline{y}\llbracket\, \underline{x}\,\rrbracket = \underline{yx} = \llbracket\, \underline{y} \boxplus \underline{x} \boxplus \underline{\Box}\llbracket\, \underline{y};\, \underline{x}\,\rrbracket\,\rrbracket \gtrless, \perp\ \underline{x}^1 = \underline{x},\ \underline{y}^1 = \underline{y} \qquad \leftrightarrow$$

$$\hat{q}_n\llbracket\hat{q}_m\rrbracket = \hat{q}_n\hat{q}_m = \hat{q}_n \boxplus \hat{q}_m \boxplus \underline{\Box}\llbracket\, \hat{q}_n;\, \hat{q}_m\,\rrbracket = \hat{q}_n \boxplus \hat{q}_m \boxplus \hat{q}_{n+m} \gtrless\ \hat{q}_n;\, \hat{q}_m\ [n, m \in \mathbf{N}].$$

In the foregoing formulae, $[\![\ \underline{\mathbf{x}} \equiv \underline{\square\mathbf{x}}\ ]\!] \ \substack{>\\=\\<}\!\!\!\!/\ \underline{\mathbf{x}}$ because the incremental ideation $\underline{\square\mathbf{x}}$, this *qualitative ideo*-increment arising from *immanent critique*, or *self-critique*, or $\underline{\mathbf{x}}$-*critique*-of-$\underline{\mathbf{x}}$; this self-corrective to $\underline{\mathbf{x}}$, is *qualitatively*, '***ideo-ontologically***' different from $\underline{\mathbf{x}}$, not merely *quantitatively* different from $\underline{\mathbf{x}}$: $\underline{\square\mathbf{x}} \ \substack{>\\=\\<}\!\!\!\!/\ \underline{\mathbf{x}}$. [Note: the computations above use the '***meta-genealogical evolute product rule***'].

Thus the ${}_{\underline{\mathbf{N}}}\underline{\mathbf{Q}}$ arithmetic models a logic of conceptual '*intra-duality*' and of '***the expanded self-reproduction of ideas***'. *Take the matter at hand.* That is, let us consider the historical '*self-evidence*' of Boolean arithmetic and algebra; the self-historical evidence-about-self of the Boolean '***ideo-onto-dynamasis***', meaning thereby the evidence already before us about the history of Boole's own thinking regarding his arithmetic and algebra of formal logic, including his own *changes-of*-thinking; the evidence of that 'self' that is the Boolean 'mathematics of logic' it*self*; the evidence of *its* own [psycho-]history, the history of Boolean algebra it*self* within Boole's own work. Is such an '*expanded reproduction of ideas*' not instantiated in Boole's own ***fundamental*** gains in cognition, e.g., regarding $\mathbf{x}^n = \mathbf{x}$, including for $\mathbf{n} = \mathbf{3}$, ***from*** his **1847** publication, in the passage above, ***to*** his **1854** re-publication? In the latter, he *strongly distinguishes* $\mathbf{x}^2 = \mathbf{x}$ *from* $\mathbf{x}^3 = \mathbf{x}$, and stipulates ***only*** $\mathbf{x}^2 = \mathbf{x}$, and no longer $\mathbf{x}^n = \mathbf{x}$ in general, nor $\mathbf{x}^3 = \mathbf{x}$ in particular, as symbolical expressions for what he terms the ***fundamental law of thought***. That is, in the latter *he* ***excludes*** $\mathbf{x}^3 = \mathbf{x}$ *as an expression of that "law", and, indeed, excludes* $\mathbf{x}^n = \mathbf{x}$ for ***all*** $\mathbf{n}$ ***except*** for $\mathbf{n} = \mathbf{2}$ [*as well as, of course, for* $\mathbf{n} = \mathbf{1}$, *for the "****reflexive law****" of identity*] -- apparently, therefore, a big and "*fundamental*" *change* -- of ***mind***, of "***laws***", of "***thought***", and of ***thought about*** no less than ***thought itself***!

Is such not also seen in the gains that resulted from the '***re-reflexions***' upon '*Boolean arithmetic*' of his followers, e.g., regarding the replacement of his "*exclusive* OR" version of the logical '+' operation by an "*inclusive* OR" version, which thus leading to the '*self-union rule*', '*self-addition rule*' or '*idempotent addition rule*' $\mathbf{x} + \mathbf{x} = \mathbf{x}$ of contemporary "Boolean algebra", symmetrically and '"dually"' paralleling its 'AND' rule, i.e., its '*self-intersection rule*' $\mathbf{x} \times \mathbf{x} = \mathbf{x}$, mirroring Boole's 'multiplicative' "***fundamental law of thought***" or "***law of duality***" in the realm of '"logical addition"'?

Thus, through such reflection and immanent critique, we are also moved from the rules-system of ***Boolean logic***, $\underline{\mathbf{E}}$, a '*Parmenidean*' formal logic of *static knowledge* of a static world; of *knowledge stasis* and of 'physis-stasis' or physical stasis; of '***ideo-onto-stasis***' or '***endo-onto-stasis***' and of [***exo-***]*onto-stasis*, based upon the postulate $\mathbf{x}^2 = \mathbf{x}$, to ${}_{\underline{\mathbf{W}}}\underline{\mathbf{Q}}$, an alternative rules-system of '***non-Boolean logic***', a '***contra-Boolean***', '***Heraclitean/progressive***' ontology-logic, 'ontological logic', or 'onto-***dynamical*** logic'; a logic of '***meta-dynamical***' and '***self-dynamizing***' ***knowledge*** in a '*Heraclitean/progressive*' world, based upon a *stronger* contrary of $\mathbf{x}^2 = \mathbf{x}$. Its '***fundamental rule***' is not a ***quantitative*** *contrary* -- ***not*** $\mathbf{x}^2 \neq \mathbf{x}$ in the sense of $\mathbf{x}^2 > \mathbf{x}$, or of $\mathbf{x}^2 < \mathbf{x}$ -- but a ***non-quantitative contrary*** -- $\mathbf{x}^2 \neq \mathbf{x}$ in the sense of $\underline{\mathbf{x}}^2 \ \substack{>\\=\\<}\!\!\!\!/\ \underline{\mathbf{x}}$. We thus enact the '***contra-Boolean***' ['ideo-onto-']***logical progression***, as an *immanent critique* of '*Boolean arithmetic*', as sketched above:

$$\underline{\mathbf{E}} \twoheadrightarrow \neg\!\!\!\lrcorner(\underline{\mathbf{E}}) = '\underline{\mathbf{E}}\ \mathit{of}\ \underline{\mathbf{E}}' = \underline{\mathbf{E}}(\underline{\mathbf{E}}) = \underline{\mathbf{E}} \circledast \underline{\mathbf{E}} = \underline{\mathbf{EE}} = \underline{\mathbf{E}}^2 = \underline{\mathbf{E}} \oplus \underline{\Delta\mathbf{E}} = \underline{\mathbf{E}} \oplus {}_{\underline{\mathbf{W}}}\underline{\mathbf{Q}} \ \substack{>\\=\\<}\!\!\!\!/\ \underline{\mathbf{E}},\ \ \mathit{because}\ \ {}_{\underline{\mathbf{W}}}\underline{\mathbf{Q}} \ \substack{>\\=\\<}\!\!\!\!/\ \underline{\mathbf{E}},$$

wherein the '$\twoheadrightarrow$' symbol is *the* '*self-movement sign*'; the '«***autokinesis***» sign', and, in the case at hand, ***the*** '*ideo-auto-kinesis sign*', which can be read off as denoting the phrase '*spontaneously* ***becomes***', or '*self-inducedly* ***becomes***'.

In a tradition dating back as far as the times of Heraclitus, Zeno, Socrates, and Plato, the ***higher*** '***other***' to deductive, formal logic -- to what Plato and Hegel called "The Understanding", «***verstand***», or «***dianoesis***» -- is called '***dialectic***'. To what extent does this '***contra-Boolean***' logic embody a '***dialectical***' logic?

Hegel's intended/attempted resumption of Plato's dialectics is centered on a special operation of negation, which Hegel termed «***aufheben***» negation: "*Das Aufheben* exhibits its true ***double meaning*** which we have observed ***in the negative***: it ***negates and*** at the same time ***preserves***." [Walter Kauffman, *Hegel: Texts and Commentaries*, Doubleday Anchor [Garden City, NY: 1966], p. 33, ***bold italics emphasis*** added]. We will here denote by '$\neg\!\!\!\lrcorner$', the ***generic*** operation of '***ontological self-negation***', as qualitatively distinct from the logical operation denoted by '$\neg$', the operation of ***propositional negation***. The operator '$\neg\!\!\!\lrcorner$' thus denotes a ***generic***, trans-Hegelian, dialectical, '***self-«aufheben»***' interpretation of the *paradigmatic conceptual sequence* within ${}_{\underline{\mathbf{W}}}\underline{\mathbf{Q}}$:

$$\{\neg[\mathbf{x}^2 = \mathbf{x}]\} \ni [\![\ \underline{\mathbf{x}}^2 \ \substack{>\\=\\<}\!\!\!\!/\ \underline{\mathbf{x}}\ ]\!] \ \Rightarrow\ [\![\ \underline{\mathbf{x}} \twoheadrightarrow \neg\!\!\!\lrcorner\underline{\mathbf{x}} = \underline{\mathbf{xx}} = '\underline{\mathbf{x}}\ \mathit{of}\ \underline{\mathbf{x}}' = \underline{\mathbf{x}}[\![\ \underline{\mathbf{x}}\ ]\!] = \underline{\mathbf{x}}^2 = \underline{\mathbf{x}} \equiv \underline{\square\mathbf{x}}\ ]\!]. \qquad (1)$$

In everyday parlance, the term '***moment***' refers to an '***element***' of time, of '***diachronicity***'. In philosophical usage, '***moment***' may refer to an '***element***' of a ***complex idea*** or ***conceptual complex***, even in a "synchronic" sense, and so we shall use it in what follows.

Within the process denoted by $\hookleftarrow\underline{\mathbf{X}} = \underline{\mathbf{X}}^2 = \underline{\mathbf{X}} \oplus \Box\underline{\mathbf{X}}$, indeed, within the RHS [Right-Hand Side] '*equitand*' of that equation -- that RHS '***non-amalgamative sum***' *of two* ***qualitatively unequal*** *terms*, $\underline{\mathbf{X}}$ and $\Box\underline{\mathbf{X}}$ -- the leftmost of those two terms, the $\underline{\mathbf{X}}$ term, represents directly the "***preservation***", "***conservation***", or '***simple reproduction***' '"moment"' of this '*self-«aufheben»*' process. The rightmost of these two terms, the 'qualitative/ontological increment' term, or $\Box\underline{\mathbf{X}}$ term, on the contrary, denotes a subtler and more indirect '"moment"' of this same '*self-«aufheben»*', '"extension"' or '***self-conservation***' of $\underline{\mathbf{X}}$, namely '***self-conservation by means of self-internalization***'. This '***self-internalization***' of the '"monads"', '"units"', or '"[onto-]logical individuals"' which form the typical population of the «***arithmoi***» denoted by $\underline{\mathbf{X}}$, creates an '"***elevation***"' effect, one which paradoxically constructs something new, something qualitatively, ontologically different than $\underline{\mathbf{X}}$, though made up out of $\underline{\mathbf{X}}$ -- out of multiplicities of the '"monads"' which constitute $\underline{\mathbf{X}}$. This '***self-internalization***' erects a new '"level"', a new '***meta-fractal***' '"scale"', a new ontological category, one which is ***not*** '"actually infinite"', but '***meta-finite***' with respect to the ['*lower*'] '***meta-fractal***' '"scale"' of $\underline{\mathbf{X}}$. Thus, the $\Box\underline{\mathbf{X}}$ '***qualitative increment***' or '***incremental ontology***' term also symbolizes the result or product of a '"***concrete or determinate qualitative or ontological self-negation***"' / *self*-'"***cancellation***"' / *self*-'"***annulment***"' / '"***self-transformation***"', or 'taxonomically', ***ontologically, qualitatively self-expanded self-reproduction*** moment of $\hookleftarrow\underline{\mathbf{X}}$ or $\underline{\mathbf{X}}[\![\underline{\mathbf{X}}]\!]$ -- of the '*self-«aufheben»*' of $\underline{\mathbf{X}}$, which both $\hookleftarrow\underline{\mathbf{X}}$ and $\underline{\mathbf{X}}[\![\underline{\mathbf{X}}]\!]$ denote. We shall see, via the examples to follow, the may particular ways in which this $\Box\underline{\mathbf{X}}$ term, which represents an opposite or '***op-posit***' to $\underline{\mathbf{X}}$, also embodies an '***elevation***', or '*meta-finite*', '*meta-fractal*', '***concrete transcendence***' '"moment"' of this '*self-«aufheben»* ***self-processing***' of $\underline{\mathbf{X}}$. The $\Box\underline{\mathbf{X}}$ term may be interpreted as denoting a '***meta-$\underline{\mathbf{X}}$***' ***made up out of multiple*** $\underline{\mathbf{X}}$s; that is, as denoting a '***meta-fractal***'-forming '*self-subsumption*' and '*self-incorporation*' of the manifold, or population, or concrete «***arithmoi***» of $\underline{\mathbf{X}}$s.

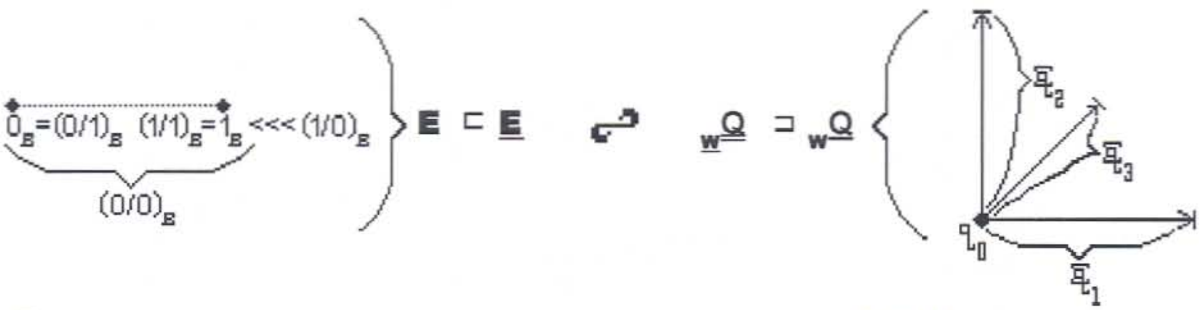
Taking the instance at hand, $\underline{\mathbf{X}} \leftrightarrow \underline{\mathbf{E}}$, we can see that we have just now worked through the following instantiation of **(1)** above --

$$\{\neg[\,\underline{\mathbf{E}}^2 = \underline{\mathbf{E}}\,]\} \ni [\,\underline{\mathbf{E}}^2 \mathbin{\dot{\wr}} \underline{\mathbf{E}}\,] \Rightarrow [\,\underline{\mathbf{E}} \rightarrow \hookleftarrow(\!(\,\underline{\mathbf{E}}\,)\!) = \underline{\mathbf{E}}(\!(\,\underline{\mathbf{E}}\,)\!) = \underline{\mathbf{E}} \otimes \underline{\mathbf{E}} = \underline{\mathbf{E}}\underline{\mathbf{E}} = \underline{\mathbf{E}}^2 = \underline{\mathbf{E}} \oplus \Delta\underline{\mathbf{E}} = \underline{\mathbf{E}} \oplus {}_{\underline{\mathbf{W}}}\underline{\mathbf{Q}} \mathbin{\dot{\wr}} \underline{\mathbf{E}}\,],$$

wherein $\underline{\mathbf{E}}^2 = \underline{\mathbf{E}} \oplus {}_{\underline{\mathbf{W}}}\underline{\mathbf{Q}} \mathbin{\dot{\wr}} \underline{\mathbf{E}}$ because $\Delta\underline{\mathbf{E}} = {}_{\underline{\mathbf{W}}}\underline{\mathbf{Q}} \mathbin{\dot{\wr}} \underline{\mathbf{E}}$.

The Space of the ${}_{\underline{\mathbf{W}}}\underline{\mathbf{Q}}$ Arithmetic as a 'meta-Boolean meta-Number-Space', made up out of a Heterogeneous Multiplicity of 'Boolean Spaces'. But in what sense and to what extent can we say that 'The ${}_{\underline{\mathbf{W}}}\underline{\mathbf{Q}}$ Arithmetic is a '***meta-fractally***' '***meta-Boolean***' Arithmetic ***made up out of multiple*** Boolean arithmetics'; that '${}_{\underline{\mathbf{W}}}\underline{\mathbf{Q}}$ is a '***meta-$\underline{\mathbf{E}}$***' ***made up out of a multitude of*** $\underline{\mathbf{E}}$s'?

Let us compare '"analytical-geometrical"' views of each the two to see:



[$\underline{\mathbf{E}}$ as a ***zero-dimensional*** or quasi-***one-dimensional*** space]

[${}_{\underline{\mathbf{W}}}\underline{\mathbf{Q}}$ as a ***potentially infinite-dimensional*** space: only the first **3** dimensions are explicit above]

We need to determine the '***contra-thesis***' of $\underline{\mathbf{E}}$, here, i.e., $\hookleftarrow\underline{\mathbf{E}} \ominus \underline{\mathbf{E}}$ or $\underline{\mathbf{E}}^2 \ominus \underline{\mathbf{E}}$, in terms of ${}_{\underline{\mathbf{W}}}\underline{\mathbf{Q}}$ rather than ${}_{\underline{\mathbf{N}}}\underline{\mathbf{Q}}$, because of the involvement of $\mathbf{0} \in \mathbf{W}$ $[\mathbf{0} \notin \mathbf{N}]$ in $\underline{\mathbf{E}}$, and the corresponding involvement of q_0 in ${}_{\underline{\mathbf{W}}}\underline{\mathbf{Q}}$, in making explicit a ***common*** '"***origin point***"' or '"***point of origin***"' for the $\hat{q}_1$ dimension, the $\hat{q}_2$ dimension, the $\hat{q}_3$ dimension, and for all of the other *potential* $\hat{q}_w$ dimensions of ${}_{\underline{\mathbf{W}}}\underline{\mathbf{Q}}$, for every $w \in \mathbf{W}$. So, we note here that $\underline{\mathbf{W}}$ is just as 'Peanic' as $\underline{\mathbf{N}}$; that $\underline{\mathbf{W}}$ is just as much a '***pure-quantitative-Peanic***, *archeonic consecuum*' as is $\underline{\mathbf{N}}$. Indeed, Peano's later version of his postulates for the '"**N**atural Numbers"' used **0**, no longer **1**, as '«***arché***»', even though, given the centuries-protracted conceptual struggle of Occidental, Mediterranean humanity with the idea of **0** ***as*** a ***number***, for 'Peanists' to term the arithmetic of $\mathbf{W} = \{0, 1, 2, 3, ...\}$ the '"**N**atural"' arithmetic, or even just a '"***Second-Nature***-al Arithmetic"', is a stretch, if the term "Natural" is to have any -- even any *implicit* -- 'Psycho-Historical' content.

The ***first-order Peano Postulates*** [*later version*] for the Standard **W**hole Numbers Arithmetic, denoted $\underline{\mathbf{W}}$, may be expressed as follows:

P1: **0** is a [**W**hole] Number, or: $\mathbf{0} \in \mathbf{W}$. Define a '**s**uccessor function': $\mathbf{s} \mid [\ \forall w \in \mathbf{W},\ \mathbf{s}(w) = w+1\ ]$.

P2: The successor of any [**W**hole] Number is a[lso a] [**W**hole] Number, or: $w \in \mathbf{W} \Rightarrow \mathbf{s}(w) \in \mathbf{W}$.

P3: No two [**W**hole] Numbers have the same successor, or: $w_1, w_2 \in \mathbf{W}\ \&\ w_1 \neq w_2 \Rightarrow \mathbf{s}(w_1) \neq \mathbf{s}(w_2)$.

P4: **0** is not the successor of any [**W**hole] number, or: $\neg\exists x \in \mathbf{W} \mid \mathbf{s}(x) = \mathbf{0}$ [i.e., $x + 1 = 0$ is an ***unsolvable equation*** within $\underline{\mathbf{W}}$].

[cf.: Reese, W., *Dictionary of Philosophy & Religion: Eastern & Western Thought*, Humanities Pr., *op. cit.*, pp. 418-419].

Boolean Logic as a '*Logic of Linearity*'. *Boolean logic* is '*linear logic*' -- a '*logic of linearity*' in which ***nonlinearity*** *reduces to linearity*. That reduction is precisely what Boole's "*law of duality*", $x^2 = x^1$, asserts: that '*self-reflexion*' is gainless; that the '***self-meditation of ideas***' yields no difference. Boole himself notes the close connexion between his arithmetic of logic and the calculus of *linear* ***differential equations***, in the following terms: "In whatever way ***an elective symbol***, considered as an ***unknown***, may be involved in a proposed equation, it is possible to assign its complete value in terms of ***the other elective symbols*** [involved in that proposed equation -- F.E.D.], considered as ***known***. It is to be observed of such equations, that from the very nature of ***elective*** symbols [i.e., because of the Boolean '"law of [exo-]duality"', $x^2 = x$, for '***election*** operations' -- F.E.D.], they are necessarily ***linear***, and that their ***solutions*** have a ***very close analogy*** with those of ***linear differential equations***, arbitrary ***elective symbols*** in the one, occupying the place of ***arbitrary constants*** in the other." [George Boole, *The Mathematical Analysis Of Logic*, *ibid.*, p. 70; ***bold** italics* ***emphasis*** added].

There are tantalizing hints that the «*insolubilia*» of formal logic/set theory [what we call '*The Standard Paradoxes*'], and the «*insolubilia*» of mathematical analysis [*e.g., those* ***nonlinear differential equations****, partial and total,* which encode our richest expressions of the known "laws" of nature today], are linked, sharing a common element, namely, that of "***nonlinearity***", i.e., of '***self-reflexivity***' or '***self-refluxivity***', that is, of '***self-functioning***', of '***self-argumenting***', or of '***self-operanding***'; the moment of the ***self-application*** of an operation: "In ***all the above contradictions*** (which are merely selections from an indefinite number) there is a ***common characteristic*** which we may describe as self-reference or ***reflexiveness***." [Bertrand Russell, Alfred North Whitehead, ***Principia Mathematica*** *to *56*, Cambridge University Press [NY: 1970], p. 61; ***bold** italics* ***emphasis*** added].

"In general, $\phi\chi$ is itself a function of two variables, ϕ and χ; of these, either may be given a constant value, and either may be varied *without reference to the other*. But in the type of propositional functions we are considering [those denoting '*standard*' *paradoxes* -- F.E.D.] ..., *the argument is itself a function of the propositional function*: Instead of $\phi\chi$, we have $\phi\{f(\phi)\}$, where $f(\phi)$ is defined as a function of ϕ. Thus *when* ϕ *is varied, the argument of which* ϕ *is asserted is varied too ... If here* ϕ *is varied, the argument is varied at the same time in a manner dependent upon the variation of* ϕ. For this reason, $\phi\{f(\phi)\}$, though it is a definite proposition when χ is assigned, is not a propositional function, in the ordinary sense, when χ is a variable [i.e., in $\phi\{f(\phi)\}$, $\chi = \{f(\phi)\}$, implying that, in this "doubtful" special case, $\phi\chi$ involves an **f-*mediated*** '*self-dependence*', '*self-operation*', or '*self-application*' of ϕ -- F.E.D.]. Propositional functions of this doubtful type may be called ***quadratic forms***, because the variable enters into them in a way analogous to that in which, in *Algebra*, a variable appears in *an expression of the* ***second degree*** [i.e., in an *algebraically nonlinear* way -- F.E.D.]." [Bertrand Russell, *The Principles of Mathematics*, W. W. Norton [NY: 1903], p. 104]. The generic expression of these paradoxes in the original Boolean algebra takes a *linear* algebraic form. However, its linearity notwithstanding, it captures *the truth-value-'limit-cycle-like'* ***self-oscillatory essence*** of all of the '***Standard Paradoxes***', starting with their ancient harbinger, the «***pseudomenon***» of Epimenides of Crete: "'***I am telling you that I am a Cretan, and also that all Cretans always lie***"', recast modernly as "**This sentence is false**".

Thus, we can converge upon ${}_{\mathbf{w}}\underline{\mathbf{\Omega}}$ from at least two distinct directions of dialectical, logical progression, one beginning with $\underline{\mathbf{W}}$, the other with $\underline{\mathbf{E}}$. Arriving from either direction at ${}_{\mathbf{w}}\underline{\mathbf{\Omega}}$ then leads us onward, continuing in the direction of that arrival, to a re-divergence; to two different '***meta-systematic dialectics***' of the «***sequelae***» of ${}_{\mathbf{w}}\underline{\mathbf{\Omega}}$, one for 'arithmetics of dialectical logics', and one for 'dialectical arithmetics' proper. The Boolean arithmetic of $\underline{\mathbf{E}}$ *self-bifurcates* into a 'non-amalgamative' 'co-knowing' of two contrary systems of logical arithmetic, $\underline{\mathbf{E}} \oplus {}_{\mathbf{w}}\underline{\mathbf{\Omega}}$, wherein ${}_{\mathbf{w}}\underline{\mathbf{\Omega}}$ '*polarly*' *opposes* and '*op-posits*' or '*sits opposite and in qualitative opposition to*' $\underline{\mathbf{E}}$. We assert an *opposition of qualities* here because ${}_{\mathbf{w}}\underline{\mathbf{\Omega}}$ forms an arithmetical 'Rules-System' of '***ideo-onto-dynamical***' *universes of discourse* and operators, *contrary to and complementing* the '***ideo-onto-statical***' *universes of discourse* and *operators* of $\underline{\mathbf{E}}$. The '"**W**hole-number"' arithmetic of $\underline{\mathbf{W}}$ likewise *self-bifurcates* into $\underline{\mathbf{W}} \oplus {}_{\mathbf{w}}\underline{\mathbf{\Omega}}$, wherein ${}_{\mathbf{w}}\underline{\mathbf{\Omega}}$ is, for a different '***direction of diametrality***' of opposition, a diametral opposite of $\underline{\mathbf{W}}$, representing the opposite extreme *within* the '*Peanic*' domain. We again assert ***opposition of qualities*** because ${}_{\mathbf{w}}\underline{\mathbf{\Omega}}$ forms an arithmetical 'Rules-System' of '***unquantified*** and ***unquantifiable qualifiers***', *contrary to and complementing* the '***unqualified quantifiers***' of $\underline{\mathbf{W}}$.

The 'Psycho-Historical' "'Revolutions'" of Arithmetic and 'The Arithmetic of Revolutions'. We can depict this multi-directional, multi-dimensional propagation of dialectical progressions as follows, with, first, the sequence:

$$\underline{E} \;\rightarrow\; \underline{E} \oplus {}_{\underline{w}}\underline{Q} \;\rightarrow\; \underline{E} \oplus {}_{\underline{w}}\underline{Q} \oplus {}_{\underline{w}}\underline{q}_{\underline{QE}} \;\rightarrow\; \underline{E} \oplus {}_{\underline{w}}\underline{Q} \oplus {}_{\underline{w}}\underline{q}_{\underline{QE}} \oplus {}_{\underline{w}}\underline{P} \;\rightarrow \ldots,$$

for the *'meta-systematic dialectical'*, *categorial-progression* method-of-exposition ***dialectic of arithmetics of logic***, with ${}_{\underline{w}}\underline{q}_{\underline{QE}}$ denoting an arithmetic interpreted for a logic that unifies key aspects of the $\underline{E}$ and ${}_{\underline{w}}\underline{Q}$ arithmetics, interpreted as arithmetics of logic, and wherein ${}_{\underline{w}}\underline{P}$ denotes an arithmetic of 'Probabilistic Logic', ***«qua»*** an arithmetic of logic for uncertain/merely Plausible inference [*wherein we use color-coded, solid-headed arrows, e.g., '$\longrightarrow$', to indicate 'lead-onto-to-next-lead-onto' progression*], and second, with the sequence for the *'meta-systematic dialectical'*, *categorial-progression* exposition ***dialectic of arithmetics of dialectics*** as well:

$${}_{\underline{w}}\underline{P}$$

$$\underline{W}$$

$${}_{\underline{w}}\underline{q}_{\underline{QE}}$$

$$\underline{W} \;\oplus\; {}_{\underline{w}}\underline{Q}$$

$$\underline{E}$$

$$\underline{W} \;\oplus\; {}_{\underline{w}}\underline{Q} \;\oplus\; {}_{\underline{w}}\underline{U}$$

$$\underline{W} \;\oplus\; {}_{\underline{w}}\underline{Q} \;\oplus\; {}_{\underline{w}}\underline{U} \;\oplus\; {}_{\underline{w}}\underline{M}$$

$$\underline{W} \;\oplus\; {}_{\underline{w}}\underline{Q} \;\oplus\; {}_{\underline{w}}\underline{U} \;\oplus\; {}_{\underline{w}}\underline{M} \;\oplus\; {}_{\underline{w}}\underline{q}_{\underline{MW}}$$

$$\underline{W} \;\oplus\; {}_{\underline{w}}\underline{Q} \;\oplus\; {}_{\underline{w}}\underline{U} \;\oplus\; {}_{\underline{w}}\underline{M} \;\oplus\; {}_{\underline{w}}\underline{q}_{\underline{MW}} \;\oplus\; {}_{\underline{w}}\underline{q}_{\underline{MQ}}$$

$$\underline{W} \;\oplus\; {}_{\underline{w}}\underline{Q} \;\oplus\; {}_{\underline{w}}\underline{U} \;\oplus\; {}_{\underline{w}}\underline{M} \;\oplus\; {}_{\underline{w}}\underline{q}_{\underline{MW}} \;\oplus\; {}_{\underline{w}}\underline{q}_{\underline{MQ}} \;\oplus\; {}_{\underline{w}}\underline{q}_{\underline{MQW}} \;=$$

$$\underline{W} \;\oplus\; {}_{\underline{w}}\underline{Q} \;\oplus\; {}_{\underline{w}}\underline{q}_{\underline{QW}} \;\oplus\; {}_{\underline{w}}\underline{M} \;\oplus\; {}_{\underline{w}}\underline{q}_{\underline{MW}} \;\oplus\; {}_{\underline{w}}\underline{q}_{\underline{MQ}} \;\oplus\; {}_{\underline{w}}\underline{q}_{\underline{MU}} \;=$$

$$\underline{W} \;\oplus\; {}_{\underline{w}}\underline{Q} \;\oplus\; {}_{\underline{w}}\underline{q}_{\underline{U}} \;\oplus\; {}_{\underline{w}}\underline{M} \;\oplus\; {}_{\underline{w}}\underline{q}_{\underline{MW}} \;\oplus\; {}_{\underline{w}}\underline{q}_{\underline{MQ}} \;\oplus\; {}_{\underline{w},\alpha}\underline{\mu} \;=$$

$$\underline{W} \;\oplus\; {}_{\underline{w}}\underline{Q} \;\oplus\; {}_{\underline{w}}\underline{U} \;\oplus\; {}_{\underline{w}}\underline{M} \;\oplus\; {}_{\underline{w}}\underline{q}_{\underline{MW}} \;\oplus\; {}_{\underline{w}}\underline{q}_{\underline{MQ}} \;\oplus\; {}_{\underline{w}}\underline{\alpha}, \quad \ldots$$

The focus of Part II., versus that of Part III., of our forthcoming treatise, entitled ***Dialectical Ideography: A Contribution to the Immanent Critique of Arithmetic***, can be summarized in terms of the following diagram. Part II., *The Meta-Evolution of Arithmetics*, is addresses the '$\downarrow$-arrowed' vertical gradient of 'meta-systematic dialectical', categorial-progression/systems-of-arithmetic progression exposition. Part III., *The Arithmetics of Meta-Evolution*, addresses the '$\rightarrow$-arrowed' horizontal gradient of such exposition:

$$\underline{N} \;\rightarrow\; \underline{N} \oplus {}_{\underline{N}}\underline{Q} \;\rightarrow\; \underline{N} \oplus {}_{\underline{N}}\underline{Q} \oplus {}_{\underline{N}}\underline{U} \;\rightarrow\; \underline{N} \oplus {}_{\underline{N}}\underline{Q} \oplus {}_{\underline{N}}\underline{U} \oplus {}_{\underline{N}}\underline{M} \;\rightarrow\; \underline{N} \oplus {}_{\underline{N}}\underline{Q} \oplus {}_{\underline{N}}\underline{U} \oplus {}_{\underline{N}}\underline{M} \;\oplus {}_{\underline{N}}\hat{\underline{q}}_{\underline{MN}} \;\rightarrow \ldots$$
$$\downarrow$$
$$\underline{W} \;\rightarrow\; \underline{W} \oplus {}_{\underline{W}}\underline{Q} \;\rightarrow\; \underline{W} \oplus {}_{\underline{W}}\underline{Q} \oplus {}_{\underline{W}}\underline{U} \;\rightarrow\; \underline{W} \oplus {}_{\underline{W}}\underline{Q} \oplus {}_{\underline{W}}\underline{U} \oplus {}_{\underline{W}}\underline{M} \;\rightarrow\; \underline{W} \oplus {}_{\underline{W}}\underline{Q} \oplus {}_{\underline{W}}\underline{U} \oplus {}_{\underline{W}}\underline{M} \;\oplus {}_{\underline{W}}\hat{\underline{q}}_{\underline{MW}} \;\rightarrow \ldots$$
$$\downarrow$$
$$\underline{Z} \;\rightarrow\; \underline{Z} \oplus {}_{\underline{Z}}\underline{Q} \;\rightarrow\; \underline{Z} \oplus {}_{\underline{Z}}\underline{Q} \oplus {}_{\underline{Z}}\underline{U} \;\rightarrow\; \underline{Z} \oplus {}_{\underline{Z}}\underline{Q} \oplus {}_{\underline{Z}}\underline{U} \oplus {}_{\underline{Z}}\underline{M} \;\rightarrow\; \underline{Z} \oplus {}_{\underline{Z}}\underline{Q} \oplus {}_{\underline{Z}}\underline{U} \oplus {}_{\underline{Z}}\underline{M} \;\oplus {}_{\underline{Z}}\hat{\underline{q}}_{\underline{MZ}} \;\rightarrow \ldots$$
$$\downarrow$$
$$\underline{Q} \;\rightarrow\; \underline{Q} \oplus {}_{\underline{Q}}\underline{Q} \;\rightarrow\; \underline{Q} \oplus {}_{\underline{Q}}\underline{Q} \oplus {}_{\underline{Q}}\underline{U} \;\rightarrow\; \underline{Q} \oplus {}_{\underline{Q}}\underline{Q} \oplus {}_{\underline{Q}}\underline{U} \oplus {}_{\underline{Q}}\underline{M} \;\rightarrow\; \underline{Q} \oplus {}_{\underline{Q}}\underline{Q} \oplus {}_{\underline{Q}}\underline{U} \oplus {}_{\underline{Q}}\underline{M} \;\oplus {}_{\underline{Q}}\hat{\underline{q}}_{\underline{MQ}} \;\rightarrow \ldots$$
$$\downarrow$$
$$\underline{R} \;\rightarrow\; \underline{R} \oplus {}_{\underline{R}}\underline{Q} \;\rightarrow\; \underline{R} \oplus {}_{\underline{R}}\underline{Q} \oplus {}_{\underline{R}}\underline{U} \;\rightarrow\; \underline{R} \oplus {}_{\underline{R}}\underline{Q} \oplus {}_{\underline{R}}\underline{U} \oplus {}_{\underline{R}}\underline{M} \;\rightarrow\; \underline{R} \oplus {}_{\underline{R}}\underline{Q} \oplus {}_{\underline{R}}\underline{U} \oplus {}_{\underline{R}}\underline{M} \;\oplus {}_{\underline{R}}\hat{\underline{q}}_{\underline{MR}} \;\rightarrow \ldots$$
$$\downarrow$$
$$\underline{C} \;\rightarrow\; \underline{C} \oplus {}_{\underline{C}}\underline{Q} \;\rightarrow\; \underline{C} \oplus {}_{\underline{C}}\underline{Q} \oplus {}_{\underline{C}}\underline{U} \;\rightarrow\; \underline{C} \oplus {}_{\underline{C}}\underline{Q} \oplus {}_{\underline{C}}\underline{U} \oplus {}_{\underline{C}}\underline{M} \;\rightarrow\; \underline{C} \oplus {}_{\underline{C}}\underline{Q} \oplus {}_{\underline{C}}\underline{U} \oplus {}_{\underline{C}}\underline{M} \;\oplus \hat{\underline{q}}_{\underline{MC}} \;\rightarrow \ldots$$
$$\downarrow$$
$$\underline{H} \;\rightarrow\; \underline{H} \oplus {}_{\underline{H}}\underline{Q} \;\rightarrow\; \underline{H} \oplus {}_{\underline{H}}\underline{Q} \oplus {}_{\underline{H}}\underline{U} \;\rightarrow\; \underline{H} \oplus {}_{\underline{H}}\underline{Q} \oplus {}_{\underline{H}}\underline{U} \oplus {}_{\underline{H}}\underline{M} \;\rightarrow\; \underline{H} \oplus {}_{\underline{H}}\underline{Q} \oplus {}_{\underline{H}}\underline{U} \oplus {}_{\underline{H}}\underline{M} \;\oplus \hat{\underline{q}}_{\underline{MH}} \;\rightarrow \ldots$$
$$\downarrow$$
$$\underline{O} \;\rightarrow\; \underline{O} \oplus {}_{\underline{O}}\underline{Q} \;\rightarrow\; \underline{O} \oplus {}_{\underline{O}}\underline{Q} \oplus {}_{\underline{O}}\underline{U} \;\rightarrow\; \underline{O} \oplus {}_{\underline{O}}\underline{Q} \oplus {}_{\underline{O}}\underline{U} \oplus {}_{\underline{O}}\underline{M} \;\rightarrow\; \underline{O} \oplus {}_{\underline{O}}\underline{Q} \oplus {}_{\underline{O}}\underline{U} \oplus {}_{\underline{O}}\underline{M} \;\oplus \hat{\underline{q}}_{\underline{MO}} \;\rightarrow \ldots$$

In the **2**-directional, **2**-dimensional 'meta-systematic' dialectical 'ideo-propagation' depicted above, $\underline{\mathbf{N}}$ denotes the arithmetical first-order axioms-system or rules-system of the standard "**N**atural" **N**umbers, $\underline{\mathbf{W}}$ that of the '"**W**hole Numbers"', $\underline{\mathbf{Z}}$ that of the "Integers" or '"Integral Numbers", $\underline{\mathbf{Q}}$ that of the "Rational Numbers", $\underline{\mathbf{R}}$ that of the "**R**eal Numbers", $\underline{\mathbf{C}}$ that of the "**C**omplex Numbers", $\underline{\mathbf{H}}$ that of the **H**amilton "Quaternions", and $\underline{\mathbf{O}}$ that of the Cayley/Graves "**O**ctonions"/"**O**ctaves". [Note: $\underline{\mathbf{Q}} \gtrless \mathbf{Q}$].

Both **Parts** employ the $\underline{\mathbf{Q}}$ ideographies to model the 'meta-systematic dialectical' categorial-/systems-progressions which they treat.

'Intra-Duality and 'Self-Changingness' / «Autokinesis». The theory-of-interpretation of the $\underline{\mathbf{Q}}$-based models set forth herein holds that the actuality of '*contra-Boolean Arithmetic*', the ontological '*meta-dynamics*' of '*contra-Boolean Processes*', in the external world, as in the conceptual worlds of the human mind, is driven by a '***self-force***', the '*self-reflexive*' or '*self-directed*' '*force*', *arising-from-self* ***and also*** *directed-back-upon-self*, of '***ontological intra-duality***' or 'ontological ***self-duality***'. The fundamental form of this '*intra-duality*' or '***self-duality***' is that '*between*' the '***subject***' aspect and the '***object***' aspect of every/any "self-same" [*ev*]entity. Any '***eventity***' is both *an* ***agent*** *of change* [including *self-change*, as well as of *change in other eventities* with which it interacts], ***an initiator of action***(s) ['***subject***'], with varying degrees/magnitudes of [***self***-]impact(s), upon its entire universe [***itself included***], and also ***a recipient of action***(s) ['***object***'], with varying degrees of [***self***-]impact(s), from its entire universe [***itself included***].

Descriptions of such an eventity's life-history, written out in the form of English sentences, should therefore place its name, or a pronoun referring [back] to that name, in the "***object***"-of-action place, "after"/'to the right of' the verb, in some sentences, and in the "***subject***"-of-action place, "before"/'to the left of' the verb, in some sentences. These sentences include single, '"***self-reflexive***"' sentences, in which the name or pronoun ['pro-name'] of this eventity will appear in ***both*** the "*subject*"-of-action and also in the "*object*"-of-action slots, thereby describing how this 'eventity' both generates the actions that characterize it, that it '***is***', and also receives these actions, '*back from itself*'. It thus '***acts upon itself***' and thereby '***changes itself***', as well as being "*acted upon*" and thereby "*changed by*", other '*selves*' -- other '*eventities*'. The verb in such a '"***self-reflexive***"' eventity's descriptive sentence should therefore also be but another name for this '*subject-object identical*'; an '*action-name*' of that 'subject-***verb***-object' 'eventity', naming a *type of activity* that arises from the 'intra-dual', 'self-dual', or 'internally-divided/self-divided', '***indivi***[sibly]-***dual***' essence or nature of this '*subject/verb/object-identical*' eventity; this '***dialectical***', or '***self-reflexive***', or '*self-refluxive*', or '*self-referential*', or "*paradoxical*", or '***ontologically/existentially*** *self-contradictory*', '***self-changing***', '***self-moving***', "*auto-kinetic*", or '*nonlinear*' eventity!

In the standard ***linear***-theory idealization of ***fixed-point*** "***equilibrium***", the eternal result of such '***self-action***' is the "***simple reproduction***", the temporal extension, or prolongation into the future, of the past "*state*" of that eventity, like unto a *Parmenidean, static, immutable, eternally unchanging, '**un**-dynamical' "Being"*. Thus, in such a Parmenidean/linear/equilibrium view, '***self-activity***' can be neglected. The ***logic*** of such an '*un-dynamical*', '***anti***-*dynamical*', '***pseudo***-dynamical' "dynamics" of ***effective non-action***; of this at most transiently-dynamical or ***cyclically***-dynamical, '***simple*** *self-reproduction*', is '***Boolean***': $\mathbf{X(X) = X}$; '$\mathbf{X}$ *of* $\mathbf{X}$' or '$\mathbf{X}$ *acting or operating upon itself*' yields just itself, $\mathbf{X}$, back again, unchanged; $\mathbf{X}^2 = \mathbf{X}$: $\Delta\mathbf{X} = 0$. But many actual processes observed in nature, many '***eventities***', as we have seen above, and as we shall see even more so below, are '***non-Boolean***' or '***meta***-*nonlinear*' in their ***logic of self-action***:

$$\underline{\mathbf{X}}\langle\!\langle\, \underline{\mathbf{X}} \,\rangle\!\rangle \neq \underline{\mathbf{X}};\ \text{'}\underline{\mathbf{X}}\ \mathit{of}\ \underline{\mathbf{X}}\text{'} \ \equiv\ \underline{\mathbf{X}} \oplus \underline{\Delta\mathbf{X}} \ \equiv\ \underline{\mathbf{X}}^2 \mid \underline{\mathbf{X}}^2 \gtrless \underline{\mathbf{X}}.$$

In such '***contra-Boolean Processes***', the *action* of the '***subject/object***' or '***eventity***' or '***system***' *back upon itself*, the ***self-application*** of the ***activity*** which it "***is***", ***changes it***[self] *quanto-qualitatively*, ***changes its ontology***, 'self-converts' ***a part of*** its '***predecessor-ontology***' into a '***successor-ontology***', and, typically, in the net, ***expands its ontology*** overall. That 'system-eventity' thereby changes the ontology of the epoch/stage of the ***universe***[-of-discourse] in which it inheres. By way of such '***contra-***' or '***non-Boolean***' processes of '***self-reflexion***/'***self-refluxion***', of '***quanto-qualitatively***', '***quanto-ontologically***' ***self-expanding*** self-reproduction, we mean principally processes of ***other-mediated*** *self-reflexion*, as opposed to processes of '***immediate self***-*reflexion*'. In this regard, $\underline{\mathbf{Q}}$ may be regarded as a '*Peanic*', '*Non-Standard-Natural*', '*contra-Boolean units*' system of arithmetic, the rules-system for a new language that provides an ideographically-symbolic, formulaic, algorithmic «***mimesis***» of such processes and their logics. The '*force*' whose existence is asserted by the '$\rightarrow$' in the *ideographical expression* $\underline{\mathbf{X}} \rightarrow \hookleftarrow\!\underline{\mathbf{X}} = \underline{\mathbf{XX}} = \underline{\mathbf{X}}\langle\!\langle\, \underline{\mathbf{X}} \,\rangle\!\rangle = \underline{\mathbf{X}}^2 = \underline{\mathbf{X}} \oplus \underline{\Delta\mathbf{X}}$, the *internal-to-*$\underline{\mathbf{X}}$ force that *moves* [$\rightarrow$] $\underline{\mathbf{X}}$ from its $\underline{\mathbf{X}}$ '*meta-state*' to its *next*, '*self-«aufheben» negation*' 'meta-state', $\hookleftarrow\!\underline{\mathbf{X}} = \underline{\mathbf{XX}} = \underline{\mathbf{X}}\langle\!\langle\, \underline{\mathbf{X}} \,\rangle\!\rangle = \underline{\mathbf{X}}^2 = \underline{\mathbf{X}} \oplus \underline{\Delta\mathbf{X}} \gtrless \underline{\mathbf{X}}$, is the '***self-force***', the '***self-reflexive force***' and the '***self-refluxive force***' of the '***subject-verb-object eventity***' denoted by $\underline{\mathbf{X}}$, the effect of an '***intra-dual***' and ever '***intra-dueling***' $\underline{\mathbf{X}}$, *qua* subject, $\underline{\mathbf{X}}\langle\!\langle\, \_ \,\rangle\!\rangle$, acting upon itself *qua* object, $\_\langle\!\langle\, \underline{\mathbf{X}} \,\rangle\!\rangle$, which process is, at its fullness or completion, denoted, in total, by $\underline{\mathbf{X}}\langle\!\langle\, \underline{\mathbf{X}} \,\rangle\!\rangle$. The expression '$\underline{\mathbf{X}}\langle\!\langle\, \underline{\mathbf{X}} \,\rangle\!\rangle$' connotes $\underline{\mathbf{X}}$ as '*being*'-'***in-and-for-itself***', i.e., as '*action*' 'within-and-***upon***-it***self***', in '***partial self-determination***'. The notation '$\underline{\mathbf{X}}\langle\!\langle\, \_ \,\rangle\!\rangle$' encompasses as well $\underline{\mathbf{X}}$ in *action* '***upon***-*other-selves*', which thus also constitute '***being for it***' as '***being-for-another***'. The notation '$\_\langle\!\langle\, \underline{\mathbf{X}} \,\rangle\!\rangle$' also encompasses $\underline{\mathbf{X}}$ as a partial '***being-in-itself***', cast as '***being-for-***[an]***other***(s)', as part of ***their*** '***other***[*ness*]'. We say '***partial self-determination***' because $\underline{\mathbf{X}} \subset \underline{\mathbf{\forall}}$, short of $\underline{\mathbf{X}} \equiv \underline{\mathbf{\forall}}$, also has an environment, and thus also constitutes '***being-for-other***[-being]***s/for-other-selves***' -- an action/actor '***being-acted-upon-by***' ***other*** *beings/selves/actions*, and thus as a ***partial*** '***being-in-itself***', or '***other-determined*** *being*'. The notation '$\underline{\mathbf{X}}$', grasped as denoting an 'autokinesis', 'self-change-inducing '***self-force***', refers to the *ontological*, '*onto-genetic*' force of $\underline{\mathbf{X}}$'s own *nature*, of its own *essence*, of its '*essence-ial*', *ontological, internal, immanent, inherent, and ineluctable* '*self-antithesis*' or '*self-opposition*', and of its concrete, contental, '*existential self-contradiction*'; of the tension between its '*object-ivity*', or action-*receiving*-character, and its '***subject-ivity***', or action-*initiating*-character; the very '***force***' of its own '*self*', of its own ***internally self-divided self***; of its own "*internally self-ravaged ground*"; of its inescapable '***intra-duality***', '***self-duality***', '***indivi-duality***', or *non-separable*, '***indivisible-duality***'.

The totality of '[*self*-]***force***' -- including the ensemble of the '***self-forces***' of all extant '*eventities*' -- is the very ***cause*** of "***Time***". For "***Time***", grasped in its concreteness -- ***not*** as a reified, hypostatized abstraction; ***not*** as the 'pseudo-Subject' of a subject-object inverting, fetishizing 'idealizationism', imposed upon the universe as if from "outside" -- is nothing other than the ensemble of all '*act*-ualized' '***self-forces***' and '***other-forces***' of all extant *act*-ualities; the totality of 'self-action' and 'other-action'/'inter-action'; the totality of '***auto-kinesis***' and '***allo-kinesis***'; the self-orchestrating cosmological concert of '***change-in-general***'. Thus, ***the real subjects***, the true agents, the true '***causors***' of "***Time***", the real 'substance' of "***Time***", are also they which constitute the ***true content*** of the cosmos: the '*self-dual*', '*ontologically self-converting*', '*existentially self-contradictory*', '*self-active*', '***self-reflexive***', '***self-refluxive***', '*self-propelling*', '*auto-kinetic*', '*quanto-qualitatively self-changing*', '*onto-dynamical system-eventities*' which we have described above.

An Example of 'Intra-Duality' and Its 'Meta-Dynamicity'. For example, it is of the nature of a *"main-sequence"* star, a *self-organized* ***«arithmos aisthetos»*** population of mainly atomic Hydrogen, **H** -- Hydrogen atom 'monads' -- to *exude "gravitational force"*, in short, to *gravitate*, hence to *self-gravitate*, which means that the star places upon itself a force for *self-contraction* and 'self-densification' / 'self-compression'. A consequence of the ensuing *self-contraction* is thus the formation and '***densification***' of a central core, and the ignition there of a *thermonuclear Hydrogen fusion reaction, once that self-gravitational self-contraction induces sufficient density/temperature in that central core.* This fusion process initiates "*stellar nucleosynthesis*", the formation of ***new*** "atomic species", of new '*atomic **ontology***', from ***old***, via *self-interaction of the manifold of the 'monads' or atoms of the* ***old*** "atomic species", together *with the explosive release of radiant energy. Each fusion-collision of Hydrogen nuclei*, e.g., of its Deuterium or Tritium isotopes, *is itself a '__Coulomb__ __singularity__'*. If we denote the "positive"/protonic '*electro-dynamic charge*' of each **H** nucleus by $+\mathbf{e}$ [*since any neutrons contribute* $\mathbf{0e}$], the moment of collision by $\mathbf{t}^*$, and the radial ***distance between*** the two colliding **H** nuclei, **H** nucleus **j** and **H** nucleus **k**, at moment **t**, by $\mathbf{r_{jk}(t)}$, then the size of the '*pure-quantitative electrodynamic force*' "between" the **2** nuclei at $\mathbf{t}^*$ is, per the standard, Coulomb idealization of it, $(+\mathbf{e} \times +\mathbf{e})/\mathbf{r_{jk}(t^*)} = \mathbf{e^2/0}$. *Thus we see, again, that this cosmos, in its '__meta__-evolutionary' __self__-__construction__, is '**made of singularities**'.*

This 'intensely active', or high-density/high-temperature, *fusion-interaction* of core Hydrogen atoms with other core Hydrogen atoms, i.e., the '*self-interaction*' or '*intra-action*' of the core population of Hydrogen atoms [part of the 'ontological category' or '***onto***' here denoted by $\underline{\mathbf{H}}$], produces a core population of Helium [part of the '***onto***' here denoted by $\underline{\mathbf{He}}$]. *Core Hydrogen, interacting with core Hydrogen, generates core Hydrogen again, plus core Helium.* Helium atoms are '***meta-Hydrogen atoms***' *made out of multiple* [***two***]

Hydrogen atoms via ***__Hydrogen__*** *fusion*: $\underline{\mathbf{H}} \rightarrow$ '$\underline{\mathbf{H}}$ *of* $\underline{\mathbf{H}}$' $= \underline{\mathbf{H}}\langle\underline{\mathbf{H}}\rangle = \rightharpoonup\langle\underline{\mathbf{H}}\rangle = \underline{\mathbf{HH}} = \underline{\mathbf{H}}^2 = \underline{\mathbf{H}} \oplus \underline{\Delta\mathbf{H}} = \underline{\mathbf{H}} \oplus \underline{\mathbf{He}},\ \nsupseteq\ \underline{\mathbf{H}}$, *since* $\underline{\mathbf{He}} \nsupseteq \underline{\mathbf{H}}$.

Thus the star's *self-movement* of '*self-gravitational* ***self-__implosion__***' induces in itself also a *contrary self-movement*, an oppositely-directed counter-movement of *atomic, thermo-nuclear, Hydrogen 'self-synthesis* ***self-__explosion__***'. These physical-spatially *oppositely-directed,* ***contrary forces*** of ***self-implosion*** and ***self-explosion*** achieve a temporary *balance*/stabilization, *until the finite endowment of current core atomic fuel, initially Hydrogen, is consumed.* Then *self-implosive self-contraction* resumes, *until the next threshold* of '***densification***' / ***ignition*** / '***re-nucleosynthesis of previous nucleosynthesis products***' is breached. *That next epoch of this star's nucleosynthesis must thus*

involve ***__Helium__*** *fusion*: $\underline{\mathbf{He}} \rightarrow\ \rightharpoonup\langle\underline{\mathbf{He}}\rangle = \underline{\mathbf{He}}\langle\underline{\mathbf{He}}\rangle = \underline{\mathbf{He}}^2 = \underline{\mathbf{He}} \oplus \underline{\Delta\mathbf{He}} = \underline{\mathbf{He}} \oplus \underline{\mathbf{C}},\ \nsupseteq\ \underline{\mathbf{He}}$, *since* $\underline{\mathbf{C}} \nsupseteq \underline{\mathbf{He}}$.

Helium is an ash, a '***__waste__*** product', a 'material *entropy*', a '"pollutant"', ***__relative__*** to the Hydrogen fusion process. However, on the contrary, Helium is a ***__resource__***, a 'material *free energy*' or 'material *__negentropy__*' -- the primary fuel -- ***__relative__*** to the Helium fusion process. This 'intensely active' *fusion-collision interaction* of core Helium atoms with other core Helium atoms, that is, the '*self-interaction*' or '*intra-action*' of the core population of Helium, produces Carbon. Core Helium, interacting with core Helium, and with the products of previous such interaction, reproduces core Helium, but also generates something new: core Carbon. Carbon atoms are '***meta-Helium atoms***' made up out of multiple [in the limiting case, in effect, out of ***three***] Helium atoms. . . . . A '*Helium based*' star, a star in its second, or Helium-fueled, fusion-epoch, is, increasingly as its existence-duration self-extends in its temporal dimension -- as its "fourth dimension" self-lengthens -- a different star, with *a different* ***core composition****, a different* ***interior atomic ontology***; a star that has moved itself off of the "*stellar main sequence*" of the Hertzsprung-Russell diagram; ***a qualitatively different system***, with ***different dynamics***; with changed ***dynamical "laws"***, from a star in its first, '*Hydrogen-fueled*' fusion-epoch of 'stellar ***self-meta***-evolution'. Just so, an *industry-dominated, 'capital-based'* human society, emerging from the womb of a '*land-based*', or '"*landed-property-based*"', *agriculture-dominated* one, is a qualitatively different human-social system, *with an expanded* ***social-relations ontology***, relative to its predecessor in that sequence/progression of human-social systems, that human-social '***meta-system***'. It is a new system, with an expanded internal ontology of human-social relations. The very ***essence***/***nature*** of a star, and of the *temporary stabilization* which gives it *temporarily* ***sustained identity and existence*** as such, as "star", is this ***immanent duality***, this ***internal-*** or '***__intra__-duality***'/'***__self__-duality***', this 'dualized', '***inside agitator***' of ***self-caused self-implosion*** and/versus ***self-caused self-explosion***.

Let us not fail to cite, here, also, the ***«ad hominem»***, '*psycho-metaphorical*', *psycho-somatic* example, always already so ready-to-hand, the 'self-[as-]evidence '***self-example***': ***it is not possible for me to stay the same***. Whether I know it or not, whether I like it or not, ***my life-history*** *cannot help but be* ***a self-critique***, a critique *of* myself *by* myself; *an* ***immanent critique*** *of my self* ***as activity***, of my self as actualized by me, of my choices in action(s). The evolving and '***meta***-evolving', '*meta-dynamical meta-system*' that is my '*self*' is a *sequence*, a *succession*, a *progression* of '*psycho-ontologically*' distinct 'self systems', of selves or '***self-identities***', *punctuated and demarcated* by '*meta-self transitions*', be they ***__self-progressive__*** or ***__self-retrogressive__***; '*__pro-temporal__*' or '*__anti-__/__ante-temporal__*'; '*__pro-chronistic__*' or '*__anti-__/__ana-chronistic__*'; ***__self-regenerative__*** $\leftrightarrow \{ \hat{\mathbf{q}}_{\mathbf{z}} \in {}_{\mathbf{z}}\underline{\mathbf{Q}} \mid \mathbf{Z} \ni \mathbf{z} > \mathbf{0} \}$, or ***__self-degenerative__*** $\leftrightarrow \{ \hat{\mathbf{q}}_{\mathbf{z}} \in {}_{\mathbf{z}}\underline{\mathbf{Q}} \mid \mathbf{Z} \ni \mathbf{z} < \mathbf{0} \}$.

The above interpretations of the '−'-signed versus the '+'-signed subscripts are entirely by conventional ascription, since the two '"minimally-interpreted"' sub-arithmetics and sub-spaces of the ${}_{\mathbf{z}}\underline{\mathbf{Q}}$ arithmetic -- its '"***positive***"'-subscript & '"***negative***"'-subscript sub-arithmetics/sub-spaces -- are entirely *mutually symmetric* and *enantiomorphic*. '*I*' am a sequence of ***__present__*** '***self-containments***' of ***__past__*** selves, in which each such successor-self or new, '***meta-self***' exceeds its predecessor-self by '***containing***' that predecessor-self, including by *remembering* and *voluntarily curbing* or *constraining* that thus *inwardly-conserved* predecessor-self, and also in the sense that each successor-self em-*bodies* the consequences, the '[self-]***refluxes***', or the '***self-refluxions***', of its predecessors; the products of the *boomeranging*, '*echoic*', lagged self-impacts of each self's past actions, '*__state__s[-of-**action**]*', or '***__dyna__tes***', upon its later actions, '"*__state__s*"'[-of-activity], or '*__dyna__tes*'.

The 'meta-dynamics' of stellar atomic-species' 'onto-dynamasis' described above, which we will assign, ultimately, to the $\hat{\mathbf{q}}_{\mathbf{masn}}$ hybrid 'onto' in the models of ***Part B.*** to follow -- though modeled above mostly narratively rather than via nonlinear differential equation ideography, and in terminology that does not 'explicitize' this fact -- is a case in point of a deep and *grand interconnexion*, hitherto little-noted, which we plan to explicate in future communications. This '*grand interconnexion*' ties together differential equation degree > 1 nonlinearity, self-reflexive action of systems, self-reflexive operation of degree > 1 unknowns/function-values, '***ontological self-conversion*** *processes*' within the ontologies of systems driven by and as that system self-activity, and the ***proneness to singularity*** of, especially, ***nonlinear*** differential equations-systems, as a signpost of both ontological conversion becoming locally complete, within the 'system-driver-locus', and of the irruption/emergence of new activity-ontology, i.e., system identity-change; '***__meta__***-dynamical' self-movement/***«autokinesis»***; '***__meta__***-evolutionary progression'; '"meta-system transition"', or '***system __self__-revolutionization***'.

The Boolean Logic and the 'contra-Boolean' $\underline{\mathbf{Q}}$ Logic Contrasted. The logic of $\mathbf{E}$ and the logic of $\underline{\mathbf{Q}}$ present to us, then, two '*contra-parallel* [ar]*rays*' regarding the import of "*nonlinearity*". One is a '*logic of linearity*'. For it, *nonlinearity* has ***no import, no impact***, $x^2 - x^1 = 0 = x^1 - x^2$. It is a Parmenidean logic of static, eternal knowledge, in which there is no history of ideas, or in which that history is always, already, all over; is forever past, forever before present, behind us and completed; and thus for which the content or substance of history, of time itself, of ***the natural-historical labor of the self-construction of the cosmos***, which, in $\underline{\mathbf{Q}}$, we model abstractly by ***a 'chronogenic', τ-powered [self-]iteration of [generic self-[«aufheben»]]operation*** -- *makes no difference/does not exist*: $[\, \forall x \in \mathbf{E},\ \forall w \in \mathbf{W} \mid w \neq 0 \,][\, x^{2^w} = x^w = x^1 = x \,]$, or,

$$[\, \forall x \in \mathbf{E} \,][\, x^1 = x^2 = x^3 = x^4 = x^5 = x^6 = x^7 = x^8 = x^9 = x^{10} = x^{11} = \ldots \,]\text{, i.e.,}$$

$$[\, 1^1 = 1^2 = 1^3 = 1^4 = 1^5 = 1^6 = 1^7 = 1^8 = 1^9 = 1^{10} = 1^{11} = \ldots \,]\text{, and;}$$

$$[\, 0^1 = 0^2 = 0^3 = 0^4 = 0^5 = 0^6 = 0^7 = 0^8 = 0^9 = 0^{10} = 0^{11} = \ldots \,].$$

The other is a '***logic of <u>meta</u>-<u>non</u>-linearity***', that is, a logic of a species of '*<u>meta</u>-algebraic*' nonlinearity which is ***<u>qualitative</u>***, *ontological*, and ***onto-dynamical***. It is also a *<u>psycho</u>-historical* [symbolic] ***logic of the histor(y)(ies) of ideas***. In it, the substance of history, namely, ***ontology-expanding meta-nonlinearity***, which we model abstractly as ***the [self-]iteration of [self-[«aufheben»]]operation, <u>does</u>*** make a difference, indeed, a ***more-than-quantitative***, i.e., a ***qualitative***, ***<u>ontological</u>*** difference --

$$\underline{x}^2 \boxminus \underline{x}^1 = \underline{\boxminus x} \neq q_0: \qquad \llbracket\, \forall \underline{x} \in {}_{\mathbf{w}}\underline{\mathbf{Q}},\ \forall w \in \mathbf{W} \mid w \neq 1 \,\rrbracket\llbracket\, \underline{x}^w \neq \underline{x}^1 \,\rrbracket \text{ or,}$$

$$\llbracket\, \forall \underline{x} \in \underline{\mathbf{Q}} \,\rrbracket\llbracket\, \underline{x}^1 \neq \underline{x}^2 \neq \underline{x}^3 \neq \underline{x}^4 \neq \underline{x}^5 \neq \underline{x}^6 \neq \underline{x}^7 \neq \underline{x}^8 \neq \underline{x}^9 \neq \underline{x}^{10} \neq \underline{x}^{11} \neq \ldots \,\rrbracket \text{--}$$

$$\llbracket\, \underline{x} = \hat{q}_1 \,\rrbracket \quad \Rightarrow \quad \llbracket\, \underline{x}^n = \textstyle\sum_{k=1,n} \hat{q}_k \,\rrbracket,\ n, k \in \mathbf{N}:$$

$$\underline{x}^1 = \hat{q}_1;$$

$$\underline{x}^2 = \hat{q}_1\llbracket\, \hat{q}_1 \,\rrbracket = \hat{q}_1 \boxplus \hat{q}_1 \boxplus \hat{q}_{1+1} = \hat{q}_1 \boxplus \hat{q}_2 \text{ [i.e., using the '\textit{meta-genealogical evolute product rule}'];}$$

$$\underline{x}^3 = \hat{q}_1\llbracket\, \hat{q}_1 \boxplus \hat{q}_2 \,\rrbracket = \llbracket\, \hat{q}_1 \boxplus \hat{q}_2 \,\rrbracket \boxplus \hat{q}_1\llbracket\, \hat{q}_2 \,\rrbracket = \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_{1+2} = \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3;$$

$$\underline{x}^4 = \hat{q}_1\llbracket\, \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \,\rrbracket = \llbracket\, \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \,\rrbracket \boxplus \hat{q}_1\llbracket\, \hat{q}_3 \,\rrbracket = \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \boxplus \hat{q}_{1+3} = \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \boxplus \hat{q}_4;$$

$$\underline{x}^5 = \hat{q}_1\llbracket\, \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \boxplus \hat{q}_4 \,\rrbracket = \llbracket \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \boxplus \hat{q}_4 \,\rrbracket \boxplus \hat{q}_1\llbracket\, \hat{q}_4 \,\rrbracket = \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \boxplus \hat{q}_4 \boxplus \hat{q}_5;$$

$$\underline{x}^6 = \hat{q}_1\llbracket\, \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \boxplus \hat{q}_4 \boxplus \hat{q}_5 \,\rrbracket = \llbracket \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \boxplus \hat{q}_4 \boxplus \hat{q}_5 \,\rrbracket \boxplus \hat{q}_1\llbracket\, \hat{q}_5 \,\rrbracket = \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \boxplus \hat{q}_4 \boxplus \hat{q}_5 \boxplus \hat{q}_6;$$

$$\underline{x}^7 = \hat{q}_1\llbracket\, \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \boxplus \hat{q}_4 \boxplus \hat{q}_5 \boxplus \hat{q}_6 \,\rrbracket = \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \boxplus \hat{q}_4 \boxplus \hat{q}_5 \boxplus \hat{q}_6 \boxplus \hat{q}_7;$$

$$\underline{x}^8 = \hat{q}_1\llbracket\, \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \boxplus \hat{q}_4 \boxplus \hat{q}_5 \boxplus \hat{q}_6 \boxplus \hat{q}_7 \,\rrbracket = \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \boxplus \hat{q}_4 \boxplus \hat{q}_5 \boxplus \hat{q}_6 \boxplus \hat{q}_7 \boxplus \hat{q}_8;$$

$$\underline{x}^9 = \hat{q}_1\llbracket\, \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \boxplus \hat{q}_4 \boxplus \hat{q}_5 \boxplus \hat{q}_6 \boxplus \hat{q}_7 \boxplus \hat{q}_8 \,\rrbracket = \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \boxplus \hat{q}_4 \boxplus \hat{q}_5 \boxplus \hat{q}_6 \boxplus \hat{q}_7 \boxplus \hat{q}_8 \boxplus \hat{q}_9;$$

$$\underline{x}^{10} = \hat{q}_1\llbracket\, \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \boxplus \hat{q}_4 \boxplus \hat{q}_5 \boxplus \hat{q}_6 \boxplus \hat{q}_7 \boxplus \hat{q}_8 \boxplus \hat{q}_9 \,\rrbracket = \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \boxplus \hat{q}_4 \boxplus \hat{q}_5 \boxplus \hat{q}_6 \boxplus \hat{q}_7 \boxplus \hat{q}_8 \boxplus \hat{q}_9 \boxplus \hat{q}_{10};$$

$$\underline{x}^{11} = \hat{q}_1 \boxplus \hat{q}_2 \boxplus \hat{q}_3 \boxplus \hat{q}_4 \boxplus \hat{q}_5 \boxplus \hat{q}_6 \boxplus \hat{q}_7 \boxplus \hat{q}_8 \boxplus \hat{q}_9 \boxplus \hat{q}_{10} \boxplus \hat{q}_{11}; \ldots \text{, and;}$$

$$\llbracket\, \underline{x} = \hat{q}_1 \,\rrbracket \quad \Rightarrow \quad \llbracket\, \underline{x}^{2^\tau} = \textstyle\sum_{k=1,\, 2^\tau} \hat{q}_k \,\rrbracket,\ k \in \mathbf{N},\ \tau \in \mathbf{W}.$$

Dialectical Diachronic Structure: *The '«Aufheben» Structure'* of '***The Cosmic Consecuum***' -- The Meaning of the Term '***Meta-Fractal***'. In recent mathematics, the term '*fractal*' refers to an idealized, *supposedly* '"*actually infinite*"', *purely-quantitative*, and *only synchronic* structure, featuring "*frac*tional dimensionality" and scaled "***self-similarity***".

In our parlance, the term '*meta-fractal*' refers to a 'generalization' of that "*fractal*" structure, one which may include:

(**1.**) '*temporal fractals*'; a *temporal*, *diachronic* dimension of '*fractality*'; a '*diachronic* ***self-similarity structure***', as well as a *synchronic* ***self-similarity*** *structure*, such that this structure may exhibit '*diachronic* self-similarity ***scaling***' in the form of a phenomenon of '*temporal acceleration*' in terms of 'units of temporal measurement constituted as units of 'self-change', i.e., as '*self-internalization*' units or monads' of '***self-«aufheben»***', «***autokinesis***». The synchronic aspect or moment of such unified, '*diachronico-synchronic*' or '*synchronico-diachronic*' self-organizing structure may thus also exhibit a 'Trotskyan' '"***uneven and combined***" ['ontologically-*hybrid-ized*'] ***self-development***'", resulting in a fluid, shifting '"***center/semi-periphery/periphery***"' physical-spatial 'self-structuration' / self-deployment. History, per such a structure, does not repeat itself, is a ***non-cyclical***, ***aperiodic***, never-repeating and indeed '*irrepeatable*', '*quanto-qualitative*' discrete '*consecuum*' of '***meta-states***', but also one which, nonetheless, exhibits a '***multi-analogous***' -- or even a '***multi***-homologous', '*quanto-qualitatively scaled*', or '*helical-vortical*' self-***similarity-recurrence-structure***.

(**2.**) 'quanto-***qualitative***' self-similarity scaling, not '*purely quantitative*' scaling;

(**3.**) ***finiteness*** or, in our terms, recurrent '***meta-finiteness***' in the extent/limits of its '***scales regress***', ***not*** '"*actual infinitude*"'.

Such '*meta-fractal*' *synchronico-diachronic structure* is the principal and 'principle[d]' structural product of '*autokinesic*', dialectical, '***self-«aufheben»***' process; the structure of the resulting, spatio-temporally manifest, dialectical or '***Qualo-Quanto-Peanic Consecua***' of this cosmos; of the '***aisthetoic***', or concrete, physical '*meta-«arithmoi»*' of the cosmological «***physis***»; of the '***self-constitution***' of the dialectical, '***Qualo-Quanto-Peanic Cumula***' of this universe.

We find also that dialectical theories of categorial, 'ideo-ontological self-progression' also exhibit this '*meta-fractal*' structure in the mental '"space"' of the «***eide***»: there are '*ideo-metafractals*' as well as '*physio-metafractals*'.

The emergence of this concept of '*meta-fractality*' can be modeled in terms of the following 'conceptual-categorial dialectic':

$$\underline{\text{fractals}} \rightarrow \underline{\text{fractals}} = \underline{\text{fractals}}(\underline{\text{fractals}}) = \underline{\text{fractals}}^{2} = \underline{\text{fractals}} \oplus \Delta\underline{\text{fractals}} = \underline{\text{fractals}} \oplus \underline{\text{meta-fractals}} \ngtrless \underline{\text{fractals}},$$

such that: $\underline{\text{fractals}} \ngtrless \underline{\text{meta-fractals}}$, *but* $\underline{\text{fractals}} \sqsubset \underline{\text{fractals}} \oplus \underline{\text{meta-fractals}}$, *&, indeed, finitary* $\underline{\text{fractals}} \sqsubset \underline{\text{meta-fractals}}$.

'*Ideo-metafractals*' are exemplified in the *first*, *second*, *ninth*, and *eleventh* dialectical models of ***Supplement B.***, respectively of the history of intellectual disciplines, of the historical genesis of ***Marxian theory***, of the categorial systematics of Hegel's «***Logik***», and of the '***meta-axiomatic***', 'logical-incompleteness-/equational-unsolvability-driven '***Gödelian dialectic***' of '***ideo-meta-evolutionary***', '***ideo-meta-dynamical***' self-progression of the *axiomatic systems* of mathematics, modeled via set-theoretical, '"logical"' constructions of ever-self-escalating Gödelian-Russellian '*logical type*', as encompassed by the immanent, 'auto-kinesic', 'self-«***aufheben***»', generic dialectical self-process of the [meta-]finitary '*set of all* ["*logical individuals*" *and/including*] *sets*.

'***Physio-metafractals***', modeled via '***Ideo-metafractals***', are exemplified in the *third* through *sixth* dialectical models of ***Supplement B.***, models of cosmological 'meta-evolution', or of '***The Dialectic Of Nature***', and, within that dialectic, of its component dialectic of '***human nature***'; of the self-construction of the ***noosphere***; of the human "patch" on, emergent within, the pre-existing biosphere; of the '*meta-biospherical*' *zone* or *region* of human settlements; of incipiently '***humanized nature***' or '***objectified human labor***'.

Supplement B.'s *fifth* dialectical model offers a special, '*psycho-historical*' combination of the two, namely, a model of the dialectic of human nature, of human-social 'self-formation', of human-social 'self-meta-evolution' which tracks, explicitly at least, only the dialectic of what Marx termed '"***the social relations of production***"'; of '***the human-social self-relations of expanding human-society*** [*including social relations*] ***self-[re-]production***' -- leaving implicit the driver of this self-progression, namely, the dialectic of what he termed '"***the social forces of production***"'; '***the human-social self-force of expanding human-society self-production***' [*sixth model*]. The '***self-«aufheben»***', '*self-internalization*' «***autokinesis***» character of all of these '***meta-fractalogenic***' dialectical processes can most easily be illustrated via the following '*partial dialectics*', or '***antithesis formations***' of the *third* model of ***Supplement B.***, all corresponding to the $\hat{q}_n \rightarrow [\![\hat{q}_n]\!] = \hat{q}_n[\![\hat{q}_n]\!] = \hat{q}_n\hat{q}_n = \hat{q}_n^{\,2} = [\![\hat{q}_n \boxplus \Delta\hat{q}_n]\!] = [\![\hat{q}_n \boxplus \hat{q}_{n+n}]\!] = [\![\hat{q}_n \boxplus \hat{q}_{2n}]\!] \ngtrless \hat{q}_n$ [$n \in \mathbb{N}$] model of self-movement or «***autokinesis***» within the generic $_{\underline{\mathbf{N}}}\underline{\Omega}$ arithmetic, and to its general '***historical algebra***' for '***antithesis formations***' of '***meta-evolutionary***' ***historical dialectic***, namely $\underline{x} \rightarrow \langle\underline{x}\rangle = \underline{x}\langle\underline{x}\rangle = \langle\underline{x}\rangle^{2} = \langle\underline{x} \oplus \underline{\Delta}\langle\underline{x}\rangle\rangle \ngtrless \underline{x}$, viz.:

a. $\underline{\text{sub-nuclears}} \rightarrow \langle\underline{\text{sub-nuclears}}\rangle = \underline{\text{sub-nuclears}}\langle\underline{\text{sub-nuclears}}\rangle = \underline{\text{sub-nuclears}}$ '*of*' $\underline{\text{sub-nuclears}} =$
$\langle\underline{\text{sub-nuclears}}\rangle^{2} = \langle\underline{\text{sub-nuclears}} \oplus \underline{\Delta}\langle\underline{\text{sub-nuclears}}\rangle\rangle = \langle\underline{\text{sub-nuclears}} \oplus \underline{\text{sub-atomics}}\rangle =$
$\langle\underline{\text{sub-nuclears}} \oplus$ *the category actualized via the self-internalization of monads of the sub-nuclears category* $\rangle$;

b. $\underline{\text{sub-atomics}} \rightarrow \langle\underline{\text{sub-atomics}}\rangle = \underline{\text{sub-atomics}}\langle\underline{\text{sub-atomics}}\rangle = \underline{\text{sub-atomics}}$ '*of*' $\underline{\text{sub-atomics}} =$
$\langle\underline{\text{sub-atomics}}\rangle^{2} = \langle\underline{\text{sub-atomics}} \oplus \underline{\Delta}\langle\underline{\text{sub-atomics}}\rangle\rangle = \langle\underline{\text{sub-atomics}} \oplus \underline{\text{atomics}}\rangle =$
$\langle\underline{\text{sub-atomics}} \oplus$ *the category actualized via the self-internalization of monads of the sub-atomics category* $\rangle$;

c. $\underline{\text{atomics}} \rightarrow \langle\underline{\text{atomics}}\rangle = \underline{\text{atomics}}\langle\underline{\text{atomics}}\rangle = \underline{\text{atomics}}$ '*of*' $\underline{\text{atomics}} = \langle\underline{\text{atomics}}\rangle^{2} =$
$\langle\underline{\text{atomics}} \oplus \underline{\Delta}\langle\underline{\text{atomics}}\rangle\rangle = \langle\underline{\text{atomics}} \oplus \underline{\text{meta-atomics}}\rangle = \langle\underline{\text{atomics}} \oplus \underline{\text{moleculars}}\rangle =$
$\langle\underline{\text{atomics}} \oplus$ *the ontological category actualized via the self-internalization of monads of the atomics category* $\rangle$;

c. $\underline{\text{moleculars}} \rightarrow \langle\underline{\text{moleculars}}\rangle = \underline{\text{moleculars}}\langle\underline{\text{moleculars}}\rangle = \underline{\text{moleculars}}$ '*of*' $\underline{\text{moleculars}} =$
$\langle\underline{\text{moleculars}}\rangle^{2} = \langle\underline{\text{moleculars}} \oplus \underline{\Delta}\langle\underline{\text{moleculars}}\rangle\rangle = \langle\underline{\text{moleculars}} \oplus \underline{\text{prokaryotic-cellulars}}\rangle =$
$\langle\underline{\text{moleculars}} \oplus$ *the category actualized via the self-internalization of monads of the moleculars category* $\rangle$; ....

Some Basic Product Rule Variants for ${}_{\underline{N}}\underline{\Omega}$ *and* ${}_{\underline{W}}\underline{\Omega}$

Four 'Meta-Dynamical' Product Rules and Their 'Gödel Numbering Subscript-Rule' Variants. We explore, in the section of ***Dialectical Ideography*** entitled *The Arithmetics of Meta-Evolution*, four alternative product rules. We also explore a 'Gödelian' variant of each of these four product rules. The latter variants employ a subscript rule inspired by "Gödel numbering"; by the use Kurt Gödel made of the *Fundamental Theorem of* ["Natural" Number] *Arithmetic* in his Incompleteness Theorem. Gödel applied that Theorem to form a unique mapping/encoding of the formulae of symbolic logic to the elements of $\mathbf{N}$. The 'subscripted' function-value $p(n)$ in the Gödelian product rule variants below, denotes/selects the nth prime number, for any $n \in \mathbf{N}$, for the $\hat{\mathfrak{q}}_n$-values they encode: $p(n) = p_n$, where p_n denotes the nth prime number.

Multiplication/Proliferation of New 'Ontological Species'/'Qualities' by Old: Four Basic Variants of the ${}_{\underline{N}}\underline{\Omega}$ Product Rules.

The following **4** product rules are basic to those versions of ${}_{\underline{N}}\underline{\Omega}$ that fulfill the '*meta-evolution equation*', $\underline{\Omega}_{\tau+1} = \underline{\Omega}_\tau^{\,2} = \underline{\Omega}_\tau\llbracket\underline{\Omega}_\tau\rrbracket = \neg\llbracket\underline{\Omega}_\tau\rrbracket$, and to its solution, the '***ontological species meta-speciation***' or '*generation equation*', $\underline{\Omega}_\tau = \underline{\Omega}_0^{\,2^\tau} = \llbracket\, \hat{\mathfrak{q}}_1 \rrbracket^{2^\tau}$. Given $j,\ k \in \mathbf{N}$, we have --

1. The '«*Aufheben*» Evolute Product' Rule: $\hat{\mathfrak{q}}_j\llbracket\, \hat{\mathfrak{q}}_k \rrbracket = \llbracket\, \hat{\mathfrak{q}}_k \boxplus \hat{\mathfrak{q}}_{j+k}\rrbracket;\quad \neq \quad \hat{\mathfrak{q}}_k\llbracket\, \hat{\mathfrak{q}}_j \rrbracket = \llbracket\, \hat{\mathfrak{q}}_j \boxplus \hat{\mathfrak{q}}_{k+j} \rrbracket;$
2. The '*Meta-Catalysis* Evolute Product' Rule: $\hat{\mathfrak{q}}_j\llbracket\, \hat{\mathfrak{q}}_k \rrbracket = \llbracket\, \hat{\mathfrak{q}}_j \boxplus \hat{\mathfrak{q}}_{j+k} \rrbracket;\quad \neq \quad \hat{\mathfrak{q}}_k\llbracket\, \hat{\mathfrak{q}}_j \rrbracket = \llbracket\, \hat{\mathfrak{q}}_k \boxplus \hat{\mathfrak{q}}_{k+j} \rrbracket;$
3. The '*Meta-Genealogical* Evolute Product' Rule: $\hat{\mathfrak{q}}_j\llbracket\, \hat{\mathfrak{q}}_k \rrbracket = \llbracket\, \hat{\mathfrak{q}}_j \boxplus \hat{\mathfrak{q}}_k \boxplus \hat{\mathfrak{q}}_{j+k} \rrbracket;\quad = \quad \hat{\mathfrak{q}}_k\llbracket\, \hat{\mathfrak{q}}_j \rrbracket = \llbracket\, \hat{\mathfrak{q}}_k \boxplus \hat{\mathfrak{q}}_j \boxplus \hat{\mathfrak{q}}_{k+j} \rrbracket;$
4. The '*Meta-Heterosis* Convolute Product' Rule: $\hat{\mathfrak{q}}_j\llbracket\, \hat{\mathfrak{q}}_k \rrbracket = \llbracket\, \hat{\mathfrak{q}}_{j+k} \rrbracket;\quad = \quad \hat{\mathfrak{q}}_k\llbracket\, \hat{\mathfrak{q}}_j \rrbracket = \llbracket\, \hat{\mathfrak{q}}_{k+j} \rrbracket.$

'***Gödelian***' **Variants of the Above Product Rules**. The four 'Gödelian' variants of these four product rules are designed to accomplish a partial 'de-confounding', or greater distinguishability of distinct ontic interaction-products, from one another. This entails an even stronger form of the non-commutativity encountered above in Rules **1** and **2**. In these Gödelian variants, the 'index' or subscript of the 'qualitative increment' portion of a product is a 'Gödel number' encoding the syntax of the 'multiplication' formula from which it arose. As a result, each product *reveals*, *contains*, or *records* its history of interactions, *path-of-formation*, *origin*, *ancestry*, or *meta-genealogy* in a 'decode-able', 'dis-entangle-able' way, and is thus '*evolute*' in a yet deeper sense.

Given $j,\ k \in \mathbf{N} \mid j < k$, we have:

1g. '<u>G</u>ödelian «*Aufheben*» Evolute Product': $\hat{\mathfrak{q}}_j\llbracket\, \hat{\mathfrak{q}}_k \rrbracket = \llbracket\, \hat{\mathfrak{q}}_k \boxplus \hat{\mathfrak{q}}_{p(j)^j \times p(k)^k} \rrbracket;\quad \neq \quad \hat{\mathfrak{q}}_k\llbracket\, \hat{\mathfrak{q}}_j \rrbracket = \llbracket\, \hat{\mathfrak{q}}_j \boxplus \hat{\mathfrak{q}}_{p(k)^j \times p(j)^k} \rrbracket;$

2g. '<u>G</u>ödelian *Meta-Catalysis* Evolute Product': $\hat{\mathfrak{q}}_j\llbracket\, \hat{\mathfrak{q}}_k \rrbracket = \llbracket\, \hat{\mathfrak{q}}_j \boxplus \hat{\mathfrak{q}}_{p(j)^j \times p(k)^k} \rrbracket;\quad \neq \quad \hat{\mathfrak{q}}_k\llbracket\, \hat{\mathfrak{q}}_j \rrbracket = \llbracket\, \hat{\mathfrak{q}}_k \boxplus \hat{\mathfrak{q}}_{p(k)^j \times p(j)^k} \rrbracket;$

3g. '<u>G</u>ödelian *Meta-Genealogical* Evolute Product': $\hat{\mathfrak{q}}_j\llbracket\, \hat{\mathfrak{q}}_k \rrbracket = \llbracket\, \hat{\mathfrak{q}}_j \boxplus \hat{\mathfrak{q}}_k \boxplus \hat{\mathfrak{q}}_{p(j)^j p(k)^k} \rrbracket;\quad \neq \quad \hat{\mathfrak{q}}_k\llbracket\, \hat{\mathfrak{q}}_j \rrbracket = \llbracket\, \hat{\mathfrak{q}}_k \boxplus \hat{\mathfrak{q}}_j \boxplus \hat{\mathfrak{q}}_{p(k)^j p(j)^k} \rrbracket;$

4g. '<u>G</u>ödelian *Meta-Heterosis* Convolute Product': $\hat{\mathfrak{q}}_j\llbracket\, \hat{\mathfrak{q}}_k \rrbracket = \llbracket\, \hat{\mathfrak{q}}_{p(j)^j \times p(k)^k} \rrbracket;\quad \neq \quad \hat{\mathfrak{q}}_k\llbracket\, \hat{\mathfrak{q}}_j \rrbracket = \llbracket\, \hat{\mathfrak{q}}_{p(k)^j \times p(j)^k} \rrbracket;$

<u>Additional Alternative Product Rules for ${}_{\underline{N}}\underline{\Omega}$ *and* ${}_{\underline{W}}\underline{\Omega}$</u>. Beyond the **8** basic product rules defined above, consider the following --

The '*<u>P</u>eano-Successor*' Product Rule Version of $\underline{\Omega}$ [${}_{\mathcal{P}}\underline{\Omega}$]. By means of this product-rule variant, one achieves an algorithmic arithmetical syntax which supports a semantic interpretation to the effect that each successive '[ideo-]*<u>onto</u>*logical category', or '*onto*', dialectically negates itself to yield its successor-category; that the self-critique or 'self-negatory' self-reflexive self-operation of every predecessor-category generates its immediate successor-category. Products of disparate ontos are null in this version of ${}_{\underline{W}}\underline{\Omega}$: there are ***<u>no</u>*** '*hybrid ontos*' in it. There are no *<u>distinct</u>* '*uni-theses*' in its syntheses. Every *successor-onto* is a '*contra-thesis*' specifically to its immediate *predecessor-onto*, as well as to all of its other predecessor-ontos. This product-rule employs a two-argument variant of the 'Peano-Successor function', s, in the subscripts of its '*qualitative increment*' terms, such that, for any $w \in \mathbf{W}$, when the mutually-multiplied 'meta-numerals' have *the same* subscript, w, then the resulting subscript of the '*qualitative increment*' term will be $s(w,\ w) = s(w) = w+1$. When, instead, the mutually-multiplied 'meta-numerals' have *different* subscripts, $w,\ m \in \mathbf{W}$, such that $w \neq m$, then the '*qualitative increment*' term's subscript will be *<u>zero</u>*: $s(w,\ m) = 0$:

$$\underline{x}^1 = \hat{\mathfrak{q}}_0;$$

$\llbracket\underline{x}^1\rrbracket^2 = \hat{\mathfrak{q}}_0\llbracket\hat{\mathfrak{q}}_0\rrbracket = \hat{\mathfrak{q}}_0 \boxplus \hat{\mathfrak{q}}_0 \boxplus \hat{\mathfrak{q}}_{s(0)} = \hat{\mathfrak{q}}_0 \boxplus \hat{\mathfrak{q}}_1$ [via a '*meta-genealogical evolute product*' version of this '*Peano Product*' rule];

$$\llbracket\underline{x}^2\rrbracket^2 = \llbracket\hat{\mathfrak{q}}_0 \boxplus \hat{\mathfrak{q}}_1\rrbracket^2 = \llbracket\hat{\mathfrak{q}}_0 \boxplus \hat{\mathfrak{q}}_1\rrbracket \boxplus \hat{\mathfrak{q}}_0\llbracket\hat{\mathfrak{q}}_1\rrbracket \boxplus \hat{\mathfrak{q}}_1\llbracket\hat{\mathfrak{q}}_1\rrbracket = \llbracket\hat{\mathfrak{q}}_0 \boxplus \hat{\mathfrak{q}}_1\rrbracket \boxplus \llbracket\hat{\mathfrak{q}}_0 \boxplus \hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_0\rrbracket \boxplus \llbracket\hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_{s(1)}\rrbracket = \hat{\mathfrak{q}}_0 \boxplus \hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2;$$

$$\llbracket\underline{x}^4\rrbracket^2 = \llbracket\hat{\mathfrak{q}}_0 \boxplus \hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2\rrbracket^2 = \llbracket\hat{\mathfrak{q}}_0 \boxplus \hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2\rrbracket \boxplus \hat{\mathfrak{q}}_0\llbracket\hat{\mathfrak{q}}_2\rrbracket \boxplus \hat{\mathfrak{q}}_1\llbracket\hat{\mathfrak{q}}_2\rrbracket \boxplus \hat{\mathfrak{q}}_2\llbracket\hat{\mathfrak{q}}_2\rrbracket = \llbracket\hat{\mathfrak{q}}_0 \boxplus \hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2\rrbracket \boxplus \llbracket\hat{\mathfrak{q}}_0 \boxplus \hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_0\rrbracket \boxplus$$

$$\llbracket\hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_0\rrbracket \boxplus \llbracket\hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_{s(2)}\rrbracket = \hat{\mathfrak{q}}_0 \boxplus \hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_3;$$

$$\llbracket\underline{x}^8\rrbracket^2 = \llbracket\hat{\mathfrak{q}}_0 \boxplus \hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_3\rrbracket^2 = \llbracket\hat{\mathfrak{q}}_0 \boxplus \hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_3\rrbracket \boxplus \hat{\mathfrak{q}}_0\llbracket\hat{\mathfrak{q}}_3\rrbracket \boxplus \hat{\mathfrak{q}}_1\llbracket\hat{\mathfrak{q}}_3\rrbracket \boxplus \hat{\mathfrak{q}}_2\llbracket\hat{\mathfrak{q}}_3\rrbracket \boxplus \hat{\mathfrak{q}}_3\llbracket\hat{\mathfrak{q}}_3\rrbracket =$$

$$\llbracket\hat{\mathfrak{q}}_0 \boxplus \hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_3\rrbracket \boxplus \llbracket\hat{\mathfrak{q}}_0 \boxplus \hat{\mathfrak{q}}_3 \boxplus \hat{\mathfrak{q}}_0\rrbracket \boxplus \llbracket\hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_3 \boxplus \hat{\mathfrak{q}}_0\rrbracket \boxplus \llbracket\hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_3 \boxplus \hat{\mathfrak{q}}_0\rrbracket \boxplus \llbracket\hat{\mathfrak{q}}_3 \boxplus \hat{\mathfrak{q}}_3 \boxplus \hat{\mathfrak{q}}_{s(3)}\rrbracket =$$

$$\hat{\mathfrak{q}}_0 \boxplus \hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_3 \boxplus \hat{\mathfrak{q}}_4; \ldots$$

This *'Peano Product'* rule provides the least-constrained variant of ${}_{\mathbf{w}}\underline{\mathbf{Q}}$ for *"systematic dialectics"*; for the categorial-progression, pedagogical exposition of dialectical theories of experienced [sub-]totalities, *«à la»* Tony Smith's account of Hegelian dialectics: "...Hegel attempted to provide an immanent ordering of the basic categories ... To see this we have first to consider what a category is. It is a principle (a universal) for unifying a manifold [a 'many-fold' -- F.*E*.*D*.] of some sort or other (different individuals, or particulars). A category thus articulates a structure with two poles, a pole of unity and a pole of differences. In Hegelian language this sort of structure, captured in some category, can be described as a unity of identity in difference, or as a reconciliation of universal and individuals. From this general notion of a category we can go on to derive three general types of categorial structures. In one the moment of unity is stressed, with the moment of differences ***implicit***. In another the moment of differences is emphasized, with the moment of unity now being ***only implicit***. In a third both unity and differences are made ***explicit together***. Hegel's next claim is that there is a systematic order immanently connecting these three categorial structures. A structure of unity in which differences are ***merely implicit*** is ***simpler*** than one in which these differences are ***explicitly introduced***; and one in which both unity and differences are ***explicit*** is yet ***more complex*** still. Similarly, the first sort of structure is the ***most abstract*** [that is, least-specified, or least "determinate" -- F.*E*.*D*.], while the other structures are successively ***more concrete*** [via additional "specifications" or "determinations" -- F.*E*.*D*.]. ... If a category is in general a ***principle*** that ***unifies*** a ***manifold***, then if a specific category only ***explicates*** the ***moment*** of unity, leaving the ***moment*** of difference ***implicit***, then there is a "contradiction" between what it inherently is *qua* category (a unifier of a manifold) and what it is ***explicitly*** (the ***moment*** of unity alone). Overcoming this contradiction requires that the initial category be "***negated***" in the sense that a ***second category*** must be formulated that makes the ***moment*** of difference ***explicit***. But when this is done the ***moment*** of difference will be emphasized at the cost of having the ***moment*** of unity made merely ***implicit***. Once again there is a contradiction between what a category inherently is and what it is ***explicitly***. Overcoming this contradiction demands that the ***second*** sort of ***category also*** be ***negated*** and ***replaced*** with a category in which both ***poles***, unity and difference, are ***each made explicit simultaneously***. Hegel is well aware that "contradiction" and "***negation***" are not being used here in the sense given to them in formal logic. ***Following a tradition that goes back to Plato***, he asserts that in the above usage "contradiction" and "***negation***" are ***logical operators*** for ordering categories systematically, as opposed to logical operators for making formal inferences. The logic with which we are concerned here is *dialectical logic*. ... The "***negation***" of the ***simple unity*** is the ***moment*** of difference that it itself ***contains implicitly***. ... But this stage of difference is itself ***one-sided*** and ***partial***. ... When ***the stage of difference*** is ***dialectically negated***, we once again have a category of unity, but now it is a ***complex unity***, one that incorporates the ***moment*** of difference ... Since a category of unity-in-difference on one level can itself prove to be a category of ***simple unity*** from a ***higher level*** perspective, ***thereby initiating another dialectical progression from unity through difference to unity-in-difference***, we can construct ***a systematic theory of categories*** by employing the ***dialectical method***. In this sort of theory we move in a step-by-step fashion from ***simple*** and ***abstract*** categories to those that are ***complex*** and ***concrete***, with ***dialectical logic*** providing the warrant for each transition." [T. Smith, *The Logic of Marx's Capital*, SUNY Press [NY: 1990], pp. 5-7, ***bold italics emphasis*** by F.*E*.*D*.]. Smith then quotes one of Hegel's own accounts of ***this dialectic of categorial cognition***: "The ***determinateness*** which was a ***result*** is itself, by virtue of the form of ***simplicity*** into which it was withdrawn, ***a fresh beginning***; as this beginning is ***distinguished from its predecessor*** precisely by that ***determinateness***, ***cognition rolls onwards from content to content***. First of all, this advance is determined as beginning from ***simple determinatenesses***, the ***succeeding ones becoming ever*** *richer and more concrete*. For ***the result contains its beginning*** and its course has enriched it by a ***fresh determinateness*** ... at each stage of its ***further determination*** it ***raises the entire mass of its preceding content***, and by its ***dialectical advance*** it not only ***does not lose anything or leave anything behind***, but ***carries along with it all it has gained*** and ***inwardly enriches and consolidates itself***." [***emphasis added*** by F.*E*.*D*.]. Per this 'Peano product' rule, each successive category is its own *'negator'*, generating its own successor category through its *'self-operation'*, that is, through *'self-multiplication'*, interpreted as signifying ***'self-reflection'***. The *mutual operation* of *disparate categories* only reproduces the ***«arché»***, for this product-rule, $\hat{\mathfrak{q}}_{0}$, as its *'qualitative* [*non*-]*increment'* / [*non*-]*incremental category*. Only *self-operation* sustains advance.

The '*Prime-Successor*' Product Rule Version of $\underline{\mathbf{Q}}$ [$_{\mathbf{x}}\underline{\mathbf{Q}}$]. By means of this product-rule version, '*non*-hybrid', '*self*-hybrid', or '***new contra-thesis ontos***' are denoted by '***meta-numerals***', '***qualifiers***' with successive *prime* numbers as their subscripts, whereas '*hybrid ontos*', except those directly involving the ***«arché»***, $\hat{\mathfrak{q}}_1$, are denoted by '***qualifier meta-numerals***' with ***composite*** numbers as subscripts. This product rule employs a 'subscripted prime-successor' function-value, denoted $\mathrm{s}_{\mathrm{p}}(\mathrm{n})$, such that, for any $\mathrm{n} \in \mathbf{N}$, if n is a repeated-subscript *prime number*, then the '*qualitative increment*' term's subscript will be *prime*, $\mathrm{s}_{\mathrm{p}}(\mathrm{n}) = \mathrm{p}_{\mathrm{n}+1}$, with $\mathrm{p}_{\mathrm{n}+1}$ denoting the $\mathbf{n+1}$st prime number in $\mathbf{N}$. When, instead, there arise mutually-multiplied '***meta-numerals***' with *non*-repeated subscripts, e.g., $\mathrm{n}, \mathrm{m} \in \mathbf{N}$, such that $\mathrm{n} \neq \mathrm{m}$, then $\mathrm{n} \times \mathrm{m}$ will be the ***composite number*** *subscript* of the '*qualitative increment*' term. The '*qualitative increment*' or '***elevation***' contribution of the multiplication of two '***qualifier sums***', as distinct from their '***conservation***' ***contribution***, arises from the operation of each term of the multiplier '***qualifier-sum***' upon only that '***qualifier***' term of the multiplicand '***qualifier-sum***' whose subscript is the '***meristemal***' or *maximal prime number extant* in it [This is but a '***prime***' product-rule version of that principle applied in other versions of the $\underline{\mathbf{Q}}$ rules for the multiplication of '***qualifier-sum multiplicands***' by '***qualifier-sum multipliers***' -- the '*meristemal qualifier* ***principle***' *of* ***non-distributive multiplication***]. A "*prime number*", recall, is a "**N**atural **N**umber" ***divisible*** evenly [i.e., with **0** remainder] only by *itself* and by **1**. We have, given $\underline{\mathbf{x}}^1 = \hat{\mathfrak{q}}_1$:

$[\![\underline{\mathbf{x}}^1]\!]^2 = \hat{\mathfrak{q}}_1[\![\hat{\mathfrak{q}}_1]\!] = \hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_{\mathrm{s_p}(1)} = \hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2$ [via a '***meta-genealogical evolute product rule***' version of this rule];

$$[\![\underline{\mathbf{x}}^2]\!]^2 = [\![\hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2]\!]^2 = [\![\hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2]\!] \boxplus \hat{\mathfrak{q}}_1[\![\hat{\mathfrak{q}}_2]\!] \boxplus \hat{\mathfrak{q}}_2[\![\hat{\mathfrak{q}}_2]\!] = [\![\hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2]\!] \boxplus [\![\hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_{1\times 2}]\!] \boxplus [\![\hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_{\mathrm{s_p}(2)}]\!] = \hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_3;$$

$$[\![\underline{\mathbf{x}}^4]\!]^2 = [\![\hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_3]\!]^2 = [\![\hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_3]\!] \boxplus \hat{\mathfrak{q}}_1[\![\hat{\mathfrak{q}}_3]\!] \boxplus \hat{\mathfrak{q}}_2[\![\hat{\mathfrak{q}}_3]\!] \boxplus \hat{\mathfrak{q}}_3[\![\hat{\mathfrak{q}}_3]\!] = [\![\hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_3]\!] \boxplus [\![\hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_3 \boxplus \hat{\mathfrak{q}}_{1\times 3}]\!] \boxplus$$
$$[\![\hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_3 \boxplus \hat{\mathfrak{q}}_{2\times 3}]\!] \boxplus [\![\hat{\mathfrak{q}}_3 \boxplus \hat{\mathfrak{q}}_3 \boxplus \hat{\mathfrak{q}}_{\mathrm{s_p}(3)}]\!] = \hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_3 \boxplus \hat{\mathfrak{q}}_6 \boxplus \hat{\mathfrak{q}}_5;$$

$$[\![\underline{\mathbf{x}}^8]\!]^2 = [\![\hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_3 \boxplus \hat{\mathfrak{q}}_6 \boxplus \hat{\mathfrak{q}}_5]\!]^2 = [\![\hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_3 \boxplus \hat{\mathfrak{q}}_6 \boxplus \hat{\mathfrak{q}}_5]\!] \boxplus \hat{\mathfrak{q}}_1[\![\hat{\mathfrak{q}}_5]\!] \boxplus \hat{\mathfrak{q}}_2[\![\hat{\mathfrak{q}}_5]\!] \boxplus \hat{\mathfrak{q}}_3[\![\hat{\mathfrak{q}}_5]\!] \boxplus \hat{\mathfrak{q}}_6[\![\hat{\mathfrak{q}}_5]\!] \boxplus \hat{\mathfrak{q}}_5[\![\hat{\mathfrak{q}}_5]\!] =$$
$$[\![\hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_3 \boxplus \hat{\mathfrak{q}}_6 \boxplus \hat{\mathfrak{q}}_5]\!] \boxplus [\![\hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_5 \boxplus \hat{\mathfrak{q}}_{1\times 5}]\!] \boxplus [\![\hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_5 \boxplus \hat{\mathfrak{q}}_{2\times 5}]\!] \boxplus [\![\hat{\mathfrak{q}}_3 \boxplus \hat{\mathfrak{q}}_5 \boxplus \hat{\mathfrak{q}}_{3\times 5}]\!] \boxplus [\![\hat{\mathfrak{q}}_6 \boxplus \hat{\mathfrak{q}}_5 \boxplus \hat{\mathfrak{q}}_{6\times 5}]\!] \boxplus$$
$$[\![\hat{\mathfrak{q}}_5 \boxplus \hat{\mathfrak{q}}_5 \boxplus \hat{\mathfrak{q}}_{\mathrm{s_p}(5)}]\!] = \hat{\mathfrak{q}}_1 \boxplus \hat{\mathfrak{q}}_2 \boxplus \hat{\mathfrak{q}}_3 \boxplus \hat{\mathfrak{q}}_5 \boxplus \hat{\mathfrak{q}}_7 \boxplus \hat{\mathfrak{q}}_6 \boxplus \hat{\mathfrak{q}}_{10} \boxplus \hat{\mathfrak{q}}_{15} \boxplus \hat{\mathfrak{q}}_{30}; \ldots$$

__The *'Generic Simile'*: How the *'Meta-Analytical Meta-Geometry'* of $_{\mathbf{N}}\underline{\mathbf{Q}}$ Provides a *'Universal Metaphor'*, *'General Analog Computation'*, or *'Algorithmic Simulation'* for *'Qualo-Peanic Successions'*, i.e., for *'Dialectical Systems-Progressions'*.__ The efficacy of $\underline{\mathbf{Q}}$ as an *'Arithmetic of Dialectic'* -- as a '[*de*]*picto-ideography*' capable of 'modeling', of being "'interpreted for'", the dialectical processes of both the *«aisthetoi»* and the *«eide»* -- principally depends upon the degree of $\underline{\mathbf{Q}}$'s success in "'capturing'" algorithmically, procedurally, and *'ideo-picto-graphically'* -- syntactically, as well as semantically -- a *'Generic Simile'* or *'Universal Metaphor'*; a 'comprehensive analogy' or *'qualitative analog-computational simulation'* for the most general features of dialectical process in nature, including in the human mind, as embodied abstractly in the paradigm of the *'Qualo-Peanic Progression'* set out above. By *'dialectical processes in general'* here, we mean to reference the whole genus of ***self-generating, immanently-driven*** or ***self-reflexively self-driven***, *'auto-kinesic'* ***progressions of systems*** and of the associated *'system-ontology'* intended by the neologisms ***'meta-evolution'***, ***'meta-dynamasis'***, ***'meta-systemasis'***, and ***'onto-dynamasis'***. This section of *Supplement A.* is scheduled, in later editions, to summarily explore the ***'meta-analytical meta-geometry'*** of the $\underline{\mathbf{Q}}$ "'arithmetic'", "'algebra'", and "'calculus'"; the *'ideo-pictography'* of the $\underline{\mathbf{Q}}$ rules-system for its successive, $_{\mathbf{W}}\underline{\mathbf{Q}}$, $_{\mathbf{Z}}\underline{\mathbf{Q}}$, $_{\mathbf{Q}}\underline{\mathbf{Q}}$ and $_{\mathbf{R}}\underline{\mathbf{Q}}$ ***'ideo-onto-logical species'***, in terms of the fitness of their respective ***'meta-analytical meta-geometries'*** to serve as a generic, algorithmic, 'computative', and 'purely-qualitative', 'ideo-pictographic' *universal metaphor for dialectic*.

__'Re-Constructive' and *'Pre-Constructive'* [*Predictive*] Modeling Using the *'Intensional-Intuitional', Heuristic Algebra* of $_{\mathbf{N}}\underline{\mathbf{Q}}$.__ The $_{\mathbf{N}}\underline{\mathbf{Q}}$ arithmetic, and its algebra, *'assigned'* or *'interpreted'* for a given modeling application, is *'semantified'* via the ***connotations*** of 'abbreviative' or "syncopated", mnemonic symbols associated to its generic symbols. The $_{\mathbf{N}}\underline{\mathbf{Q}}$ calculus is, in that sense, an "'***intuitional***'", "'***intensional***'", and "'***heuristic***'", rather than an "'***extensional***'", calculus. Its symbols represent the 'intuitive intensions' or 'meanings' of, rather than "'extensions'" [exhaustive list-specifications of the elements or attributes of] the categories which they denote. The generic structure of, for instance, the evolute-product-rule variants of the $_{\mathbf{N}}\underline{\mathbf{Q}}$ arithmetic/algebra, encode and codify general principles, typically unnoted in their full generality, that may describe a vast diversity of historical processes, located within many different scales of the ***'temporal fractal structure'***, or ***'diachronic meta-fractal scaled self-similarity structure'***, of the 'Peanic' ***cosmological 'consecuum-cumulum'***. Examples of such general principles of ***'dialectical analogy'*** include: (**1**.) the "'***original accumulation***'" $\frac{\dagger}{\dagger}$ ***'reproductive accumulation'*** principle; (**2**.) the "'***formal subsumption***'" $\frac{\dagger}{\dagger}$ "'***real subsumption***'" principle, and; (**3**.) the "'***uneven and combined*** ['*hybrid*'] ***development***'" principle. Cognizance as to how each of these principles plays out in terms of the $_{\mathbf{N}}\underline{\mathbf{Q}}$ syntax may often lead to insights toward formation of hypotheses and models able to 'reconstruct' remote epochs of the history of humanity, and of the cosmos generally, otherwise virtually inaccessible, not only, of course, in terms of direct observation, but even in terms of otherwise readily-discernible 'mediated' or indirect observations of contemporary consequences of those past epochs' activity, conserved into present. The "'***original accumulation***'" $\frac{\dagger}{\dagger}$ ***'reproductive accumulation'*** principle will, in *Supplement B.*, be illustrated, to demonstrate this dimension of the scientific utility of the $_{\mathbf{N}}\underline{\mathbf{Q}}$ language, using the "'***original***'" or "'***primitive***'" versus ***'reproductive' accumulation*** of ***capital***, and of ***prokaryotic*** "***living organisms***". If the $_{\mathbf{N}}\underline{\mathbf{Q}}$ ***'algorithmic-heuristic'*** can shed light back upon the past, for model-based, *'retro-dictive'* *re*-construction of the structure of past ***'meta-states'*** of the cosmos, can it also, by the same -- or a similar -- token, cast light ahead, for the model-based and *pre*-dictive '*pre*-construction' of future ***'meta-states'*** of the cosmos, likely to arise if the ***actual*** 'self-iteration' of the cosmic ontology continues along its "'time-honored'" 'meta-fractal', '*«aufheben»*'-/'self-internalization'-generated, dialectical course? One can ***mechanically*** iterate forward the heuristically-represented ontology of any given $_{\mathbf{N}}\underline{\mathbf{Q}}$-formulated *universe-of-discourse*, about as far beyond the recognized representation of its present 'meta-state' as one might wish, exploiting the mechanism of the $_{\mathbf{N}}\underline{\mathbf{Q}}$ algorithm. The capability to ***interpret*** the ***meaning*** of the new symbols generated from the old in that way, denoting never-yet directly-experienced ***'meta-states'*** of that *universe*, is quite another matter. If one does not iterate too far beyond present experience -- say restricting one's ambition to the next epoch alone, hypotheses as to the possible future may be evoked by this ***'heuristic-organonic algebraic method'***, of ***'solving for the successor system'***. One such candidate hypothesis is a ***'heuristic derivation'*** of a relatively detailed, concretized concept of the next "***social relation of*** [*human-society self-re-*]***production***", successor to the ***capital-relation***, as the next subsuming organizing principle of human society, namely, ***'the social relation of generalized equity'***, with ***'externality-equity'*** as ***«arché»***, seeding a succeeding ***social*** [*re-*]***formation*** which we term ***'Equitism'*** or ***'Equitarian Society'***; 'solving' the partial dialectic:

$$\underline{\text{Capitalism}} \rightarrow \rightharpoondown\!\langle\, \underline{\text{Capitalism}} \,\rangle = \underline{\text{Capitalism}}\langle\, \underline{\text{Capitalism}} \,\rangle = \underline{\text{Capitalism}}\text{ 'of }\underline{\text{Capitalism}} = \langle\, \underline{\text{Capitalism}} \,\rangle^2 =$$

$$\langle\, \underline{\text{Capitalism}} \oplus \underline{\Delta}\langle\, \underline{\text{Capitalism}} \,\rangle \,\rangle = \langle\, \underline{\text{Capitalism}} \oplus \underline{\textit{Meta-Capitalism}} \,\rangle = \langle\, \underline{\text{Capitalism}} \oplus \underline{\textit{Equitism}} \,\rangle.$$

'Equitism' is both a *collective-property, **public-property instantiation of the Coase Theorem***, and an ultimate fruition of the "'***equity***'" or "'***equitable jurisprudence***'" ***tradition of law***, in contradistinction to the common law and statutory law traditions. ***'Equitism'*** is an immanent-/self-expansion of *joint-stock-company stockholder democracy* principles, inherent in the ***capital-relation***, to encompass the *'institution-ization'* of generalized, comprehensive "'***stakeholder democracy***'", starting with constitutional recognition of *a new ontological category* of equities, ***'externality equities'***. ***'Equitism'*** generalizes core, capital-equity logic to encompass ***economic democracy***, starting with *public, democratic econo-political governance* of capitalism's externalities. The ***'Equitarian Reforms'*** are a ***'constitutionalization'*** and ***'juridicalization'***, into social law, of an immanent critique, or self-critique, of capital, theoretical and practical. The institutional infrastructure of generalized equity is a scaled self-similarity structure, a *'synchronic meta-fractal'*, of *economic governance bodies*, based in *publicly-elected* ***public directors***, serving in the second ***houses*** of local ***bi-cameral boards of directors***, the ***public stakeholders'*** ***«camera»*** or ***externality-equities «camera»***, in all local enterprises with sufficient externalities impact; co-managing, with the traditional, ***internality-equities board***, the ***'externalities budgets'/operating plans*** of each such enterprise, with constitutionally and legislatively ceded co-authority to do so, *'adjudicate-able'/'arbitrate-able'* in case of deadlock. Arising therefrom, perhaps, at first, as extra-constitutional "NGOs": local, regional, national, and, eventually, global, *base-elected* ***associations of public directors***, coordinating externalities social management policy at *'meta-enterprise'* levels, constituting a ***fourth***, *'econo-political'*, ***branch*** of government, in ***sustained*** *'quadruple-power'* with ***«aufheben»***-*conserved/transformed executive, legislative, and judicial branches, with checks-and-balances between every pair of branches*, for a *human-geographical 'de-abstractification'/'re-determination* ***containment***' of abstract capital; an ***«aufheben»-conserving/-negating real subsumption*** of the ***capital-equity*** or ***'internality-equity'*** relation, its markets, and its "'market failures'", within the ***democratized relations of production*** of ***the generalized equity relation***, including all of the *new ontological classes* of '*non*-internality-equity' emergent from their ***'externality-equity'*** ***«arché»*** [*fifth model*].

Transition to *Supplement B*. ***Supplement B*.** of this primer presents dialectical ideography in use, via the construction of distinct ideographic '***dialectical models***' of both conceptual & physical dialectical processes, expressed in the algebras of either $\underline{\mathbf{Q}}$ or ${}_{a}\underline{\mu} \equiv \underline{\mathbf{q}}$:

1. An ${}_{\mathbf{N}}\underline{\mathbf{Q}}$ model of the 'Historical Dialectic' of human intellectual history -- of '"The History Of Ideas"' and of '"Ideologies"' -- in the sense of the dialectical, «*aufheben*» / evolute self-progression-of-emergence of the fundamental ***fields*** / *disciplines* of human intellectual inquiry, beginning with '**Mythopoeia**', or $\underline{\mathbf{M}}$, as «*arché*», thence progressing/'meta-fractally self-iterating' through formalized '**Religion**', or $\underline{\mathbf{R}}$, to '**Philosophy**', or $\underline{\mathbf{P}}$, to '**Science**', or $\underline{\mathbf{S}}$, ***plus their hybrids***, on to '**Psycho-History**', or $\underline{\Psi}$.
2. An ${}_{\mathbf{N}}\underline{\mathbf{Q}}$ model of the 'Historical Dialectic' of '*18th Century, mainly French Mechanical Materialism*' and of '*19th Century, mainly German Dialectical Idealism*', leading to the Marxian/Engelsian/Dietzgenian synthesis.
3. A 'Taxonomy Level **1**' ${}_{\mathbf{N}}\underline{\mathbf{Q}}$ model of the 'Historical Dialectic' of 'Natural History' itself, as a whole; i.e., of the 'Dialectic' of the Cosmological «***Physis***» -- '***The Dialectic of Nature***' itself -- as a 'meta-organic' dynamical and 'meta-dynamical' totality.
4. A 'Taxonomy Level **2**' ${}_{\mathbf{N}}\underline{\mathbf{Q}}$ model of the 'Historical Dialectic' of Human-Social Formation -- '***The Dialectic of Human Nature***', from the viewpoint of the historical-ontological self-progression of '***human socio-econo-political demography***'; a model of human social forma***tion*** as a self-progression of human-social formation***s***, grasped as '***meta-geological***' geographical ***formations***.
5. A 'Taxonomy Level **2**' ${}_{\mathbf{N}}\underline{\mathbf{Q}}$ model of the 'Historical Dialectic' of Human-Social Formation -- '***The Dialectic of Human Nature***', from the viewpoint of the historical '***meta-evolution***' of 'human-***social relations of*** [*human society self-re-*]***production***'.
6. A 'Taxonomy Level **2**' ${}_{\mathbf{N}}\underline{\mathbf{Q}}$ model of the 'Historical Dialectic' of Human-Social Formation -- '***The Dialectic of Human Nature***', from the viewpoint of the historical '***meta-evolution***' of 'human-***social forces of*** [*human society self-re-*]***production***', grasped via the psycho-historical order in which humanity is able to ***appropriate*** the ontology of nature generated in the ***pre-human***[oid] epochs of '*Cosmo-Auto-Genesis*' / '***Dialectic Of Nature***', which recapitulates the order of emergence of that ontology ***in reverse***.
7. An $\underline{\mathbf{q}}$ model of the "*micro-economic*", '*micro-historical* dialectic' of the '***meta-evolution***' of a capitalist firm / "*individual capital*".
8. The '[Meta-]Finitary ***Set Of All Sets*** 'Idea-Eventity' as *General Paradigm of Dialectics*, and *The Immanent Critique of Set Theory*.
9. An ${}_{\mathbf{N}}\underline{\mathbf{Q}}$ model of the 'Categorial Dialectic' of Book One of Hegel's «***Wissenschaft der Logik***», "***The Doctrine Of Being***".
10. An ${}_{\mathbf{N}}\underline{\mathbf{Q}}$ model of the 'Categorial Dialectic' of Volumes I - III of Marx's «***Das Kapital***»: "***A Critique Of Political Economy***".
11. A model of 'meta-Gödelian'/'meta-Platonic'/'meta-deductive' '***Meta-Axiomatics***' -- of the '*ideo-meta-evolutionary*' dialectical self-progression of Axioms-Systems, driven by the ineluctable Gödel self-incompleteness of each Axiom-System, arising as the immanence of *Diophantine-number-unsolvable Diophantine equations* within the *language* & *numbers-ontology* of each such System.

These models are all excerpts from a planned [meta-]systematic compendium of dialectical models, the ***Encyclopedia Dialectica***. ***Encyclopedia Dialectica*** is envisioned as a new kind of repository of human knowledge, in which each 'topic' or 'entry' is defined and '***modeled***' not only narratively, or 'phonogramically', but also '***ideo-picto-gram-ically***', that is, in which each '***ontological genus***' of '[ev]entities' covered by this ***Encyclopedia***, together with its principal '***ontological species and sub-species***', is described via a '[***meta-***]***systematic dialectical***', categorial-progression expository spectrum of increasingly-complex, increasingly-concrete, increasingly-specific, '***historical-dialectical models***' -- '[meta-]dynamical' and '[meta-]evolutionary' -- cast in each major language arising in the 'meta-systematic dialectical' progression of systems of dialectical ideography, such that each such successive '***historical-dialectical***' model provides representations of: (**1**) a reconstructed genesis of that eventity-category; (**2**) its present '[meta-]state'; (**3**) its 'preconstructed' future '[meta-]states', and; (**4**) its interconnexions with all other covered categories of eventities, to the extent *of current comprehension*, and *of the descriptive capacity native to each successive dialectical ideographic 'language'*, and in which the presentations of these '[ev]entity-category' dialectical models are ordered in their natural-historical order of appearance in the natural history of this cosmos. The degree of our fulfillment of this plan -- the remoulding of manifold research notes, drafts of special-topic monographs, and separate treatises into the integrated whole of this ***Encyclopedia*** -- will depend upon circumstances. It is a work that cannot be completed in any case, but which may be readily taken up, in the future, by others so moved, in «*aufheben*» relationship to the partially-completed contents, criteria, standards, and methods left behind. We have provided herein, on the following page, a '***Table of Similes***' juxtaposing four of the central '***historical-dialectical***' and '***metasystematic-dialectical***' models of ***Dialectical Ideography***, and of ***Encyclopedia Dialectica***, as a preview of the applications to be found in ***Supplement B***. Background on the notations employed in these models is provided below. Model **#3** addresses ***The [Historical] Dialectic Of Nature***, at **taxonomy level 1**, as summarized 'meta-temporally' via the 'self-iteration' formula $\langle\!\langle\, {}^{1}_{\forall}\underline{\mathbf{n}} \,\rangle\!\rangle^{2^{\tau\uparrow}}$. The '*pre-sub*script', in this case, '$\forall$', denotes the 'All'; the cosmological totality. The '*pre-super*script', **1**, denotes '**taxonomy level *one***', the universal level. The symbol '${}^{1}_{\forall}\underline{\mathbf{n}}$' intends the «*arché*» ontological category of '"pre-**n**uclear particles"', e.g., '"mesons"', plus everything else extant from that epoch of cosmological 'meta-evolution', and before, even if unknown to present science. Model **#5** addresses ***The [Historical] Dialectic Of The Human-Social Relations Of Production***, at **taxonomy level 2**, as summarized via the formula $\langle\!\langle\, {}^{2}_{h}\underline{\mathbf{A}} \,\rangle\!\rangle^{2^{\tau\uparrow}}$. The '*pre-sub*script', **h**, restricts the *universe of discourse* to a particular category within level **2**, namely, that of '${}^{1}_{\forall}\underline{\mathbf{h}}$', the level one ontological category of '**h**umanity'. The '*pre-super*script', **2**, denotes '**taxonomy level *two***'. Thus '${}^{2}_{h}$' as a whole designates the ***first*** level of ontological ***sub***-categories within level **1** 'onto' ${}^{1}_{\forall}\underline{\mathbf{h}}$. The symbol '${}^{2}_{h}\underline{\mathbf{A}}$' intends the «*arché*» *socio-ontological category* of the social relations of **h**uman-societal self-re-production, namely that of the '***nearly non***-production **A**ppropriation of raw products of nature by [proto-]**h**umans', with *minimal* human improvement *for* human consumption/'use-value-added'. The two '***Meta-Systematic Dialectical Models***' of the conceptual 'meta-evolutions' of systems of arithmetic are principally addressed in the book ***Dialectical Ideography*** -- in its Part II., for '***The Historical Dialectic Of The Standard Arithmetics***', and in its Part III., for '***The Meta-Systematic Dialectic Of Some Non-Standard Arithmetics***'. The denotations of the specific symbols $\underline{\mathbf{N}}$, $\underline{\mathbf{W}}$, $\underline{\mathbf{Z}}$, $\underline{\mathbf{Q}}$, and $\underline{\mathbf{R}}$, used in the former model, for the standard arithmetics, have already been suggested above. In general, we use symbols of the form $\underline{\mathbf{X}}$ to denote the *richer, vaster-in-scope* '***first-order***' specifications of the ***rules-systems*** for arithmetics [*of the corresponding number-spaces, denoted by symbols like* $\mathbf{X}$], which, being 'Gödel-incomplete' *only syntactically*, encompass "*Non-Standard Models*", and symbols like $\underline{\mathbf{X}}$ to denote narrower, '***first-&-higher-order***' specifications of the corresponding arithmetical rules-systems, which encompass only their "*Standard Models*" -- at the cost of 'complete' Gödel-incompleteness, *both* syntactical and semantical.

Table of Similes for the 3*rd* and 5*th* Dialectical Models of *Supplement B*. in Relation to Dialectical Models of the Self-Progression of the *Standard Arithmetics* versus of the *Dialectical Arithmetics*, for epochs $\tau=0$ to $\tau=5$

${}_{N}\underline{Q}$ *Generic Ontic Qualifiers'* Dialectic: Epoch τ Ideo-Phono-Gramic '*Character*-izations' of $[\![\hat{q}_1]\!]^{2^{\tau}}$				Model #3: *Dialectic Of Nature*, taxonomy level 1, $\langle {}^{1}_{\vee}\underline{n} \rangle^{2^{\tau}}$		Model #5: *Historical Dialectic Of Human-Social Relations Of Production*, taxonomy level 2, $\langle {}^{2}_{h}\underline{A} \rangle^{2^{\tau}}$		Model: *The Meta-Systematic Dialectic Of The Standard Arithmetics*, $(\underline{N})^{2^{\tau}}$		Model: *Meta-Systematic Dialectic Of The Dialectical Arithmetics*, $(\underline{N})^{2^{\tau}}$	
O	τ	Generic Qualifier	Narrative '*Character-ization*'	'*Character* -ization'	Narration	Char.	Narration	Char.	Narration	Char.	Narration
Σ	0	$\hat{q}_1$	first thesis, «*arché*» thesis, or initial thesis	${}^{1}_{\vee}\underline{n} = \hat{q}_n$	*sub-nuclear monads* [e.g., *mesons*]	${}^{2}_{h}\underline{A} = \hat{q}_A$	'*Predation-relation*'; raw Approp. of nature's products	$\underline{N} = \hat{q}_N$	*Rules-System of* "*Natural*" *Numbers*, $\mathbf{N} = \{1, 2, 3, ...\}$	$\underline{N} = \hat{q}_N$	*unqualified quantifiers*
	1	$\hat{q}_2$	first '*contra-thesis*'	${}^{1}_{\vee}\underline{s} = \hat{q}_{nn}$	*sub-atomic monads* [e.g., *protons*]	${}^{2}_{h}\underline{G} = \hat{q}_{AA}$	'*Goods-relation*'; deeper, re-/cross-Appropriation	$\underline{A} = \hat{q}_{NN}$	**0**, '*no units*' -- '*no monads*' -- *as a* '*count*'; **A**ught numbers	${}_{N}\underline{Q} = \hat{q}_{NN}$	*unquantifiable, ontic or ontological Qualifiers*
	2	$\hat{q}_3$	first full '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{sn}$	...*galaxies* [*starless*]...	${}^{2}_{h}\hat{q}_{GA}$	'*real subsumption*', **A** by **G**	$\underline{W} = \hat{q}_{AN}$	"**W**hole" Numbers Arithmetic	${}_{N}\underline{U} = \hat{q}_{QN}$	*ontic quanto-qualifiers*
		$\hat{q}_4$	second '*contra-thesis*'	${}^{1}_{\vee}\underline{a} = \hat{q}_{ss}$	*atomic monads*	${}^{2}_{h}\underline{C} = \hat{q}_{GG}$	*Commodity-relation*	$\underline{M} = \hat{q}_{AA}$	*Minus Numbers*, {...−2, −1}	${}_{N}\underline{M} = \hat{q}_{QQ}$	*Metrical qualifiers*
	3	$\hat{q}_5$	first partial '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{an}$	possible hybrid '*onto*'	${}^{2}_{h}\hat{q}_{CA}$	'*real subsumption*', **A** by **C**	$\hat{q}_{MN}$	{1, 2,...} as {+1, +2,...}	$\hat{q}_{MN}$	*quantifiable metricals*
		$\hat{q}_6$	second partial '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{as}$	possible hybrid '*onto*'	${}^{2}_{h}\hat{q}_{CG}$	'*real subsumption*', **G** by **C**	$\hat{q}_{MA}$	**0** translated/converted to **±0**	$\hat{q}_{MQ}$	*heterog.* **2**-*level subscr.*
		$\hat{q}_7$	second full '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{asn}$	...*stars* [*planetless*]...	${}^{2}_{h}\hat{q}_{CGA}$	'*real subsump.*', **G\A** by **C**	$\underline{Z} = \hat{q}_{MW}$	"Integers", {...−1, ±0, +1...}	${}_{N}\underline{\alpha} = \hat{q}_{MU}$	*state-variable qualifiers*
		$\hat{q}_8$	third '*contra-thesis*'	${}^{1}_{\vee}\underline{m} = \hat{q}_{aa}$	*molecular monads* [e.g., H_2O molecules]	${}^{2}_{h}\underline{M} = \hat{q}_{CC}$	*Money-relation*	$\underline{F} = \hat{q}_{MM}$	*Fractions*, {...−1/2...+1/2...} [com.-fragment sub-monads]	${}_{N}\underline{A} = \hat{q}_{MM}$	**A**: *metadynamical meta-system qualifiers*
	4	$\hat{q}_9$	third partial '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{mn}$	possible hybrid '*onto*'	${}^{2}_{h}\hat{q}_{MA}$	'*real subsumption*', **A** by **M**	$\hat{q}_{FN}$	{1, 2,...} as {1/1, 2/1,...}	$\hat{q}_{AN}$	*populations of systems*
		$\hat{q}_{10}$	fourth partial '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{ms}$	possible hybrid '*onto*'	${}^{2}_{h}\hat{q}_{MG}$	'*real subsumption*', **G** by **M**	$\hat{q}_{FA}$	**0** translated/converted to **0/1**	$\hat{q}_{AQ}$	**A** *w/het.* **2**-*level subscr.*
		$\hat{q}_{11}$	fifth partial '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{msn}$	possible hybrid '*onto*'	${}^{2}_{h}\hat{q}_{MGA}$	'*real subsump.*', **G\A** by **M**	$\hat{q}_{FW}$	{0, 1, 2,...}:{0/1,1/1,2/1,...}	$\hat{q}_{AU}$	*quanto-qualifier subscr.*
		$\hat{q}_{12}$	sixth partial '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{ma}$	..."'*molecular clouds*'"...	${}^{2}_{h}\hat{q}_{MC}$	'*real subsumption*', **C** by **M**	$\hat{q}_{FM}$	{−3/1, −2/1, −1/1...}	$\hat{q}_{AM}$	*metricals as subscripts*
		$\hat{q}_{13}$	seventh partial '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{man}$	possible hybrid '*onto*'	${}^{2}_{h}\hat{q}_{MCA}$	'*real subsump.*', **C\A** by **M**	$\hat{q}_{FMN}$	{+1/1, +2/1, +3/1...}	$\hat{q}_{AMN}$	*quantified metr. subscr.*
		$\hat{q}_{14}$	eighth partial '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{mas}$	possible hybrid '*onto*'	${}^{2}_{h}\hat{q}_{MCG}$	'*real subsump.*', **C\G** by **M**	$\hat{q}_{FMA}$	**±0** translated to **±0/1**	$\hat{q}_{AMQ}$	**A** *w/het.* **3**-*level subscr.*
		$\hat{q}_{15}$	third full '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{masn}$	...*proto-planetary disks*...	${}^{2}_{h}\hat{q}_{MCGA}$	'*real subsump.*', **C\G\A** by **M**	$\underline{Q} = \hat{q}_{FZ}$	"Rational" Arithmetic System	${}_{N}\underline{\beta} = \hat{q}_{A\alpha}$	**A** *w/state-vector subscr.*
		$\hat{q}_{16}$	fourth '*contra-thesis*'	${}^{1}_{\vee}\underline{p} = \hat{q}_{mm}$	*prokaryotic monads* [e.g., *bacteria*]	${}^{2}_{h}\underline{K} = \hat{q}_{MM}$	«*Kapital*»-*relation*	$\underline{D} = \hat{q}_{FF}$	'**D**iagonals'/"*Irrationals*" [incom.-fragment sub-units]	${}_{N}\underline{B} = \hat{q}_{AA}$	*super*^{1}-*system* / **B***i-level meta-system qualifiers*
	5	$\hat{q}_{17}$	ninth partial '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{pn}$	possible hybrid '*onto*'	${}^{2}_{h}\hat{q}_{KA}$	'*real subsumption*', **A** by **K**	$\hat{q}_{DN}$	{1.00..., 2.00..., 3.00..., ...}	$\hat{q}_{BN}$	*super-systems populs.*
		$\hat{q}_{18}$	tenth partial '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{ps}$	possible hybrid '*onto*'	${}^{2}_{h}\hat{q}_{KG}$	'*real subsumption*', **G** by **K**	$\hat{q}_{DA}$	**0** translated to **0.0000000...** .	$\hat{q}_{BQ}$	**B** *w/het.* **2**-*level subscr.*
		$\hat{q}_{19}$	eleventh partial '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{psn}$	possible hybrid '*onto*'	${}^{2}_{h}\hat{q}_{KGA}$	'*real subsump.*', **G\A** by **K**	$\hat{q}_{DW}$	{0.00..., 1.00..., 2.00..., ...}	$\hat{q}_{BU}$	*quanto-qualifier subscr.*
		$\hat{q}_{20}$	twelfth partial '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{pa}$	possible hybrid '*onto*'	${}^{2}_{h}\hat{q}_{KC}$	'*real subsumption*', **C** by **K**	$\hat{q}_{DM}$	{..., −3.000..., −2.000..., ...}	$\hat{q}_{BM}$	**B** *w/metrical subscr.*
		$\hat{q}_{21}$	thirteenth partial '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{pan}$	possible hybrid '*onto*'	${}^{2}_{h}\hat{q}_{KCA}$	'*real subsump.*', **C\A** by **K**	$\hat{q}_{DMN}$	{+1.000..., +2.000..., ...}	$\hat{q}_{BMN}$	*quantified metr. subscr.*
		$\hat{q}_{22}$	fourteenth partial '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{pas}$	possible hybrid '*onto*'	${}^{2}_{h}\hat{q}_{KCG}$	'*real subsump.*', **C\G** by **K**	$\hat{q}_{DMA}$	**±0** translated to **±0.000000...**	$\hat{q}_{BMQ}$	**B** *w/het.* **3**-*level subscr.*
		$\hat{q}_{23}$	fifteenth partial '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{pasn}$	possible hybrid '*onto*'	${}^{2}_{h}\hat{q}_{KCGA}$	'*real subsump.*', **C\G\A** by **K**	$\hat{q}_{DZ}$	{...,−1.0...,±0.0...,+1.0...,...}	$\hat{q}_{BMU}$	**B**: *subscr. state-vectors*
		$\hat{q}_{24}$	sixteenth partial '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{pm}$	possible hybrid '*onto*'	${}^{2}_{h}\hat{q}_{KM}$	'*real subsumption*', **M** by **K**	$\hat{q}_{DF}$	{...−0.50...±0.00...+0.50...}	$\hat{q}_{BA}$	**B**: *subscr. sys. qualifier*
		$\hat{q}_{25}$	seventeenth partial '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{pmn}$	possible hybrid '*onto*'	${}^{2}_{h}\hat{q}_{KMA}$	'*real subsump.*', **M\A** by **K**	$\hat{q}_{DFN}$	{1.000...2.000...3.000}	$\hat{q}_{BAN}$	**B**: *subscr. sys. populs.*
		$\hat{q}_{26}$	eighteenth partial '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{pms}$	possible hybrid '*onto*'	${}^{2}_{h}\hat{q}_{KMG}$	'*real subsump.*', **M\G** by **K**	$\hat{q}_{DFA}$	**0/1** translated to **0.00000...** .	$\hat{q}_{BAQ}$	*subs. sys. w/subs. quals.*
		$\hat{q}_{27}$	nineteenth partial '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{pmsn}$	possible hybrid '*onto*'	${}^{2}_{h}\hat{q}_{KMGA}$	'*real subsump.*', **M\G\A** by **K**	$\hat{q}_{DFW}$	{...0.000...1.000...2.000...}	$\hat{q}_{BAU}$	*sub. sub. quanto-quals.*
		$\hat{q}_{28}$	twentieth partial '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{pma}$	...*proto-stromatolites*...	${}^{2}_{h}\hat{q}_{KMC}$	'*real subsump.*', **M\C** by **K**	$\hat{q}_{DFM}$	{...−3.00...−2.00...−1.00...}	$\hat{q}_{BAM}$	*subs. subs. metr. quals.*
		$\hat{q}_{29}$	twenty-first partial '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{pman}$	possible hybrid '*onto*'	${}^{2}_{h}\hat{q}_{KMCA}$	'*real subsump.*', **M\C\A** by **K**	$\hat{q}_{DFMN}$	{...+1.00...+2.00...+3.00...}	$\hat{q}_{BAMN}$	*sub.*^{2} *mtr. quanto-quals.*
		$\hat{q}_{30}$	twenty-second partial '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{pmas}$	possible hybrid '*onto*'	${}^{2}_{h}\hat{q}_{KMCG}$	'*real subsump.*', **M\C\G** by **K**	$\hat{q}_{DFMA}$	**±0/1** translated to **±0.0000...**	$\hat{q}_{BAMQ}$	**B** *w/het.* **4**-*level subscr.*
		$\hat{q}_{31}$	fourth full '*uni-thesis*'	${}^{1}_{\vee}\hat{q}_{pmasn}$	...*primordial oceans*...	${}^{2}_{h}\hat{q}_{KMCGA}$	'*real subsump.*', **M\C\G\A** by **K**	$\underline{R} = \hat{q}_{DQ}$	System of "**R**eal" Arithmetic	${}_{N}\underline{\gamma} = \hat{q}_{B\beta}$	*multiple metadynamical meta-system subscripts*
		$\hat{q}_{32}$	fifth '*contra-thesis*'	${}^{1}_{\vee}\underline{e} = \hat{q}_{pp}$	*eukaryotic monads* [e.g., *algae*]	${}^{2}_{h}\underline{E} = \hat{q}_{KK}$	*Generalized-Equity-relation* ⊐'*externality-equities*'	$\underline{I} = \hat{q}_{DD}$	"**I***maginaries*": *monad* = i, ...(−1.0)i...(±0.0)i...(+1.0)i...	${}_{N}\underline{\Gamma} = \hat{q}_{BB}$	*super*^{2}-*system* / *tri-level meta-system qualifiers*