Encyclopedia Dialectica 'Fractment':

Generic Dialectical Interpretation of the ${}_{\mathbf{N}}\underline{\underline{\mathbf{Q}}}$ Ideography

---

Introduction: Characterization of $\underline{\underline{\mathbf{Q}}}$. The $\underline{\underline{\mathbf{Q}}}$ ideography as defined herein is an arithmetic of 'pure '*unquantifiable* ***qualifiers***', i.e., of '*non-addable*', or "*additively idempotent*" 'qualifiers'. Therein, $\underline{\underline{\mathbf{Q}}}$ contrasts with $\underline{\underline{\mathbf{N}}}$, the arithmetic or arithmetical ***rules-system*** of the "Natural" Numbers, denoted **N**, and defined [≡] by **N** ≡ **{1, 2, 3,...}**. This is because the $\underline{\underline{\mathbf{N}}}$ arithmetic is an ideography of '*pure, unqualified* ***quantifiers***', thus much the opposite of $\underline{\underline{\mathbf{Q}}}$. Nonetheless, the $\underline{\underline{\mathbf{Q}}}$ 'meta-numbers' conform to the same "first-order" axioms [the "Peano Postulates"] that were designed to characterize the "Natural" numbers alone. Indeed, the $\underline{\underline{\mathbf{Q}}}$ arithmetic is a "Non-Standard Model" of the "Natural" Numbers. The possibility of such "Non-Standard Models" is implied by the joint applicability of the Gödel Completeness and Incompleteness Theorems at the "first-order" level. Thus, $\underline{\underline{\mathbf{N}}}$ is pregnant with a 'self-duality', one whose range spans the opposition between 'pure quantifiers' and 'pure qualifiers'. Our symbol $\underline{\underline{\mathbf{N}}}$ denotes specifically the "first-order" rules-system of "Natural" arithmetic. "First-order" rules make assertions only about individual numbers or about variables that "range over" individual numbers. They do not 'detach', and make assertions about, 'characteristics' shared in common by specific sub-'populations' of numbers. We usually "interpret" these 'qualifiers', the 'meta-numerals' of the $\underline{\underline{\mathbf{Q}}}$ 'meta-arithmetic', i.e., we usually "assign" them, in our use of the $\underline{\underline{\mathbf{Q}}}$ language for psycho-historical modeling applications, as '***ontological***' qualifiers; as "*kind* of being" or "*kind* of idea" ***qualifiers***. The arithmetical ideography that we denote by ${}_{\mathbf{N}}\underline{\underline{\mathbf{Q}}}$ is the beginning version of the $\underline{\underline{\mathbf{Q}}}$ arithmetics. It is an "*intensional*" [onto-]*logical ideography*, like that of Boole. Its symbols denote "intensions", "meanings", or "memes". It is also a "heuristic" tool, for conceptual discovery and self-clarification. As the **N** in its 'pre-subscript' suggests, ${}_{\mathbf{N}}\underline{\underline{\mathbf{Q}}}$ denotes an 'meta-arithmetic' built on the **N** "Natural" numbers; an arithmetic of 'meta-numerals' made up out of **N**-numerals **{1, 2, 3,...}**, by a kind of '*self-subsumption*' of the $\underline{\underline{\mathbf{N}}}$ arithmetic: ${}_{\mathbf{N}}\underline{\underline{\mathbf{Q}}} \equiv \{ \mathbf{N}\hat{\underline{\mathbf{q}}}_{\mathbf{N}}{}^{\mathbf{N}} \}$. Given the 'non-addability' or *additive idempotency* and multiplicative closure of the $\hat{\underline{\mathbf{q}}}_{\mathbf{n}}$, the latter simplifies to $\{ (1)\hat{\underline{\mathbf{q}}}_{\mathbf{N}}{}^{(1)} \}$ and to $\{ \hat{\underline{\mathbf{q}}}_{\mathbf{N}}{}^{(1)} \}$, i.e., to ${}_{\mathbf{N}}\underline{\underline{\mathbf{Q}}} \equiv \{ \hat{\underline{\mathbf{q}}}_{\mathbf{N}} \}$ or ${}_{\mathbf{N}}\underline{\underline{\mathbf{Q}}} \equiv \{\hat{\underline{\mathbf{q}}}_1, \hat{\underline{\mathbf{q}}}_2, \hat{\underline{\mathbf{q}}}_3,...\}$. The $\hat{\underline{\mathbf{q}}}$ symbol is the core 'meta-numeral' ideogram for all of the $\underline{\underline{\mathbf{Q}}}$ 'meta-numbers', and stands for what they all have in common. The caret 'overscore' or "hat" ideogram-element, '^', signifies the 'unit' status of each $\underline{\underline{\mathbf{Q}}}$ 'unit-qualifier'; that its "vector length" or "modulus" is one standard unit of axial extent. The underscore, '_', signifies that each ${}_{\mathbf{N}}\underline{\underline{\mathbf{Q}}}$ unit is a '***qualifier***', or is 'Contra-Boolean' in its "squaring" or self-multiplicative behavior, i.e., that for all **n** in **N**, $\hat{\underline{\mathbf{q}}}_{\mathbf{n}}{}^{2} \neq \hat{\underline{\mathbf{q}}}_{\mathbf{n}}{}^{1} = \hat{\underline{\mathbf{q}}}_{\mathbf{n}}$, meaning that a ${}_{\mathbf{N}}\underline{\underline{\mathbf{Q}}}$-number squared differs from itself 'unsquared' not in the pure-quantitative sense $\hat{\underline{\mathbf{q}}}_{\mathbf{n}} \cdot \hat{\underline{\mathbf{q}}}_{\mathbf{n}} \gtrless \hat{\underline{\mathbf{q}}}_{\mathbf{n}}$, but in a *non-quantitative*, ***qualitative*** sense: $\hat{\underline{\mathbf{q}}}_{\mathbf{n}}\hat{\underline{\mathbf{q}}}_{\mathbf{n}} \not\gtrless \hat{\underline{\mathbf{q}}}_{\mathbf{n}}$, because $\hat{\underline{\mathbf{q}}}_{2\mathbf{n}} \perp \hat{\underline{\mathbf{q}}}_{\mathbf{n}}$. Via the product rules of ${}_{\mathbf{N}}\underline{\underline{\mathbf{Q}}}$, the ${}_{\mathbf{N}}\underline{\underline{\mathbf{Q}}}$ meta-number ***sequence of series*** is generated by $\{ [\hat{\underline{\mathbf{q}}}_1]^{\tau} \}$, $\tau \in \mathbf{N}$, and takes on its values in ascending ['time-like'] order, so that $[\hat{\underline{\mathbf{q}}}_1]^{\tau} = \underline{\sum}_{\mathbf{k}=1,\tau}\hat{\underline{\mathbf{q}}}_{\mathbf{k}}$, wherein the single-underscored sigma symbol signifies the operation of '*pure-qualitative summation*'.

Our standard, generic dialectical interpretation of the ${}_{\underline{N}}\underline{\underline{Q}}$ arithmetic is given below. This generic interpretation encompasses both 'diachronic' or "historical dialectics" and 'synchronic' or "systematic dialectics", as well as our own method of '*metasystem*atic dialectics', in which each category in such a '*metasystem*atic' categorial progression denotes an [ev]entity which is a 'system' in its own right, so that it is the '*metasystem*' [sub-]totality that determines, and that is also determined by, this progression of systems that is reconstructed in thought by such a dialectical categorial progression.

Generic Dialectical Interpretation of the Sequence: $[\,\hat{\mathfrak{q}}_1\,] \rightarrow [\,\hat{\mathfrak{q}}_1 + \hat{\mathfrak{q}}_2\,] \rightarrow [\,\hat{\mathfrak{q}}_1 + \hat{\mathfrak{q}}_2 + \hat{\mathfrak{q}}_3\,]$.

The interpretation below employs the syntax '**a** ↔ **b**' to signify the ***interpretation*** of **a** by **b**, or the *interpretive* ***assignment*** *of* **a** *to* **b**, the syntax '**x** → **y**' to signify 'meta-evolution', 'revolution', 'temporal implication', or '[self-]***becoming***', the 'essence-ial' or ***self***-transformation of **x** into **y**, and; the '+' sign generalized so as to encompass also '*non-amalgamative*', '***heterogeneous***', or '*poly-qualinomial*' addition, as in '**apples** + **oranges**', wherein summands do not "reduce" to single values, as they do in the more familiar cases of '***homogeneous***' addition, e.g., **1** + **2** = **3**, or **1·cm.** + **2·cm.** = **3·cm.**.

Our generic dialectical assignments of the first three ${}_{\underline{N}}\underline{\underline{Q}}$ meta-numerals, $\hat{\mathfrak{q}}_1$, $\hat{\mathfrak{q}}_2$ & $\hat{\mathfrak{q}}_3$, are as follows:

$[\,\hat{\mathfrak{q}}_1\,]^\tau$, $\tau = 1$: $[\,\hat{\mathfrak{q}}_1\,]^\tau = [\,\hat{\mathfrak{q}}_1\,]^1 = [\,\hat{\mathfrak{q}}_1\,] \leftrightarrow$ [**initiating posit**];
$[\,\hat{\mathfrak{q}}_2\,]^1 \leftrightarrow$ [**first contra-posit**] ≡ counter-posit to $\hat{\mathfrak{q}}_1$, or **contra**-$\hat{\mathfrak{q}}_1$, so --

$[\,\hat{\mathfrak{q}}_1\,]^\tau$, $\tau = 2$: $[\,\hat{\mathfrak{q}}_1\,]^\tau = [\,\hat{\mathfrak{q}}_1\,]^2 = [\,\hat{\mathfrak{q}}_1 + \hat{\mathfrak{q}}_2\,]^1 \leftrightarrow$ [**posit** + **contra-posit**] ≡ **first posit / contra-posit cumulum**;
$[\,\hat{\mathfrak{q}}_3\,]^1 = [\,\hat{\mathfrak{q}}_{2+1}\,] \leftrightarrow$ [**first hybrid-posit**] ≡ hybrid of $\hat{\mathfrak{q}}_2$ and $\hat{\mathfrak{q}}_1$, so --

$[\,\hat{\mathfrak{q}}_1\,]^\tau$, $\tau = 3$: $[\,\hat{\mathfrak{q}}_1\,]^\tau = [\,\hat{\mathfrak{q}}_1\,]^3 = [\,\hat{\mathfrak{q}}_1 + \hat{\mathfrak{q}}_2 + \hat{\mathfrak{q}}_3\,]^1 \leftrightarrow$ **first triad cumulum**, namely --
[**posit** + **contra-posit** + **hybrid-posit**].

Contexts of systematic-dialectical exposition of an argument or theory brook the further interpretation:
$[\,\hat{\mathfrak{q}}_1\,]^\tau$, $\tau = 1$: $[\,\hat{\mathfrak{q}}_1\,]^1 = [\,\hat{\mathfrak{q}}_1\,] \leftrightarrow$ [**thesis**];
$[\,\hat{\mathfrak{q}}_1\,]^\tau$, $\tau = 2$: $[\,\hat{\mathfrak{q}}_1\,]^2 = [\,\hat{\mathfrak{q}}_1 + \hat{\mathfrak{q}}_2\,] \leftrightarrow$ [**thesis** + **antithesis**];
$[\,\hat{\mathfrak{q}}_1\,]^\tau$, $\tau = 3$: $[\,\hat{\mathfrak{q}}_1\,]^3 = [\,\hat{\mathfrak{q}}_1 + \hat{\mathfrak{q}}_2 + \hat{\mathfrak{q}}_3\,] \leftrightarrow$ [**thesis** + **antithesis** + **synthesis**].

'***Metafractal***-scaling cumula', or 'qualitative self-similarity regresses' are historical and conceptual progressions which involve ever higher unit[ie]s, 'meta-unities' or 'meta-monads' made up out of previously emerged 'units' or 'monads', e.g., molecules as 'meta-atoms' made up out of atoms, etc. Note: '***Monad***' connotes not just "***unit***" or "***element***" but also ["holonomic"] ***microcosm*** of the macrocosm / totality as well [where the "hol" is also '*inside*' each part of itself]:
$[\,\hat{\mathfrak{q}}_1\,]^\tau$, $\tau = 1$: $[\,\hat{\mathfrak{q}}_1\,]^1 = [\,\hat{\mathfrak{q}}_1\,] \leftrightarrow$ [**monad**];
$[\,\hat{\mathfrak{q}}_1\,]^\tau$, $\tau = 2$: $[\,\hat{\mathfrak{q}}_1\,]^2 = [\,\hat{\mathfrak{q}}_1 + \hat{\mathfrak{q}}_2\,] \leftrightarrow$ [**monad** + **meta-monad**];
$[\,\hat{\mathfrak{q}}_1\,]^\tau$, $\tau = 3$: $[\,\hat{\mathfrak{q}}_1\,]^3 = [\,\hat{\mathfrak{q}}_1 + \hat{\mathfrak{q}}_2 + \hat{\mathfrak{q}}_3\,] \leftrightarrow$ [**monad** + **meta-monad** + **monads' hybrid**].

Tony Smith describes the Hegelian "systematic dialectics" of *categories* and of the exposition of dialectical theories of experienced [sub-]totalities by means of *dialectical categorial progressions* in the following terms: "...Hegel attempted to provide an immanent ordering of the basic categories ... To see this we have first to consider what a category is. It is a principle (a universal) for unifying a manifold of some sort or other (different individuals, or particulars). A category thus articulates a structure with two poles, a pole of unity and a pole of differences. In Hegelian language this sort of structure, captured in some category, can be described as a unity of identity in difference, or as a reconciliation of universal and individuals. From this general notion of a category we can go on to derive three general types of categorial structures. In one the moment of unity is stressed, with the moment of differences implicit. In another the moment of differences is emphasized, with the moment of unity now being only implicit. In a third both unity and differences are made explicit together. Hegel's next claim is that there is a systematic order immanently connecting these three categorial structures. A structure of unity in which differences are merely implicit is ***simpler*** than one in which these differences are explicitly introduced; and one in which both unity and differences are explicit is yet ***more complex*** still. Similarly, the first sort of structure is the ***most abstract*** [least-specified; least "determinate" -- **F.E.D.**], while the other structures are successively ***more concrete*** [via additional "specifications" or "determinations" -- **F.E.D.**]. ... If a category is in general a principle that unifies a manifold, then if a specific category only explicates the moment of unity, leaving the moment of difference implicit, then there is a "contradiction" between what it inherently is *qua* category (a unifier of a manifold) and what it is explicitly (the moment of unity alone). Overcoming this contradiction requires that the initial category be "negated" in the sense that a second category must be formulated that makes the moment of difference explicit. But when this is done the moment of difference will be emphasized at the cost of having the moment of unity made merely implicit. Once again there is a contradiction between what a category inherently is and what it is explicitly. Overcoming this contradiction demands that the second sort of category also be negated and replaced with a category in which both poles, unity and difference, are each made explicit simultaneously. Hegel is well aware that "contradiction" and "negation" are not being used here in the sense given to them in formal logic. Following a tradition that goes back to Plato, he asserts that in the above usage "contradiction" and "negation" are logical operators for ordering categories systematically, as opposed to logical operators for making formal inferences. The logic with which we are concerned here is *dialectical logic*. ... The "negation" of the ***simple unity*** is the moment of difference that it itself contains implicitly. ... But this stage of difference is itself one-sided and partial. ... When ***the stage of difference*** is dialectically negated, we once again have a category of unity, but now it is a ***complex unity***, one that incorporates the moment of difference ... Since a category of unity-in-difference on one level can itself prove to be a category of simple unity from a higher level perspective, thereby initiating another dialectical progression from unity through difference to unity-in-difference, we can construct a systematic theory of categories by employing the dialectical method. In this sort of theory we move in a step-by-step fashion from ***simple*** and ***abstract*** categories to those that are ***complex*** and ***concrete***, with dialectical logic providing the warrant for each transition." [T. Smith, *The Logic of Marx's Capital*, SUNY Press [NY: 1990], pp. 5-7, ***bold italics emphasis*** by **F.E.D.**]. Smith then quotes one of Hegel's own accounts of this dialectic of categorial cognition: "The ***determinateness*** which was a result is itself, by virtue of the form of simplicity into which it was withdrawn, a fresh beginning; as this beginning is distinguished from its predecessor precisely by that determinateness, cognition rolls onwards from content to content. First of all, this advance is determined as beginning from simple ***determinatenesses***, the succeeding ones becoming ever *richer and more concrete*. For the result contains its beginning and its course has enriched it by a ***fresh determinateness*** ... at each stage of its ***further determination*** it raises the entire mass of its preceding content, and by its dialectical advance it not only does not lose anything or leave anything behind, but carries along with it all it has gained and inwardly enriches and consolidates itself." [***emphasis added***].

The above-stated description leads to the following interpretation in terms of its categorial typology:

$[\,\hat{\underline{q}}_1\,]^{\tau},\ \tau = 1$: $[\,\hat{\underline{q}}_1\,]^1 = [\,\hat{\underline{q}}_1\,] \quad \leftrightarrow \quad$ [**simple unity**];

$[\,\hat{\underline{q}}_1\,]^{\tau},\ \tau = 2$: $[\,\hat{\underline{q}}_1\,]^2 = [\,\hat{\underline{q}}_1 + \hat{\underline{q}}_2\,] \quad \leftrightarrow \quad$ [**simple unity** + **simple difference**];

$[\,\hat{\underline{q}}_1\,]^{\tau},\ \tau = 3$: $[\,\hat{\underline{q}}_1\,]^3 = [\,\hat{\underline{q}}_1 + \hat{\underline{q}}_2 + \hat{\underline{q}}_3\,] \quad \leftrightarrow \quad$ [**unity** + **difference** + **complex unity**].

Algorithmic, Syntactical Rules for the $_{\underline{\underline{N}}}\underline{\underline{Q}}$ Arithmetic of Dialectics. The interested reader can replicate the above-asserted 'pure-qualitative calculations' of the expansions of $\hat{\underline{q}}_1^{\,2}$ and $\hat{\underline{q}}_1^{\,3}$ by applying the following five rules of the rules-system of the $_{\underline{\underline{N}}}\underline{\underline{Q}}$ ideography. For all numbers **j**, **k**, **m**, **n**, τ in the 'space' of the **N**aturals, or, in ideograms, $[\,\forall\ \mathbf{j, k, m, n,}\ \tau \in \mathbf{N}\,]$, those rules are:

1. $[\,[[\,\mathbf{j} \neq \mathbf{k}\,]$ i.e., $[\,\mathbf{j} \gtrless \mathbf{k}\,]]\ \Rightarrow\ [[\,\hat{\underline{q}}_j \gtrless \hat{\underline{q}}_k\,]\ \&\ [\,\hat{\underline{q}}_j \perp \hat{\underline{q}}_k\,]]\,]$, with '$\Rightarrow$' for '*formally implies*'; i.e., distinct subscripts signify qualitatively distinct 'meta-numbers', mutually-perpendicular 'meta-vectors' ['dialectors']; ontologically distinct individual identities; distinct, mutually-orthogonal unit ray-segments;

2. $[\,[\,\mathbf{j} \gtrless \mathbf{k}\,]\ \Rightarrow\ [[\nexists \ell \in \mathbf{N}]\ |\ \hat{\underline{q}}_j + \hat{\underline{q}}_k = \hat{\underline{q}}_\ell\,]]\,]$, with '$\nexists$' for '*does not exist*' & '|' for '*such that*'; i.e., sums of qualitatively distinct meta-numbers are irreducible; such 'heterogeneous' or 'inhomogeneous' qualifier-sums or 'poly-qualinomial' cumulum-sums do not 'amalgamate' to single meta-number values in $_{\underline{\underline{N}}}\underline{\underline{Q}}$, as do sums of distinct 'pure-quantifier' numbers in $\underline{\underline{\mathbf{N}}}$, e.g., $(\mathbf{1} + \mathbf{2}) = \mathbf{3}$.

3. $[\,\hat{\underline{q}}_j + \hat{\underline{q}}_j = \hat{\underline{q}}_j\,]$,
i.e., the meta-numbers of $_{\underline{\underline{N}}}\underline{\underline{Q}}$ are "additively idempotent" as are the 'logical numbers' of the contemporary Boolean algebra of logic: $(\mathbf{0} + \mathbf{0}) = \mathbf{0}$ and $(\mathbf{1} + \mathbf{1}) = \mathbf{1}$. Thus, $_{\underline{\underline{N}}}\underline{\underline{Q}}$ meta-numbers are 'non-additive' or 'unquantifiable'. They manifest in 'unitary presence' or 'unitarity of presence'; i.e., in non-multiplicity, or in 'ontological / categorial parsimony' vs. in pleonasm or 'redundancy'.

4. The «***aufheben***» '***evolute***' ***product*** rule of non-distributive, 'nonlinear' multiplication, in which the multiplicand [the argument of the function, the operand, the starting point of the process or 'calculative *movement*'] always adds-back as part of the product and thus re-appears in that product, ***conserving*** the thing "negated" in the result of the "negation"-operation. Given $[\,\mathbf{j} < \mathbf{k} < \mathbf{m} < \mathbf{n}\,]$:

$$[\,\hat{\underline{q}}_j \cdot \hat{\underline{q}}_k \equiv \hat{\underline{q}}_j \times \hat{\underline{q}}_k \equiv \hat{\underline{q}}_j[\,\hat{\underline{q}}_k\,] \equiv \hat{\underline{q}}_j \text{ "of" } \hat{\underline{q}}_k \equiv \hat{\underline{q}}_j\hat{\underline{q}}_k = [\,\hat{\underline{q}}_k + \hat{\underline{q}}_{j+k}\,]\,];$$

$$[\,\hat{\underline{q}}_k \cdot \hat{\underline{q}}_k \equiv \hat{\underline{q}}_k \times \hat{\underline{q}}_k \equiv \hat{\underline{q}}_k[\,\hat{\underline{q}}_k\,] \equiv \hat{\underline{q}}_k \text{ "of" } \hat{\underline{q}}_k \equiv \hat{\underline{q}}_k\hat{\underline{q}}_k = [\,\hat{\underline{q}}_k + \hat{\underline{q}}_{k+k}\,] = [\,\hat{\underline{q}}_k + \hat{\underline{q}}_{2k}\,]\,];$$

$$\hat{\underline{q}}_j[\,\hat{\underline{q}}_k + \hat{\underline{q}}_m\,] = [\,[\hat{\underline{q}}_k + \hat{\underline{q}}_m\,] + \hat{\underline{q}}_j[\,\hat{\underline{q}}_k\,] + \hat{\underline{q}}_j[\,\hat{\underline{q}}_m\,]\,] = [\,\hat{\underline{q}}_k + \hat{\underline{q}}_m + \hat{\underline{q}}_{j+k} + \hat{\underline{q}}_{j+m}\,];$$

$$[\,\hat{\underline{q}}_k + \hat{\underline{q}}_m\,][\,\hat{\underline{q}}_k + \hat{\underline{q}}_m\,] = [\,[\hat{\underline{q}}_k + \hat{\underline{q}}_m\,] + \hat{\underline{q}}_m[\,\hat{\underline{q}}_k\,] + \hat{\underline{q}}_m[\,\hat{\underline{q}}_m\,]\,] = [\,\hat{\underline{q}}_k + \hat{\underline{q}}_m + \hat{\underline{q}}_{m+k} + \hat{\underline{q}}_{2m}\,];$$

$$[\,\hat{\underline{q}}_k + \hat{\underline{q}}_j\,][\,\hat{\underline{q}}_m + \hat{\underline{q}}_n\,] = [\,[\hat{\underline{q}}_m + \hat{\underline{q}}_n\,] + \hat{\underline{q}}_k[\,\hat{\underline{q}}_m\,] + \hat{\underline{q}}_k[\,\hat{\underline{q}}_n\,]\,] = [\,\hat{\underline{q}}_m + \hat{\underline{q}}_n + \hat{\underline{q}}_{k+m} + \hat{\underline{q}}_{k+n}\,].$$

5. $[\hat{\mathbf{q}}_1]^{\tau+1} \equiv [\hat{\mathbf{q}}_1]^1 \times [\hat{\mathbf{q}}_1]^{\tau}$, but not generally $[\hat{\mathbf{q}}_1]^{\tau} \times [\hat{\mathbf{q}}_1]^1$.

By these rules --

$$[\hat{\mathbf{q}}_1]^1 \equiv \hat{\mathbf{q}}_1^{\,1} \equiv \hat{\mathbf{q}}_1;$$

$$[\hat{\mathbf{q}}_1]^2 = \hat{\mathbf{q}}_1[\hat{\mathbf{q}}_1] = [\hat{\mathbf{q}}_1 + \hat{\mathbf{q}}_{1+1}] = [\hat{\mathbf{q}}_1 + \hat{\mathbf{q}}_2];$$

$$[\hat{\mathbf{q}}_1]^3 = \hat{\mathbf{q}}_1[\hat{\mathbf{q}}_1 + \hat{\mathbf{q}}_2] = [[\hat{\mathbf{q}}_1 + \hat{\mathbf{q}}_2] + \hat{\mathbf{q}}_1[\hat{\mathbf{q}}_1] + \hat{\mathbf{q}}_1[\hat{\mathbf{q}}_2]]$$

$$= [\hat{\mathbf{q}}_1 + \hat{\mathbf{q}}_2 + \hat{\mathbf{q}}_2 + \hat{\mathbf{q}}_3] = [\hat{\mathbf{q}}_1 + \hat{\mathbf{q}}_2 + \hat{\mathbf{q}}_3];$$

so $[\hat{\mathbf{q}}_1]^1 \rightarrow [\hat{\mathbf{q}}_1]^2 \rightarrow [\hat{\mathbf{q}}_1]^3$ means $[\hat{\mathbf{q}}_1] \rightarrow [\hat{\mathbf{q}}_1 + \hat{\mathbf{q}}_2] \rightarrow [\hat{\mathbf{q}}_1 + \hat{\mathbf{q}}_2 + \hat{\mathbf{q}}_3]$.

The $\hat{\mathbf{q}}_k$ don't stop at $\hat{\mathbf{q}}_3$. What, then, are our '*generic* dialectical interpretations' for $\hat{\mathbf{q}}_4$ and beyond? We address this question below by moving from "*genus*" to "*species*", i.e., from *generic* to *specific* interpretations of $_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{Q}}}$, via two illustrations of "models" or *specific* applications of $_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{Q}}}$; two exemplary meta-system models of historical sequences / conceptual progressions of systems ***of ideas***. The first example will also enable us to evoke an 'essence-ial' definition of '***Psycho-History***', as an outcome of an immanent critique of 'the pre-psycho-historical' Sciences. After assimilating the data of the experience of these two, disparate '***species***', readers may attain a better "feeling" for the dialectical '***genus***' of each of $_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{Q}}}$'s $\hat{\mathbf{q}}_k$s, $\mathbf{k} > \mathbf{3}$.

A "cook book" catalog and tabular rendition of the generic dialectical meanings of $_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{Q}}}$ meta-numerals, $\hat{\mathbf{q}}_1$ through $\hat{\mathbf{q}}_{16}$, is provided in the last two pages of this 'Fractment', and readers are welcome to skip to it immediately now. We have provided the lengthy intervening material on the two 'specific' examples that follow for those readers who seek a deeper understanding of the uses of this 'Dialectical Arithmetic'. A study of these two living examples of the application of its method may serve them well.

Example A: Toward A $\underline{\underline{\mathbf{Q}}}$ Dialectical Model of the "History of Ideas" / "History of Disciplines". The 'resolution' or 'acuity' of such a model will depend crucially on the 'partition principle', the 'ontological taxonomy', the 'framework of categorization' into which the modeler chooses to sub-divide the chosen universe of discourse. Suppose we apply the $_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{Q}}}$ language to model the 'ideo-phenomenology' of the 'ideo-ontology' of the ***cognitive*** psycho-history of humanity in terms of the "History Of Ideas", and in terms of the historical order of emergence of the broad disciplinary categories or "fields" of idea-systems. That is, suppose we posit a primitive undifferentiated unity of explanation-making, theory-making, or story-creation, i.e., the largely oral and even 'gestural' category of "Mythology" or "Mythopoieia", as the «***arché***» proto-field or proto-discipline category of human ideation, followed [so far] by doctrinal, codified, scriptural and institutionalized "Religion", then by "Philosophy", and then by "Science" --

$\hat{\underline{q}}_1 \leftrightarrow \underline{\mathbf{M}} \equiv$ Mythology or Mythopoieia, stipulated starting point, or «***arché***»;

$\hat{\underline{q}}_2 \leftrightarrow \underline{\mathbf{R}} \equiv$ Religion, a 'meta-Mythology' made up out of multiple Mythologies;

$\hat{\underline{q}}_4 \leftrightarrow \underline{\mathbf{P}} \equiv$ Philosophy, a 'meta-Religion', made up out of multiple Religions;

$\hat{\underline{q}}_8 \leftrightarrow \underline{\mathbf{S}} \equiv$ Science, a 'meta-Philosophy', made up out of multiple Philosophies.

We then have the categorial progression $[\,\hat{\underline{q}}_1\,]^{\tau} \leftrightarrow [\,\underline{\mathbf{M}}\,]^{\tau}$ as --

$$[\,\hat{\underline{q}}_1\,]^1 = [\,\hat{\underline{q}}_1\,] \leftrightarrow [\,\underline{\mathbf{M}}\,];$$

$$[\,\hat{\underline{q}}_1\,]^2 = [\,\hat{\underline{q}}_1 + \hat{\underline{q}}_2\,] \leftrightarrow [\,\underline{\mathbf{M}} + \underline{\mathbf{R}}\,];$$

$$[\,\hat{\underline{q}}_1\,]^3 = [\,\hat{\underline{q}}_1 + \hat{\underline{q}}_2 + \hat{\underline{q}}_3\,] \leftrightarrow [\,\underline{\mathbf{M}} + \underline{\mathbf{R}} + \hat{\underline{q}}_{RM}\,];$$

$$[\,\hat{\underline{q}}_1\,]^4 = [\,\hat{\underline{q}}_1 + \hat{\underline{q}}_2 + \hat{\underline{q}}_3 + \hat{\underline{q}}_4\,] \leftrightarrow [\,\underline{\mathbf{M}} + \underline{\mathbf{R}} + \hat{\underline{q}}_{RM} + \underline{\mathbf{P}}\,];$$

$$[\,\hat{\underline{q}}_1\,]^5 = [\,\hat{\underline{q}}_1 + \hat{\underline{q}}_2 + \hat{\underline{q}}_3 + \hat{\underline{q}}_4 + \hat{\underline{q}}_5\,] \leftrightarrow [\,\underline{\mathbf{M}} + \underline{\mathbf{R}} + \hat{\underline{q}}_{RM} + \underline{\mathbf{P}} + \hat{\underline{q}}_{PM}\,];$$

$$[\,\hat{\underline{q}}_1\,]^6 = [\,\hat{\underline{q}}_1 + \hat{\underline{q}}_2 + \hat{\underline{q}}_3 + \hat{\underline{q}}_4 + \hat{\underline{q}}_5 + \hat{\underline{q}}_6\,] \leftrightarrow [\,\underline{\mathbf{M}} + \underline{\mathbf{R}} + \hat{\underline{q}}_{RM} + \underline{\mathbf{P}} + \hat{\underline{q}}_{PM} + \hat{\underline{q}}_{PR}\,];$$

$$[\,\hat{\underline{q}}_1\,]^7 = [\,\hat{\underline{q}}_1 + \hat{\underline{q}}_2 + \hat{\underline{q}}_3 + \hat{\underline{q}}_4 + \hat{\underline{q}}_5 + \hat{\underline{q}}_6 + \hat{\underline{q}}_7\,] \leftrightarrow [\,\underline{\mathbf{M}} + \underline{\mathbf{R}} + \hat{\underline{q}}_{RM} + \underline{\mathbf{P}} + \hat{\underline{q}}_{PM} + \hat{\underline{q}}_{PR} + \hat{\underline{q}}_{PRM}\,];$$

$$[\,\hat{\underline{q}}_1\,]^8 \leftrightarrow [\,\underline{\mathbf{M}} + \underline{\mathbf{R}} + \hat{\underline{q}}_{RM} + \underline{\mathbf{P}} + \hat{\underline{q}}_{PM} + \hat{\underline{q}}_{PR} + \hat{\underline{q}}_{PRM} + \underline{\mathbf{S}}\,];$$

$$[\,\hat{\underline{q}}_1\,]^9 \leftrightarrow [\,\underline{\mathbf{M}} + \underline{\mathbf{R}} + \hat{\underline{q}}_{RM} + \underline{\mathbf{P}} + \hat{\underline{q}}_{PM} + \hat{\underline{q}}_{PR} + \hat{\underline{q}}_{PRM} + \underline{\mathbf{S}} + \hat{\underline{q}}_{SM}\,];$$

$$[\,\hat{\underline{q}}_1\,]^{10} \leftrightarrow [\,\underline{\mathbf{M}} + \underline{\mathbf{R}} + \hat{\underline{q}}_{RM} + \underline{\mathbf{P}} + \hat{\underline{q}}_{PM} + \hat{\underline{q}}_{PR} + \hat{\underline{q}}_{PRM} + \underline{\mathbf{S}} + \hat{\underline{q}}_{SM} + \hat{\underline{q}}_{SR}\,];$$

$$[\,\hat{\underline{q}}_1\,]^{11} \leftrightarrow [\,\underline{\mathbf{M}} + \underline{\mathbf{R}} + \hat{\underline{q}}_{RM} + \underline{\mathbf{P}} + \hat{\underline{q}}_{PM} + \hat{\underline{q}}_{PR} + \hat{\underline{q}}_{PRM} + \underline{\mathbf{S}} + \hat{\underline{q}}_{SM} + \hat{\underline{q}}_{SR} + \hat{\underline{q}}_{SRM}\,];$$

$$[\,\hat{\underline{q}}_1\,]^{12} \leftrightarrow [\,\underline{\mathbf{M}} + \underline{\mathbf{R}} + \hat{\underline{q}}_{RM} + \underline{\mathbf{P}} + \hat{\underline{q}}_{PM} + \hat{\underline{q}}_{PR} + \hat{\underline{q}}_{PRM} + \underline{\mathbf{S}} + \hat{\underline{q}}_{SM} + \hat{\underline{q}}_{SR} + \hat{\underline{q}}_{SRM} + \hat{\underline{q}}_{SP}\,];$$

$$[\,\hat{\underline{q}}_1\,]^{13} \leftrightarrow [\,\underline{\mathbf{M}} + \underline{\mathbf{R}} + \hat{\underline{q}}_{RM} + \underline{\mathbf{P}} + \hat{\underline{q}}_{PM} + \hat{\underline{q}}_{PR} + \hat{\underline{q}}_{PRM} + \underline{\mathbf{S}} + \hat{\underline{q}}_{SM} + \hat{\underline{q}}_{SR} + \hat{\underline{q}}_{SRM} + \hat{\underline{q}}_{SP} + \hat{\underline{q}}_{SPM}\,];$$

$$[\,\hat{\underline{q}}_1\,]^{14} \leftrightarrow [\,\underline{\mathbf{M}} + \underline{\mathbf{R}} + \hat{\underline{q}}_{RM} + \underline{\mathbf{P}} + \hat{\underline{q}}_{PM} + \hat{\underline{q}}_{PR} + \hat{\underline{q}}_{PRM} + \underline{\mathbf{S}} + \hat{\underline{q}}_{SM} + \hat{\underline{q}}_{SR} + \hat{\underline{q}}_{SRM} + \hat{\underline{q}}_{SP} + \hat{\underline{q}}_{SPM} + \hat{\underline{q}}_{SPR}\,];$$

$$[\,\hat{\underline{q}}_1\,]^{15} \leftrightarrow [\,\underline{\mathbf{M}} + \underline{\mathbf{R}} + \hat{\underline{q}}_{RM} + \underline{\mathbf{P}} + \hat{\underline{q}}_{PM} + \hat{\underline{q}}_{PR} + \hat{\underline{q}}_{PRM} + \underline{\mathbf{S}} + \hat{\underline{q}}_{SM} + \hat{\underline{q}}_{SR} + \hat{\underline{q}}_{SRM} + \hat{\underline{q}}_{SP} + \hat{\underline{q}}_{SPM} + \hat{\underline{q}}_{SPR} + \hat{\underline{q}}_{SPRM}\,];$$

$$[\,\hat{\underline{q}}_1\,]^{16} \leftrightarrow [\,\underline{\mathbf{M}} + \underline{\mathbf{R}} + \hat{\underline{q}}_{RM} + \underline{\mathbf{P}} + \hat{\underline{q}}_{PM} + \hat{\underline{q}}_{PR} + \hat{\underline{q}}_{PRM} + \underline{\mathbf{S}} + \hat{\underline{q}}_{SM} + \hat{\underline{q}}_{SR} + \hat{\underline{q}}_{SRM} + \hat{\underline{q}}_{SP} + \hat{\underline{q}}_{SPM} + \hat{\underline{q}}_{SPR} + \hat{\underline{q}}_{SPRM} + \underline{\Psi}?\,];\ \ldots$$

Note the «***aufheben***» structure or '***evolute***' structure of the above, ever-accumulating multi-ideo-ontic '***cumula***' or 'non-amalgamative sums' of categories of 'ideo-ontology'. An "***evolute***" spiral shell, as it grows, grows up out of the level of its original growth, into new, higher levels, so that its preceding growth is not hidden from horizontal view by its succeeding growth, as it is in a "***convolute***" spiral shell, in which all growth takes place on the same, original level, the later 'whorls' thus covering over the earlier 'whorls' if viewed horizontally.

In the categorial progression which is ideographically 'notated', 'algebraicized', or 'character-ized' above, the new does not 'cover over' the old, but ***"at each stage of its further determination it raises the entire mass of its preceding content, and by its dialectical advance it not only does not lose anything or leave anything behind, but carries along with it all it has gained and inwardly enriches and consolidates itself"*** [Hegel]. The old is fully conserved, at least in *possibility*, if not always fully in actuality. For we interpret the spaces modeled by the $_{\underline{N}}\underline{\underline{\mathbf{Q}}}$ language as ***Possibility Spaces***, not ***Actuality Spaces***. That $\hat{\underline{\mathbf{q}}}_{\mathbf{RM}}$ is first extant in the progression in step $\tau = \mathbf{3}$, means that [finite] manifestation of the 'intension' or category of ideo-ontology signified by $\hat{\underline{\mathbf{q}}}_{\mathbf{RM}}$ is ***Possible*** in steps $\tau \geq \mathbf{3}$, though not in steps $\tau < \mathbf{3}$, but does not guarantee that this possibility will be ***actualized*** in a given historical instantiation of $[\,\underline{\mathbf{M}}\,]^{\tau}$. This model suggests an agenda for a historiography that it could serve: to attempt to map the authors, schools, and intellectual movements of the Terran past to the hybrid and non-hybrid terms and categories of the $[\,\underline{\mathbf{M}}\,]^{\tau}$expansion. The first historical transitions, per this model, were those from the oral/dance story-making and collective story-telling of "***Mythology***" or "popular religion", to the separation of elite[-only] literacy-based, bureaucratized, institutionalized, ritualized, 'literaturized', codified, sacred-text-based, text-fetishistic, doctrinaire, dogmatic, inquisitorial ***Religion***. They are, for most of us, mostly lost to detailed knowledge, lost in the mists of our deep, proto-literate past. So we pass over them, for now, per the purpose of this 'fractment', in our narrative gloss, here, on this 'ideo-ontological', ideo-onto-*dynamical*', ideographical 'model of ideo-history':

$$[\,\underline{\mathbf{M}}\,]^{1} \rightarrow [\,\underline{\mathbf{M}}\,]^{2} \rightarrow [\,\underline{\mathbf{M}}\,]^{3} \text{ or } [\,\underline{\mathbf{M}}\,] \rightarrow [\,\underline{\mathbf{M}} + \underline{\mathbf{R}}\,] \rightarrow [\,\underline{\mathbf{M}} + \underline{\mathbf{R}} + \hat{\underline{\mathbf{q}}}_{\mathbf{RM}}\,] \leftrightarrow$$

$$[\,\hat{\underline{\mathbf{q}}}_{1}\,]^{1} \rightarrow [\,\hat{\underline{\mathbf{q}}}_{1}\,]^{2} \rightarrow [\,\hat{\underline{\mathbf{q}}}_{1}\,]^{3} \text{ or } [\,\hat{\underline{\mathbf{q}}}_{1}\,] \rightarrow [\,\hat{\underline{\mathbf{q}}}_{1} + \hat{\underline{\mathbf{q}}}_{2}\,] \rightarrow [\,\hat{\underline{\mathbf{q}}}_{1} + \hat{\underline{\mathbf{q}}}_{2} + \hat{\underline{\mathbf{q}}}_{3}\,].$$

But consider the next spate of transitions. Per this model, ***Philosophy*** arises as the antithesis to the whole cumulum of these first transitions. Recall that many of the ancient Greek philosophers, including Socrates, were executed or exiled on charges including "atheism" against the ancient pagan 'mytho-religion' of the Greek gods. This model calls for the later emergence or 'extantcy' of a category of syntheses of ***Philosophy*** and ***Religion***, denoted $\hat{\underline{\mathbf{q}}}_{\mathbf{PR}} \leftrightarrow \hat{\underline{\mathbf{q}}}_{\mathbf{6}}$. ¿*Perhaps* the *Summa Theologiae* doctrines of Thomas Aquinas fit into this category? After the syntheses of mytho-religion and philosophy, this model calls for the emergence of a new antithesis category, ***Science***, or $\underline{\mathbf{S}} = \underline{\Delta\mathbf{P}} = \underline{\Delta\Delta\mathbf{R}} = \underline{\Delta\Delta\Delta\mathbf{M}}$. The model then asserts the subsequent emergence of seven categories of 'partial synthesis' or 'partial hybridization' of these partitions of ideo-ontology, culminating in a 'grand synthesis' of ***Mythopoieia***, ***Religion***, ***Philosophy***, and ***Science***, denoted $\hat{\underline{\mathbf{q}}}_{\mathbf{SPRM}} \leftrightarrow \hat{\underline{\mathbf{q}}}_{\mathbf{15}}$, which is, no doubt, not yet fully extant even in our own time. Beyond that, if this progression continues, the model predicts, in step **16**, $[\,\underline{\mathbf{M}}\,]^{16}$, the emergence of a new antithesis category, field, or discipline of human cognition, denoted with a '***?***' above, and equal to $\underline{\Delta^{1}\mathbf{S}} = \underline{\Delta^{2}\mathbf{P}} = \underline{\Delta^{3}\mathbf{R}} = \underline{\Delta^{4}\mathbf{M}}$. Is the envisionment of this category of the psycho-history of human thought so far beyond the horizon of our times as to make its symbol in this model, $\underline{\Delta\mathbf{S}}$, uninterpretable by us -- beyond our ken? Or does $\underline{\Delta\mathbf{S}} = \underline{\Psi}$, denoting the field of ***Psycho-History*** itself -- the comprehensive 'self-***historicization***' of all of the sciences, including mathematics, and including the science of the human psyche, thus also coupled with the comprehensive 'self-***psychologization***' of all of the sciences; the rigorous accounting for the "lawfully" developing 'ingredience' of historically-developing collective-subjective human ***mentality*** in all of the theoretical products of humans? The former, 'self-historicization' hypothesis is grounded in a broad-based, growing trend within recent human thought, as traced in such studies as *The Discovery Of Time* by Toulmin and Goodfield.

Marx and Engels wrote, in *The German Ideology*, "We know only a single science, the science of history". In *Capital*, Marx wrote of "historically specific" social-evolutionary "laws of motion", valid only within the limits of one epoch of social 'meta-evolution' -- not the same before that epoch begins, nor after it ends. He thus sees the "laws" of [social] change as self-forming within a historical and 'meta-evolutionary' self-change of "laws", and, moreover, within a '***meta-lawful***' and a ***qualitative*** self-change of [social] "laws" from one such social-evolutionary epoch to the next, not just a ***quantitative*** change of "laws". In the *Grundrisse*, Marx wrote of the "material force" of human social conscience ['*con-scientia*'], of humanity's collective 'psyche-ology': "with the slave's awareness that he *cannot be the property of another*, with his consciousness of himself as a person, the existence of slavery becomes a merely artificial, vegetative existence, and ceases to be able to prevail as the basis of production." [*Grundrisse: Foundations of the Critique of Political Economy*, Pelican [London: 1973], p. 463]. In the *Economic-Philosophic Manuscripts* of 1844, Marx wrote "It can be seen that the history of *industry* and industry as it *objectively* exists is an *open* book of the *human faculties*, and a human psychology which can be sensuously apprehended. This history has not so far been conceived in relation to human nature, but only from a superficial utilitarian point of view, since in the condition of alienation it was only possible to conceive real human faculties and *human* species-action in the form of general human existence, as religion, or as history in its abstract, general aspect as politics, art and literature, etc. *Everyday material industry*...shows us, in the form of *sensuous useful objects*,... the *essential human faculties* transformed into objects. No psychology for which this book, i.e., the most tangible and accessible part of history, remains closed, can become a *real* science with a genuine content." Today's official Natural Science retains the 'Parmenidean hangover' of viewing natural regularities, natural "laws", and most natural conditions, as fixed and static -- the latter being an assumption to which even Einstein succumbed in what he later assessed to be his "greatest blunder": adding the "cosmological constant" to his equations for General Relativity to 'staticize' its model universe because, without it, they ***predicted*** the "expanding universe" later encountered observationally by Hubble. Official Science assumes a time-independent 'repeatability' of experiments, with the same average results. But what if the "laws" of nature, and, more specifically, their universal "constants", like Newton's **G**, are actually variables, ***functions of history***, and thus change with time [a possibility of ***quantitative*** change of natural "laws" which Dirac and Poincaré explored], even if over time-scales mostly beyond human observation to-date? "Mach's Principle" held the inertia of each body to result from its gravitic interaction with ***the totality of mass-bodies*** in the universe, thus changing as ***a function of the history of the universe*** if that history involved movements sufficiently redistributing those mass-bodies. Volterra formulated integral equations which predicted the next state of a system as a function of its entire history, of the integration of all of its past states. The phenomenon of "hysteresis", or history-dependent reactivity, in which a system's further reactions depend upon its history of previous reactions, received attention. Perhaps the assumption of memory-less "state-determinism" in the state-spaces of contemporary dynamical systems theory, that two systems of the same kind, attaining a given state, must have the same next state, no matter how different their past state-trajectories; that state-space trajectories can never cross, is inadequate. Perhaps the apparent indeterminacy of next states in Quantum Mechanics is an artefact of that theory's assumptions of linearity and memory-less state-determinism? Perhaps the apparent "***randomicity***" of "quantum" behavior ***is actually its 'historicity'*** -- due to the impact of the differing cumula of the past "entanglements" of "identical" "quanta" on their present/recent behavior? [Super-]String theory, one of the leading contemporary candidates for the "Unified Field Theory" of all physical forces, makes gradual quantitative change of physical "laws" an inherent feature of its model of the universe, via the "dilaton field". Meanwhile, on the more philosophical frontiers of the natural and social sciences, Benedetto Croce held that all knowledge is historical knowledge, that historical knowledge is complete knowledge. Bergson argued for a new, less abstract, more 'contental' conception of time, which he termed "concrete duration". Chardin argued that all phenomena investigated by science must henceforth be viewed «*sub specie evolutionis*». The "time reversible" character of the equations which formulate the "laws" of nature in physics -- despite the apparently irreversible character of macroscopic process and macroscopic time ["time's arrow"] as humans experience it -- came under increasingly critical scrutiny, by Blum, Reichenbach, Prigogine, Davies, and many others.

For us, the phenomenon of '**onto-dynamasis**' forms the key consideration with respect to the 'quanto-***qualitative***' evolution and 'meta-evolution" of the "laws" of nature, including of the "laws" of human-social nature and its evolution and 'meta-evolution' within the dynamical and 'meta-dynamical' historical totality of nature. An escalating expansion of ***qualitative*** "degrees of freedom", of ***behavioral possibilities***, of the ***kinds of action*** available, accumulates with the net ***self-expansion of the ontology*** of the cosmos in the past that humanity has reconstructed from its present evidence. The historical progression from a cosmos populated by eventities organized only up to the level of "sub-nuclear particles", to one that includes also "sub-atomic particles", to one that includes also "atoms", to one that includes also "molecules", to one that includes also "prokaryotic cells", to one that includes also "eukaryotic cells", to one that includes also "multicellular organisms", to one that includes also "animal societies", to one that includes also "humanoid meta-societies"...qualitatively expands the universe of "events", "processes" and "activities" which "laws" of nature must describe.

"OK", you might say, "all of the above is an argument for the self-transformation of Science in the sense of its comprehensive 'self-***historicization***'. That's an argument for the "***History***" component in $\underline{\Delta \mathbf{S}} = \underline{\Psi} \equiv$ **Psycho-*History***. But where does the "Psyche" come into it?

We hold that $\underline{\Psi} \equiv$ **Psycho-History** must denote a single science, a ***science of history*** that encompasses ***the history of science***, i.e., that includes ***a science of the history of science***, a '***science of the sciences***' as '***meta-science***', '***science squared***' or '***science of the second degree***', employing the material of the present, the '***pre***-sent'; the material "sent from the past", to rigorously '***re-construct***' the past and to '***pre-construct***' the future, and thus also ***a science of human subjectivity***, of '**subject-*hood***', of '**subject-*ness***', and of the "***objectifications***" thereof, and this in a double sense --

(**1**) **Psycho-History** -- with respect to the future, and to those portions of the past during which *the history of* ***human nature*** has been part of *the history of* ***nature*** -- must account for this history of human subjects, the activity of humanity, as part of the history and the activity of cosmic Nature as a totality. There human subjectivity is part of the side of the observed and theorized as well as of the side of the observers and the theorizers.

(**2**) Yet even when considering scientific theories of contemporary *extra*-human nature, as well as of pre-*homo-sapiens* 'Natural History', i.e., of *pre*-human nature, where human societies and human subjects are not among the objects, not part of the side of the objectivity being theorized -- observed or reconstructed -- nevertheless, ***psyches are involved***: human subjectivity remains on the side of the observors and theorizers. A true **Psycho-History** must account for the human -- the human 'subject-ive', socio-psychological-cultural, historical-cognitive-developmental, '***ideo-ontodynamical***', and ideological -- ingredience; the ingredience of historically 'meta-evolving' human ***mentality***, in the scientific theories that humans make.

The history of human ideas, the history of human knowledge, the history of science -- of this ***humanity's*** sciences -- is part of the "historical material". Human subjectivity, via its objectifications, is empirical, is part of the data, part of the "external objectivity", the "objective world" of the past which a true **Psycho-History** must 're-construct', and part of the potential "objective world" of the future which such a **Psycho-History** must 'pre-construct'. We may also glimpse, in these considerations, why a ***strategic*** function is essential to any true **Psycho-History**, worthy of the name -- the formation, implementation, and on-going mid-course correction of a multi-generational ***strategy for the human species***, and its «***sequelae***»; a strategy which, in scope, is global, and, indeed, cosmological***!***

A dialectical universe -- an «***aufheben***», 'evolute', *cumulative* and '***onto-dynamic***' universe -- is one in which exact re-'petition' is impossible. Second and later 'petition' can only occur in a cosmos already altered by the impact of first 'petition'. Thus, the assumptions of current Science regarding experimental results' "replication" and "reproducibility' are only expected in short-term approximation. In a dialectical universe -- a self-moving, self-developing, self-reflexive, self-refl*ux*ive universe -- novelty, new '[ev]entities', new 'monads', new ontology often arises by means of the '***meta-fractal-ogenic***' processes that we term '***self-incorporation***', '***self-internalization***', or '***self-subsumption***'. These are the processes, of '***self-surroundment***', or '***self-environment***', of '***self-densification***', and of '***self-confrontation***', by which populations of atoms, in the process of expandedly reproducing themselves, also form molecules -- '***meta-atoms***' made up out of atoms; by which populations of molecules, in expandedly reproducing themselves, also form prokaryotic cells -- '***meta-molecules***' made up out of molecules; by which populations of prokaryotic cells, expandedly reproducing themselves, also form eukaryotic cells -- '***meta-prokaryotes***' made up out of prokaryotes; by which populations of eukaryotic cells, expandedly reproducing themselves, also form multicellular organisms -- '***meta-eukaryotes***' made up out of eukaryotes; by which populations of 'meta-biotic', multicellular organisms, the "metazoa", expandedly reproducing themselves, also form animal societies -- '***meta-meta-zoa***' made up out of 'meta-zoa'; and by which populations of animal societies, expandedly reproducing themselves, also form 'proto-humanoid societies' -- '***meta-societies***' made up out of 'inter-symbiotic', 'mutually-domesticating' and gradually 'self-domesticating' multicellular animal societies and multicellular plant communities, the matrix within which humanoid language and self-awareness come to birth. Such 'meta-fractals', or scale-regressing quanto-***qualitative*** self-similarity structures, exhibit scale-regression both synchronically and diachronically, both spatially and temporally. Synchronically, this scaling results from 'uneven and combined meta-evolution', and the diffusion of new developments, including 'hybridizations' with old, across the full spectrum of regions of differential rates of advancement, from its regions of first-emergence to its regions of last-emergence. Diachronically, this scaling manifests in '***temporal acceleration***', the ever-shorter durations separating successive epochs of self-incorporation. For the abstract, absolutist, reified, and fetishized notions of "Time" still prevailing in official Science, the concept of non-constant celerity of time, **dt/dt ≠ const..**, is senseless. However, a concept of '***temporal acceleration***' becomes possible when one defines time concretely, identifying it with localizations of the ensemble of the concerted cosmic activities of all [ev]entities. A core historical intellectual enterprise of humanity, namely, the 'meta-evolution' of mathematics, also exhibits, this time in the "internal", mental world of 'subjective objectivity', of inter-subjective, 'psyche-ological', mathematical 'idea-objects', this scaling, '***meta-fractal***', qualitative, 'ideo-ontological' ***self-similarity structure***, in the form of the progression of "logical types", of '***meta-sets***' made up out of sets [of sets of sets...], as set forth abstractly and ahistorically by Kurt Gödel, in his account of the completeness-incompleteness 'meta-dynamic'. Each given formal 'epoch' of arithmetical axiomatic-systems' 'meta-evolution' exhibits logical incompleteness, as revealed by the "'well-formed" 'formability', within it, of unsolvable equations, mapping to true 'theorems', ***not provable from its axioms***, which are propositions asserting that unsolvability. 'Self-incorporations' of the sets encoding the kinds of number extant in that system, to form new 'meta-sets' [of sets...] -- new '***ideo***-ontology' -- encode ***new kinds of numbers***, basing *new, additional axioms*, rendering those 'theorems' provable via the new axioms. But the resulting, new, qualitatively different, '***ideo-ontologically*** *different*', axiomatic system will always have its own, new unsolvable equations, and their corresponding 'unprovable theorems', so that the self-incorporation of sets, the formation of new kinds of numbers, and of new axioms, hence of new axiomatic systems, is always driven to re-ensue anew. The result is a '***meta-fractal progression***', and a '***meta-fractal ideo-cumulum***', of mathematical systems, i.e., an ever 'ideo-ontologically' self-expanding mathematical '***ideo-meta-system***', made up out of these successive systems; out of their '***cumulum***'.

This abstracted process, '***The Gödelian Dialectic***', or '***The Gödelian Ideo-Metadynamic***', calls out to be mapped onto actual human psycho-history. That task is taken up, in its arithmetical part, in Part II. of *Dialectical Ideography*, in the section entitled *The Meta-Evolution of Arithmetics*. It thus also addresses '*The Psycho-History of Arithmetics*'. The scientific theories produced by humanity are also 'psycho-artefacts', reflecting humanity's developing subjectivity, i.e., reflecting 'human natural history' as the history of the self-development of human[ized] nature, including of human «***mentalité***». A 'subjectless' objectivity, observation of a pure objectivity or purely-objective world without any observor; an objectivity by itself, devoid of any observing subject(s) as (a) part(s) thereof, does not exist as a generator of scientific theories. Nor is the detailed conceptual anatomy of any major scientific theory any sort of one-to-one mapping of the observations, the data that the theorizers observe; any sort of uniquely determined, empirically-prescribed formulation, forced in the last detail by the objectivity they measure and record. Qualitatively different, "Non-Standard" alternatives are always available. Scientific theories always embody choices and creative conceptual features which can only be accounted for psycho-historically. The subjectivity-projection of the social self-identity of the observor is party to what that observer perceives, thus in that observer's theoretical accounts of the objectivity being theorized. The observer partly constitutes that which the observer observes. And 'scientific' theories to date always conserve in themselves, however unconsciously, some aspect of their ultimate «***arché***» in **M**, in myth-making, in explanation by story-telling, always remaining partly infected with "ideology" in Marx's sense, with unconscious social-relations-reflecting ideological content. We need an explicit $\underline{\Psi}$ for many reasons, including to aid our cognizance of this fact. The very ***possibility*** of a given scientific theory, including of consensus assent to it within the scientific community, presupposes a requisite cognitive readiness in the communities of human theory-makers, and in their surrounding social audiences. It is not only the genomic, genotypic limitations and specificities of human theorizers, which, we hold, do not vary very systematically across human history to-date, that impact the formation of scientific theories and their histories. It is also their culturally-acquired, phenomic/phenotypic, 'meme-nomic'/'memetic' limitations and specificities, which do vary, in a broadly cumulative and progressive fashion, along the diachronic direction. It is 'constitutively' unlikely, in the history of mathematical theories, for the system of the "Complex" numbers to arise before the system of the positive *and negative* "Rationals", or for the "Rational" numbers' system to arise before that of the "Natural numbers"; no more likely than is the appearance of a prokaryotic cell prior to that of the molecules out of which it is constituted. Likewise, there is a necessary but more subtle 'constitutive' ordering in the succession of scientific theories. A theory of gravity like Einstein's General Relativity is unlikely to arise prior to one like that of Newton, or even prior to a field theory of electromagnetism like that of Maxwell. This may be due, in part, *but only in part*, to the new mathematical tools that undergird many advances in scientific theory, given the 'cumulative' character of mathematical advance as briefed above. 'Meta-scientific' explanation of this '***ideo-meta-genealogy***' of scientific theories belongs partly to '***memetics***', thus also partly to **Psycho-History**. The psycho-historical, cognitive readinesses for scientific revolution/'meta-evolution', addressed here, are also a function of an overall psycho-historical '[meta]-evolution' of collective human ***mentality***. Mentalities are, in part, a function of the instrumentalities they employ, beginning with the human body itself. New theory formation is driven partly by theory/data discrepancies revealed by more accurate measurements enabled by the innovation of more sensitive instruments, driven, in turn, partly by the deepening needs of human social-reproductive praxis, especially under prodding by the profit-motive within the epoch of the competition-driven capital-praxis. But new instrumentalities have qualitative as well as quantitative repercussions. Marx wrote the following on this 'self-refluxive' nature of human social-reproductive activity: "The act of reproduction itself changes not only the objective conditions -- e.g., transforming village into town, the wilderness into agricultural clearings, etc. -- but the producers change with it, by the emergence of new qualities, by transforming and developing themselves in production, forming new powers and new conceptions, new modes of intercourse, new needs, and new speech." [E. Hobsbawm, translator, *Pre-Capitalist Economic Formations*, International Publishers [NY: 1965], p. 93].

"Technology", which, as Marx held [see earlier quote], is also a reflection, an extension, an outering, an externalization, and an explicitization -- an '***object**-ification*' -- of the human psyche, and thus of human 'psyche-ology', also plays a key role in the historical formation of new cognitive readinesses for theoretical advance, its use reacting back upon the mentalities which fashion and use it, changing them further, in ways such as those explored by Innis, McLuhan, Logan, and Schmandt-Besserat.

The social state / stage of human subjectivity unconsciously projects itself into and encrypts itself in the theories it produces. The character of the 'monads' we see in, ascribe to, or conceptually / mentally carve out of nature are not given uniquely in the phenomenologies as recorded by our experiments and instruments. The 'monadizations' in our models of nature also, in part, reflect our own prevailing 'self-monadizations', our own 'self-models' or internal 'models-of-self', the character of our own social self-identities. Consider the social-self's self-atomization in the various stages of exchange-value-induced dissolution of the primordial kinship-based-communities, up to and including its historical extremity in the capital-nexus and the «***bellum omnium contra omnes***» of our 'alienation-based' [selling-based], ever-more "de-tribalized" and even 'de-family-ized' society. We hold that this self atomization gives rise, in part, to the various versions of reductionist atomism. Our own internal, semi-conscious sense of self-identity is our inescapable model for all of the identities that we perceive in the external worlds of our interpretations of our experience. $\underline{\Psi}$ is needed to help found a scientific and trans-scientific understanding of the history of $\underline{\mathbf{S}}$ itself, for all of these reasons. The extreme 'ideality", the high degree of idealization and schematism of this $[\ \underline{\mathbf{M}}\ ]^{\tau}$ model brooks great homeomorphic defect when it is compared against, for example, the observed [reconstructed] intellectual psycho-history of Occidental humanity. This is in part because that history exhibits, not a single progression from $\underline{\mathbf{M}}$ to $\underline{\mathbf{M}} + \underline{\mathbf{R}}$ to $\underline{\mathbf{M}} + \underline{\mathbf{R}} + \hat{\underline{\mathbf{q}}}_{RM}$ to $\underline{\mathbf{M}} + \underline{\mathbf{R}} + \hat{\underline{\mathbf{q}}}_{RM} + \underline{\mathbf{P}} + \hat{\underline{\mathbf{q}}}_{PM} + \ldots + \underline{\mathbf{S}} + \ldots$, but rather a broken progression, collapsed and interrupted, then later resumed. It exhibits instead a progression from $[\ \underline{\mathbf{M}}\ ]^{1}$ through $[\ \underline{\mathbf{M}}\ ]^{8}$, followed by a catastrophic interruption of positive or expanding social reproduction, and a prolonged descent into negative or contracting social reproduction, via the Roman-imperialization of Christianity, the violent, genocidal suppression of ancient pan-Hellenic "pagan" learning, the fall of the Roman empire, and the ≈**1000**-year Inquisition and "Dark Age" that followed. The $[\ \underline{\mathbf{M}} + \underline{\mathbf{R}} + \hat{\underline{\mathbf{q}}}_{RM} + \underline{\mathbf{P}} + \hat{\underline{\mathbf{q}}}_{PM} + \hat{\underline{\mathbf{q}}}_{PR} + \hat{\underline{\mathbf{q}}}_{PRM} + \underline{\mathbf{S}}\ ]$ 'cumulum' of Antiquity lapsed back into a profoundly retrograde version of $[\ \underline{\mathbf{M}} + \underline{\mathbf{R}} + \hat{\underline{\mathbf{q}}}_{RM}\ ]$, if, predominantly, into a new development within $\underline{\mathbf{R}}$, that of state-power-wielding, theocratic-totalitarian pseudo-Christianity. Only with restoration of acceleratedly expanding social reproduction, and with the easing of the "Dark Age" into the "Dim Age" of the European pre-Renaissance and beyond, did a second wave of $[\ \underline{\mathbf{M}}\ ]^{4+}$ take-off. More adequate models require ontological "interaction terms", denoting hybrid ontological formations, for the interaction of re-emergent Western European $\underline{\mathbf{P}}$ and incipient $\underline{\mathbf{S}}$ and their hybrids with those of the Islamic civilization, plus with those of the fossilized, preserved remains of the Hellenic civilization of Mediterranean antiquity which informed and inspired both. That added complexity renders this example unsuitable to be further pursued within the purposes of this 'Fractment'. So we move on to a second example, 'closer to hand'. This second example arises via the '*self evidence*' of the ${}_{\underline{N}}\underline{\underline{\mathbf{Q}}}$ system of arithmetic, *the evidence of this system itself* in the context of its dialectical antecedents and consequents, and thus in the context of the arithmetical 'meta-system' of which it forms a part. I.e., we next construct an abridgement of our ${}_{\underline{N}}\underline{\underline{\mathbf{Q}}}$ dialectical model of a pedagogical, expository dialectical progression of systems of dialectical arithmetic that includes the ${}_{\underline{N}}\underline{\underline{\mathbf{Q}}}$ system itself, and beyond.

Example B: Self-Included Modeling [A $\underline{\underline{\mathbf{Q}}}$ Model of the Dialectic of the Dialectical Arithmetics Including $\underline{\underline{\mathbf{Q}}}$]. Below we model that *progression of categories* which is also the *progression of systems of arithmetic* that constitute the arithmetical 'meta-system' which we term '***dialectical ideography***'. This model plays out not in the actual, historical ***order of discovery*** of these systems, but in a pedagogically-designed, programmed and rehearsed 'micro-history'; an expositorily-advantaged ***order of presentation***. We thus construct a ['meta-']systematic dialectic of the dialectical arithmetics. We construct it and present it via a *specific interpretation* or application of the '*heuristic intensional calculus*' of ${}_{\underline{\underline{\mathbf{N}}}}\underline{\underline{\mathbf{Q}}}$, ${}_{\underline{\underline{\mathbf{N}}}}\underline{\underline{\mathbf{Q}}}$ being also one of the systems / categories in the categorial progression / systems progression of this [meta-]systematic dialectical argument. This exposition dialectically 'derives' the dialectical ideographies via and as an ***immanent critique*** of Standard "Natural" arithmetic, rooted in the 'intra-duality' of that system of arithmetic [again, we denote the *first-order* rules-system of that arithmetic via the 'intension' or 'intensional symbol' '$\underline{\underline{\mathbf{N}}}$'].

Step 1: $\vdash [\,\underline{\underline{\mathbf{N}}}\,] \leftrightarrow \hat{\underline{\underline{\mathbf{q}}}}_1$, Category of Simple Unity [«***arché***» assertion [$\vdash$]]. Thus, $\underline{\underline{\mathbf{N}}}$ is the «***arché***» category / system of our categorial / [meta-]system[atic] progression. Therefore, the summary expressions for the total dialectical process of this '***meta-system***', this '***Dialectic of Arithmetics***' are

(1) $[\,\underline{\underline{\mathbf{N}}}\,]^{\tau}$, for step-by-step, category-by-category, system-by-system progression, and **(2)** $[\,\underline{\underline{\mathbf{N}}}\,]^{2^{\tau}}$ for 'epoch'-to-next-'epoch', one-conceptual-revolution-to-the-next-conceptual-revolution, antithesis-to-next-antithesis progression. The ${}_{\underline{\underline{\mathbf{N}}}}\underline{\underline{\mathbf{Q}}}$ dialectical ideography is applied herein as an 'heuristic ***in***tensional' arithmetic and algebra. There is no guarantee that the full connotations of any interpreted, 'intensional' ideogram are the same for one human mind as for another, without further, exhaustive, 'extensional' specification, *exterior to* the native ${}_{\underline{\underline{\mathbf{N}}}}\underline{\underline{\mathbf{Q}}}$ language. As the $[\,\underline{\underline{\mathbf{N}}}\,]^{\tau}$ progression turns up increasingly concrete, complex, and determinate arithmetico-algebraic systems, the systems in the sequence gain progressively in their explicitude, and in their capacity for ***ex***tensional specificity. Each successor system exceeds its predecessors in its power to describe 'meta-fractal' cumula, i.e., '[quanto-]qualitatively self-similar' or 'evolute helical', 'multi-meta-ontic', 'multi-meta-monadic', heterogeneous accumulations of self-coordinating, self-organizing 'neo-ideo-ontology'. Each yields models with increasing degrees of descriptive richness, specificity, complexity, or concreteness, all as parties to '***The Generalized Cumulum Hypothesis***'. Assigning $\hat{\underline{\underline{\mathbf{q}}}}_1 \leftrightarrow \underline{\underline{\mathbf{N}}}$, we obtain the following progression of 'ideo-ontological emergences' [$\underline{\uparrow}$] from $\tau = \mathbf{1}$ through $\tau = \mathbf{7}$:

$$
\begin{aligned}
[\,\hat{\underline{\underline{\mathbf{q}}}}_1\,]^1 &= [\,\hat{\underline{\underline{\mathbf{q}}}}_1\,] && \leftrightarrow [\,\underline{\underline{\mathbf{N}}}\,];\\
[\,\hat{\underline{\underline{\mathbf{q}}}}_1\,]^2 &= [\,\hat{\underline{\underline{\mathbf{q}}}}_1 + \hat{\underline{\underline{\mathbf{q}}}}_2\,] && \leftrightarrow [\,\underline{\underline{\mathbf{N}}} + \underline{\underline{\mathbf{Q}}}\,];\\
[\,\hat{\underline{\underline{\mathbf{q}}}}_1\,]^3 &= [\,\hat{\underline{\underline{\mathbf{q}}}}_1 + \hat{\underline{\underline{\mathbf{q}}}}_2 + \hat{\underline{\underline{\mathbf{q}}}}_3\,] && \leftrightarrow [\,\underline{\underline{\mathbf{N}}} + \underline{\underline{\mathbf{Q}}} + \underline{\underline{\mathbf{U}}}\,];\\
[\,\hat{\underline{\underline{\mathbf{q}}}}_1\,]^4 &= [\,\hat{\underline{\underline{\mathbf{q}}}}_1 + \hat{\underline{\underline{\mathbf{q}}}}_2 + \hat{\underline{\underline{\mathbf{q}}}}_3 + \hat{\underline{\underline{\mathbf{q}}}}_4\,] && \leftrightarrow [\,\underline{\underline{\mathbf{N}}} + \underline{\underline{\mathbf{Q}}} + \underline{\underline{\mathbf{U}}} + \underline{\mathrm{M}}\,];\\
[\,\hat{\underline{\underline{\mathbf{q}}}}_1\,]^5 &= [\,\hat{\underline{\underline{\mathbf{q}}}}_1 + \hat{\underline{\underline{\mathbf{q}}}}_2 + \hat{\underline{\underline{\mathbf{q}}}}_3 + \hat{\underline{\underline{\mathbf{q}}}}_4 + \hat{\underline{\underline{\mathbf{q}}}}_5\,] && \leftrightarrow [\,\underline{\underline{\mathbf{N}}} + \underline{\underline{\mathbf{Q}}} + \underline{\underline{\mathbf{U}}} + \underline{\mathrm{M}} + \hat{\underline{\underline{\mathbf{q}}}}_{\underline{\mathrm{MN}}}\,];\\
[\,\hat{\underline{\underline{\mathbf{q}}}}_1\,]^6 &= [\,\hat{\underline{\underline{\mathbf{q}}}}_1 + \hat{\underline{\underline{\mathbf{q}}}}_2 + \hat{\underline{\underline{\mathbf{q}}}}_3 + \hat{\underline{\underline{\mathbf{q}}}}_4 + \hat{\underline{\underline{\mathbf{q}}}}_5 + \hat{\underline{\underline{\mathbf{q}}}}_6\,] && \leftrightarrow [\,\underline{\underline{\mathbf{N}}} + \underline{\underline{\mathbf{Q}}} + \underline{\underline{\mathbf{U}}} + \underline{\mathrm{M}} + \hat{\underline{\underline{\mathbf{q}}}}_{\underline{\mathrm{MN}}} + \hat{\underline{\underline{\mathbf{q}}}}_{\underline{\mathrm{MQ}}}\,];\\
[\,\hat{\underline{\underline{\mathbf{q}}}}_1\,]^7 &= [\,\hat{\underline{\underline{\mathbf{q}}}}_1 + \hat{\underline{\underline{\mathbf{q}}}}_2 + \hat{\underline{\underline{\mathbf{q}}}}_3 + \hat{\underline{\underline{\mathbf{q}}}}_4 + \hat{\underline{\underline{\mathbf{q}}}}_5 + \hat{\underline{\underline{\mathbf{q}}}}_6 + \hat{\underline{\underline{\mathbf{q}}}}_7\,] && \leftrightarrow [\,\underline{\underline{\mathbf{N}}} + \underline{\underline{\mathbf{Q}}} + \underline{\underline{\mathbf{U}}} + \underline{\mathrm{M}} + \hat{\underline{\underline{\mathbf{q}}}}_{\underline{\mathrm{MN}}} + \hat{\underline{\underline{\mathbf{q}}}}_{\underline{\mathrm{MQ}}} + \hat{\underline{\underline{\mathbf{q}}}}_{\underline{\mathrm{MQN}}}\,].
\end{aligned}
$$

The question is, what should the above '*superpositions of intensions*' or '*meme cumula*' ***mean***, per our '*generic dialectical interpretation of* ${}_{\underline{\underline{\mathbf{N}}}}\underline{\underline{\mathbf{Q}}}$', given the meaning we stipulated for their «***arché***», $\underline{\underline{\mathbf{N}}}$?

Step 2. **Category of Simple Difference:** $\uparrow[_{\underline{\underline{N}}}\underline{\underline{Q}}] \leftrightarrow \hat{\underline{\underline{q}}}_2$. By the rules of $_{\underline{\underline{N}}}\underline{\underline{Q}}$ arithmetic, given earlier, we have, as $\tau=\mathbf{1} \rightarrow \tau=\mathbf{2}$, $\underline{\underline{\mathbf{N}}} \rightarrow \underline{\underline{\mathbf{N}}}[\underline{\underline{\mathbf{N}}}] = \sim\underline{\underline{\mathbf{N}}} = \underline{\underline{\mathbf{N}}}\underline{\underline{\mathbf{N}}} = \underline{\underline{\mathbf{N}}}^2 = \underline{\underline{\mathbf{N}}} + \Delta\underline{\underline{\mathbf{N}}} \leftrightarrow \hat{\underline{\underline{q}}}_1 \rightarrow \hat{\underline{\underline{q}}}_1[\hat{\underline{\underline{q}}}_1] = \sim[\hat{\underline{\underline{q}}}_1] = \hat{\underline{\underline{q}}}_1\hat{\underline{\underline{q}}}_1 = \hat{\underline{\underline{q}}}_1^{\,2} = \hat{\underline{\underline{q}}}_1 + \Delta\hat{\underline{\underline{q}}}_1 = \hat{\underline{\underline{q}}}_1 + \hat{\underline{\underline{q}}}_2$. Thus, in this model, $\hat{\underline{\underline{q}}}_2 \leftrightarrow {}_{\underline{\underline{N}}}\underline{\underline{Q}}$, and $_{\underline{\underline{N}}}\underline{\underline{Q}} = \Delta\underline{\underline{\mathbf{N}}}$. If we hold that, in the $\underline{\underline{\mathbf{N}}}[\underline{\underline{\mathbf{N}}}]$ 'self-«***aufheben***»' [~] operation, $\underline{\underline{\mathbf{N}}}$ extracts, "selects", or "elects", by virtue of the 'immanent ambiguity' of $\underline{\underline{\mathbf{N}}}$, the internal, implicit 'self-difference', 'immanent alternative', or 'intra-dual' out of itself, out of $\underline{\underline{\mathbf{N}}}$, and posits it externally, explicitly, as $_{\underline{\underline{N}}}\underline{\underline{Q}}$, then $_{\underline{\underline{N}}}\underline{\underline{Q}}$ must be a *qualitative* ***opposite*** of $\underline{\underline{\mathbf{N}}}$. So too, if we hold that $_{\underline{\underline{N}}}\underline{\underline{Q}}$ must be the '***anti***-thesis' to $\underline{\underline{\mathbf{N}}}$ as 'thesis'. Our ideogram $\underline{\underline{\mathbf{N}}}$ denotes an arithmetic of 'pure, *unqualified* ***quantifiers***'. Therefore, $_{\underline{\underline{N}}}\underline{\underline{Q}}$ here must denote an arithmetic of 'pure, *unquantified* ***qualifiers***'. It should denote the initial, $_{\underline{\underline{N}}}\underline{\underline{Q}}$ version of $\underline{\underline{Q}}$, since it arises from, and therefore 'inherits from', $\underline{\underline{\mathbf{N}}}$. This $_{\underline{\underline{N}}}\underline{\underline{Q}}$ should also be 'Peanic', since $\underline{\underline{\mathbf{N}}}$ is. I.e., it should conform to the first order Peano postulates for "Natural" arithmetic. It thus must be a "Non-Standard Model" of these first order Peano axioms, describing 'pure qualifiers', which we interpret as ***ontological*** *qualifiers*, as opposed to the 'pure quantifiers' of "Standard" $\mathbf{N}$. It is and it does! [see *Intro. Letter* for more on the 'Peanicity' of the $_{\underline{\underline{N}}}\underline{\underline{Q}}$]. Note: $\underline{\underline{\mathbf{N}}} \neq \underline{\mathbf{N}} \neq \mathbf{N}$. The first, $\underline{\underline{\mathbf{N}}}$ with double underscore, denotes a "first-order" axiomatics of the "Natural Numbers", which contains the Standard / Non-Standard intra-duality. The second, $\underline{\mathbf{N}}$, denotes a "higher order" axiomatics, confining it to the "Standard Model" and to Gödelian logical incompleteness. The third denotes the 'points-set' or 'space' of the "Natural Numbers", $\mathbf{N} \equiv \{\mathbf{1, 2, 3,...}\}$.

Transition. There is an imbalance, an asymmetry, a ***dis***-equilibrium in the contrast, and the stand-off, between $\underline{\underline{\mathbf{N}}}$ and $_{\underline{\underline{N}}}\underline{\underline{Q}}$, that makes their opposition incomplete, and that thus draws thought onwards toward their synthesis. The $\underline{\underline{\mathbf{N}}}$ arithmetic is one of unqual***ified*** quantifiers. The $_{\underline{\underline{N}}}\underline{\underline{Q}}$ arithmetic is one of unquanti***fiable*** qualifiers. The '*un...fi-**able***' epithet is a *stronger* 'restrictor' than is '*un...fi-**ed***'. Thus $\underline{\underline{\mathbf{N}}}$ and $_{\underline{\underline{N}}}\underline{\underline{Q}}$ are not ***exact*** contraries. The $\hat{\underline{\underline{q}}}_n$ are truly unquanti***fiable***. Defined as additively idempotent, the $\hat{\underline{\underline{q}}}_n$ ***cannot*** express homogeneous multiplicity, only unity. But the $\mathbf{n} \in \mathbf{N}$ are merely unqual***ified*** in their $\underline{\underline{\mathbf{N}}}$ form. They aren't qualified in that guise, but they ***can*** be, e.g., as in **3cm.**, **3i**, $\mathbf{3}\hat{\mathbf{e}}_4$, $\mathbf{3}\hat{\mathbf{x}}$, etc. The $\hat{\underline{\underline{q}}}_n$ ***cannot*** be quantified, by definition and by 'dialectical descent'. But might not conceptual possibility encompass (an)other arithmetical system(s) of [ontic] qualifiers that ***can*** be quantified, and in a way yielding greater specificity/determinateness of expressive power? Yes, it does! We call it $\underline{\underline{\mathbf{U}}}$.

Step 3. **Category of Complex Unity:** $\uparrow[_{\underline{\underline{N}}}\underline{\underline{U}}] \leftrightarrow \hat{\underline{\underline{q}}}_3$. Per our '*generic dialectical interpretation of* $_{\underline{\underline{N}}}\underline{\underline{Q}}$', the next category / system of arithmetic should be a ***synthesis*** of the first two, a "higher, complex unity" of $\underline{\underline{\mathbf{N}}}$ and $_{\underline{\underline{N}}}\underline{\underline{Q}}$. That is, $\hat{\underline{\underline{q}}}_3$ here should stand for to $_{\underline{\underline{N}}}\hat{\underline{\underline{q}}}_{\underline{\underline{QN}}}$. We have given $_{\underline{\underline{N}}}\hat{\underline{\underline{q}}}_{\underline{\underline{QN}}}$ the added nickname $\underline{\underline{\mathbf{U}}}$, or, more specifically, $_{\underline{\underline{N}}}\underline{\underline{\mathbf{U}}}$, since $\underline{\underline{\mathbf{N}}}$ is its 'heritage', the 'ultimate ancestor', or «***arché***» in its 'ideo-meta-genealogy'. It stands for an arithmetical rules-system of ***quantifiable*** qualitative unit[ie]s, which we denote generically by $\hat{\underline{\underline{\mathbf{u}}}}$: $_{\underline{\underline{N}}}\underline{\underline{\mathbf{U}}} \equiv \{\, \mathbf{N}\hat{\underline{\underline{\mathbf{u}}}}_{\mathbf{N}}^{\,\mathbf{N}} \,\} = \{\, \mathbf{N}\hat{\underline{\underline{\mathbf{u}}}}_1, \mathbf{N}\hat{\underline{\underline{\mathbf{u}}}}_2, \mathbf{N}\hat{\underline{\underline{\mathbf{u}}}}_3, \ldots \,\}$.

The ${}_{\mathbf{N}}\underline{\mathbf{U}}$ are 'addable' or additively ***non***-idempotent units, ***un***like the $\hat{\underline{\mathbf{q}}}$ units, but they are still multiplicatively 'Contra-Boolean' units, ***like*** the $\hat{\underline{\mathbf{q}}}$ units. We interpret the $\hat{\underline{\mathbf{u}}}$ units, again, as ***ontological*** *qualifier* units, as we did the $\hat{\underline{\mathbf{q}}}$ unities. The generic ***irreducible*** ${}_{\mathbf{N}}\underline{\underline{\mathbf{U}}}$ meta-numeral is thus $\mathbf{n}\cdot\hat{\underline{\mathbf{u}}}_{\mathbf{k}}$ or just $\mathbf{n}\hat{\underline{\mathbf{u}}}_{\mathbf{k}}$, where $\mathbf{n}$ denotes a quantifier [$\mathbf{n}, \mathbf{k} \in \mathbf{N}$], and where $\hat{\underline{\mathbf{u}}}_{\mathbf{k}}$ denotes, of course, an [ontological] ***unit-qualifier*** [i.e., analytical-geometrically, a directed line segment, or 'finite ray', of unit axial length, or modulus, 1]. Note: One can very well write out $\mathbf{n}\cdot\hat{\underline{\mathbf{q}}}_{\mathbf{k}}$ or $\mathbf{n}\hat{\underline{\mathbf{q}}}_{\mathbf{k}}$, but, due to the additive idempotency rule of ${}_{\mathbf{N}}\underline{\underline{\mathbf{Q}}}$, it ***reduces*** immediately to just $\hat{\underline{\mathbf{q}}}_{\mathbf{k}}$. Note: In 'meta-***dynamical***' modeling contexts, $\mathbf{n}\cdot\hat{\underline{\mathbf{u}}}_{\mathbf{k}}$ becomes $\mathbf{m}_{\mathbf{k}}(\tau)\cdot\hat{\underline{\mathbf{u}}}_{\mathbf{k}}$.

Transition. There is an ***inadequacy*** in our so-far-extant ideo-cumulum, $[\ \underline{\underline{\mathbf{N}}} + {}_{\mathbf{N}}\underline{\underline{\mathbf{Q}}} + {}_{\mathbf{N}}\underline{\underline{\mathbf{U}}}\ ]$, with respect to its explicit conceptual development of the universe of arithmetics of qualifiers, of the theory of arithmetical qualification, or of qualifier arithmetics. We have elaborated systems of 'meta-numbers' interpretable as **(1)** unquantifiable, and **(2)** as quantifiable ***ontological*** qualifiers. We have yet, however, to systematically explore systems of meta-numbers interpretable as ***metrical*** qualifiers -- i.e., to explore the possibility of a ***fully-ideographic*** arithmetic [and algebra!] for "***dimensional analysis***", a field which has, heretofore, been left behind at the "***rhetorical***" or "***syncopated***" stage of arithmetico-algebraic development. Note that "dimensions" and their "metrical unit[ie]s" also exhibit the 'Contra-Boolean' characteristic, e.g., **inch** $\times$ **inch** = **in.**^{2} = **sq. inch** $\not\succ$ **in.**^{1} = **linear inch**; **cm.**^{2} $\not\succ$ **cm.**^{1} $\not\succ$ **cm.**^{3}; **gm.**^{2} $\not\succ$ **gm.**^{1} $\not\succ$ **gm.**^{3}; **sec.**^{2} $\not\succ$ **sec.**^{1} $\not\succ$ **sec.**^{3}, etc. Indeed, for *any* such metrical unit, we have the progression: $\underline{\mathbf{unit}}^1 \not\succ \underline{\mathbf{unit}}^2 \not\succ \underline{\mathbf{unit}}^3 \not\succ \underline{\mathbf{unit}}^4 \not\succ \ldots$. Thus we see '***metrical*** *qualifiers*' as the [immediate, next] '***other***' of '*ontological* ***qualifiers***'. Note: The $[\ \hat{\underline{\mathbf{q}}}_1\ ]^{\tau}$ summary formulation of dialectical category-/system-progression makes τ tell the number of categories extant, emerged, or 'explicitized' as of step τ in the dialectical argument. We have identified another 'standard' summary formulation of the dialectic within the ${}_{\mathbf{N}}\underline{\underline{\mathbf{Q}}}$ dialectical ideography, in which τ plays a different role, and assumes *different values* than for $[\ \hat{\underline{\mathbf{q}}}_1\ ]^{\tau}$ in specifying *the same value* of the 'cumulum'. This alternative 'standard' formulation is the $[\ \hat{\underline{\mathbf{q}}}_1\ ]^{2^{\tau}}$ formulation. Thus, for example, $[\ \hat{\underline{\mathbf{q}}}_1\ ]^{2^0}$ gives the same 'cumulum' value as $[\ \hat{\underline{\mathbf{q}}}_1\ ]^1$, namely, the value $[\ \hat{\underline{\mathbf{q}}}_1\ ]$; $[\ \hat{\underline{\mathbf{q}}}_1\ ]^{2^1}$ gives the same value of the 'cumulum' as $[\ \hat{\underline{\mathbf{q}}}_1\ ]^2$, namely, the value $[\ \hat{\underline{\mathbf{q}}}_1 + \hat{\underline{\mathbf{q}}}_2\ ]$; and $[\ \hat{\underline{\mathbf{q}}}_1\ ]^{2^2}$ gives the same value as $[\ \hat{\underline{\mathbf{q}}}_1\ ]^4$, namely, $[\ \hat{\underline{\mathbf{q}}}_1 + \hat{\underline{\mathbf{q}}}_2 + \hat{\underline{\mathbf{q}}}_3 + \hat{\underline{\mathbf{q}}}_4\ ]$. By virtue of product rules **4**., given on page **4**, above, the intermediate operational-arithmetical, algorithmic processes of the $[\ \hat{\underline{\mathbf{q}}}_1\ ]^{2^{\tau}}$ formulation are more readily interpretable as *dialectical* processes, whereas the $[\ \hat{\underline{\mathbf{q}}}_1\ ]^{\tau}$ formulation is so interpretable, in our view, only in its results, which are thus reached via a syntactical path that passes through dialectical 'unsemantifiability' or 'uninterpretability'.

Moreover, $[\ \hat{\mathbf{q}}_1\ ]^{2^\tau}$ progresses in steps -- units of "stages', epochs', 'revolutions' or 'meta-evolutions'; 'meta-dynamical self-movements'; 'sums' or 'series' ['cumula'] of 'ontos' -- which always ***include*** the synthesis term for the last-epoch antithesis, the category of complex unity, but which always also ***culminate in*** the emergence of the next 'contra-thesis' term / 'onto', the next category of higher difference, signifying the ever-re-production and ever-re-emergence of the «***arché***» 'self-duality' or 'ontological self-contradiction' on a new, yet-higher 'plane'. The $[\ \hat{\mathbf{q}}_1\ ]^{2^\tau}$ progression is thus '***a progression of antitheses***'. We reference the $[\ \hat{\mathbf{q}}_1\ ]^{2^\tau}$ 'formation' herein beginning with **Step. 4**.

Step 4. Category of Higher / Renewed Self-Difference: $\uparrow[{}_{\mathbf{N}}\mathrm{M}] \leftrightarrow \hat{\mathbf{q}}_4$. By rules of ${}_{\mathbf{N}}\mathbf{Q}$ arithmetic, we have, *during* $\tau = \mathbf{1} \rightarrow \tau = \mathbf{2}$, *within* the $[\ \mathbf{N}\ ]^{2^\tau}$ *process* $[\ \mathbf{N} + {}_{\mathbf{N}}\mathbf{Q}\ ] \rightarrow [\ \mathbf{N} + \mathbf{Q} + \mathbf{U} + \mathrm{M}\ ]$, the *sub*-process ${}_{\mathbf{N}}\mathbf{Q} \rightarrow {}_{\mathbf{N}}\mathbf{Q}[{}_{\mathbf{N}}\mathbf{Q}] = \sim[{}_{\mathbf{N}}\mathbf{Q}] = {}_{\mathbf{N}}\mathbf{Q}\cdot{}_{\mathbf{N}}\mathbf{Q} = {}_{\mathbf{N}}\mathbf{Q}^2 = {}_{\mathbf{N}}\mathbf{Q} + \Delta[{}_{\mathbf{N}}\mathbf{Q}] \leftrightarrow \hat{\mathbf{q}}_2 \rightarrow \hat{\mathbf{q}}_2[\hat{\mathbf{q}}_2] = \sim[\hat{\mathbf{q}}_2] =$ $\hat{\mathbf{q}}_2\hat{\mathbf{q}}_2 = \hat{\mathbf{q}}_2{}^2 = \hat{\mathbf{q}}_2 + \Delta\hat{\mathbf{q}}_2 = \hat{\mathbf{q}}_2 + \hat{\mathbf{q}}_{2+2} = \hat{\mathbf{q}}_2 + \hat{\mathbf{q}}_4$. Thus, in this model, $\hat{\mathbf{q}}_4 \leftrightarrow \mathrm{M}$, and $\mathrm{M} = \Delta[{}_{\mathbf{N}}\mathbf{Q}]$. If we hold that, in the ${}_{\mathbf{N}}\mathbf{Q}[{}_{\mathbf{N}}\mathbf{Q}]$ 'self-«***aufheben***»' operation, ${}_{\mathbf{N}}\mathbf{Q}$ extracts, "selects", or "elects", by virtue of the 'immanent ambiguity' of ${}_{\mathbf{N}}\mathbf{Q}$, the inherent, internal, ***implicit*** 'self-difference', 'self-otherness', 'immanent alternative', or 'intra-dual' out of itself, out of ${}_{\mathbf{N}}\mathbf{Q}$, and posits it externally, ***explicitly***, then M must be a kind of *qualitative* ***opposite*** of ${}_{\mathbf{N}}\mathbf{Q}$. Our ideogram ${}_{\mathbf{N}}\mathbf{Q}$ denotes a system of arithmetic interpretable as one of 'pure, *unquantifiable* ***ontological*** *qualifiers*'. Thus, we hold that M should denote an arithmetical system of 'pure, *unquantifiable* ***non-****ontological qualifiers*' in the form of 'pure, *unquantifiable* ***metrical*** *qualifiers*'. It should denote an initial, ${}_{\mathbf{N}}\mathrm{M}$ version of M, as it arises from, and thereby 'inherits from', $\mathbf{N}$ & ${}_{\mathbf{N}}\mathbf{Q}$. This time it is *not*, as it *was* in $\mathbf{N}\cdot\mathbf{N} - \mathbf{N} = {}_{\mathbf{N}}\mathbf{Q}$, the self-ambiguity of ***quantifier*** *ideography* [& of quantifier *non*-qualification] ***vs. qualifier*** *ideography* [& qualifier *non*-quantification] that is to be 'ideo-externalized' & 'ideo-objectified'. This time the self-ambiguity within the ${}_{\mathbf{N}}\mathbf{Q}$, qualifier-side comes to the fore in ${}_{\mathbf{N}}\mathbf{Q}\cdot{}_{\mathbf{N}}\mathbf{Q} - {}_{\mathbf{N}}\mathbf{Q} = {}_{\mathbf{N}}\mathrm{M}$. The syntax of the algorithmic, calculative process itself, set out in interpreted, intensional ideograms, even captures some of the flavor of this meaning -- this flow of dialectical argumentation -- in itself:

$$[\ \mathbf{N}\ ]^{2^2} = [\ \mathbf{N}+{}_{\mathbf{N}}\mathbf{Q}\ ]\cdot[\ \mathbf{N}+{}_{\mathbf{N}}\mathbf{Q}\ ] = [[\ \mathbf{N}+{}_{\mathbf{N}}\mathbf{Q}\ ] + {}_{\mathbf{N}}\mathbf{Q}\cdot[\ \mathbf{N}\ ] + {}_{\mathbf{N}}\mathbf{Q}\cdot[\ {}_{\mathbf{N}}\mathbf{Q}\ ]] = [\hat{\mathbf{q}}_{\mathbf{N}} + \hat{\mathbf{q}}_{\mathbf{Q}} + \hat{\mathbf{q}}_{\mathbf{QN}} + \hat{\mathbf{q}}_{\mathbf{QQ}}] =$$

$$[\ \mathbf{N}+\mathbf{Q}+\mathbf{U}+\mathrm{M}\ ] \leftrightarrow [\ \hat{\mathbf{q}}_1+\hat{\mathbf{q}}_2\ ]\cdot[\ \hat{\mathbf{q}}_1+\hat{\mathbf{q}}_2\ ] = [[\hat{\mathbf{q}}_1+\hat{\mathbf{q}}_2\ ] + \hat{\mathbf{q}}_2\cdot[\hat{\mathbf{q}}_1\ ] + \hat{\mathbf{q}}_2\cdot[\hat{\mathbf{q}}_2]] = [\ \hat{\mathbf{q}}_1+\hat{\mathbf{q}}_2+\hat{\mathbf{q}}_3+\hat{\mathbf{q}}_4\ ].$$

That is, the self-reflexion or immanent [self-]critique of the $[\ \mathbf{N}+{}_{\mathbf{N}}\mathbf{Q}\ ]$ 'antithesis ideo-cumulum' is equivalent to the 'self-«***aufheben***»' conservation of that ideo-cumulum, *plus* the 'elevation'-negation of that ideo-cumulum in the form of the ${}_{\mathbf{N}}\mathbf{Q}$-critique or subsumption / assimilation of $\mathbf{N}$, which consists of the 'composite-formation', 'complex-formation'', or 'collision-fusion' of the ${}_{\mathbf{N}}\mathbf{Q}$ and $\mathbf{N}$ principles, *plus* the ${}_{\mathbf{N}}\mathbf{Q}$-critique or subsumption / assimilation of ${}_{\mathbf{N}}\mathbf{Q}$ itself, the 'self-complexification', or '*self*-collision' and '*self*-fusion' of the ${}_{\mathbf{N}}\mathbf{Q}$ principle, whose outcome we identify as *the principle of* '***metrical*** *qualification*'; of '***metrical*** *ideography*'.

Thus too, this ${}_{\underline{N}}\underline{\underline{M}}$ should also be 'Peanic", as is $\underline{\underline{\mathbf{N}}}$. I.e., it should conform to the first four, first-order Peano postulates for "Natural" arithmetic. It must thus also be a "Non-Standard Model" of these four first-order Peano axioms, describing a new realm of 'pure qualifiers', which we interpret to be one of ***metrical*** *qualifiers*, in contrast to those of the 'pure ***quantifiers***' of "Standard" $\underline{\underline{\mathbf{N}}}$, and of the 'pure ***ontological*** qualifiers' of ${}_{\underline{N}}\underline{\underline{\mathbf{Q}}}$ and ${}_{\underline{N}}\underline{\underline{\mathbf{U}}}$. As well, ${}_{\underline{N}}\underline{\underline{M}}$ should be 'Contra-Boolean', also like ${}_{\underline{N}}\underline{\underline{\mathbf{Q}}}$ and ${}_{\underline{N}}\underline{\underline{\mathbf{U}}}$. It inherits its 'Peanicity' from its «***arché***», $\underline{\underline{\mathbf{N}}}$, and its 'Contra-Boolean' character from ${}_{\underline{N}}\underline{\underline{\mathbf{Q}}}$.

The ${}_{\underline{N}}\underline{\underline{M}}$ ideography is a higher analogue, arising on a more complex, more specified, more determinate ideo-qualitative, 'ideo-meta-fractal' scale, of the ${}_{\underline{N}}\underline{\underline{\mathbf{Q}}}$ ideography. The space of the ${}_{\underline{N}}\underline{\underline{\mathbf{Q}}}$ arithmetical ideography is ${}_{\underline{N}}\underline{\underline{\mathbf{Q}}} \equiv \{ \hat{\underline{\underline{q}}}_1, \hat{\underline{\underline{q}}}_2, \hat{\underline{\underline{q}}}_3, \ldots \}$. That of ${}_{\underline{N}}\underline{\underline{M}}$ is ${}_{\underline{N}}\underline{\underline{M}} \equiv \{ \hat{\underline{\underline{m}}}_1, \hat{\underline{\underline{m}}}_2, \hat{\underline{\underline{m}}}_3, \ldots \}$, where the calculative behaviors of the $\hat{\underline{\underline{m}}}_k$ express unquantifiable / additively-idempotent addition rules, e.g., $[\ \hat{\underline{\underline{m}}}_k + \hat{\underline{\underline{m}}}_k = \hat{\underline{\underline{m}}}_k\ ]$, and «***aufheben***» 'evolute' multiplication rules $[\ \hat{\underline{\underline{m}}}_k \times \hat{\underline{\underline{m}}}_k = \hat{\underline{\underline{m}}}_k + \hat{\underline{\underline{m}}}_{2k}$, etc. $]$, just like those of ${}_{\underline{N}}\underline{\underline{\mathbf{Q}}}$. The $\underline{\underline{M}}$ or $\Delta\underline{\underline{\mathbf{Q}}}$ system of arithmetic represents just the barest emergence of the ***metrical qualifier*** ideo-ontology, with a very limited range of applications. In the meta-systematic dialectical argument, it serves essentially only to posit the principle of the analogy between the principle of '***onto***-*dynamasis*' or "***non***-*ontostasis*' as codified in ${}_{\underline{N}}\underline{\underline{\mathbf{Q}}}$, and that of '***metrico***-*dynamasis*', the principle of ${}_{\underline{N}}\underline{\underline{M}}$. That is, we interpret $[\ \hat{\underline{\underline{m}}}_1\ ]^k$ as summarizing the '***non-amalgamative sum of ideas***' or '***heterogeneous ideo-cumulum***' within ${}_{\underline{N}}\underline{\underline{M}}$ that results form the assignment of $\hat{\underline{\underline{m}}}_1$ to a specific '*metric*', "*unit of measurement*", or '***metrical unity***'. For example, for $\mathbf{k} > \mathbf{5}$, we have --

$[\ \hat{\underline{\underline{m}}}_1\ ]^k = [\ \hat{\underline{\underline{m}}}_1 + \hat{\underline{\underline{m}}}_2 + \hat{\underline{\underline{m}}}_3 + \hat{\underline{\underline{m}}}_4 + \hat{\underline{\underline{m}}}_5 + \ \ldots\ + \hat{\underline{\underline{m}}}_k\ ] \leftrightarrow [\ \underline{\mathbf{unit}}\ ]^k$ for some '*metrical monad*' =
$[\ \underline{\mathbf{unit}} + \underline{\mathbf{meta\text{-}unit}} + \underline{\mathbf{meta\text{-}meta\text{-}unit}} + \underline{\mathbf{meta\text{-}meta\text{-}meta\text{-}unit}} + \ldots + \underline{\mathbf{meta}^{k-1}\mathbf{\text{-}unit}}\ ]$,

e.g., for $\underline{\mathbf{unit}} = \mathbf{cm.}$, $[\mathbf{cm.}]^k = [\ \mathbf{linear\ cm.} + \mathbf{square\ cm.} + \mathbf{cubic\ cm.} + \ldots + \text{'}\mathbf{k\text{-}ic}\text{'}\ \mathbf{cm.}\ ]$.
Notice, via the above formulae, that: **(1)** The '***metrico-dynamasis***' interpretation of $[\ \hat{\underline{\underline{m}}}_1\ ]^k$ losses the historical, diachronic, and evolutionary or 'meta-evolutionary' content and connotations of the '***onto-dynamasis***' interpretation of $[\ \hat{\underline{\underline{q}}}_1\ ]^\tau$. That is, we are *not* proposing here, e.g., in the case of 'physical spatial "Length" units' like '**cm.**', that physical space '[meta-]evolves' historically in the sequence: $[\ \mathbf{cm.}^1\ ] \rightarrow [\ \mathbf{cm.}^1 + \mathbf{cm.}^2\ ] \rightarrow [\ \mathbf{cm.}^1 + \mathbf{cm.}^2 + \mathbf{cm.}^3\ ] \rightarrow \ldots$, that is, in the epochs: $[\ \underline{\mathbf{1\text{-}D\ space}}\ ] \rightarrow [\ \underline{\mathbf{2\text{-}D\ space}}\ ] \rightarrow [\ \underline{\mathbf{3\text{-}D\ space}}\ ] \rightarrow \ldots$; **(2)** The '***metrico-dynamasis***' interpretation of $[\ \hat{\underline{\underline{m}}}_1\ ]^k$ can only express this '*self-elaboration*' into a series of ***qualitatively*** *distinct* metrical units -- this '*meta-genealogy*' of 'units of metric' or of the 'monads of measurement' -- for a ***single*** unit of measure or 'metrical monad'. It can only interpret 'one metric at a time'; **(3)** There is no way, in ${}_{\underline{N}}\underline{\underline{M}}$, to formulate 'compound' units of measure, such as the classical $[\ \mathbf{gm.} \cdot \mathbf{cm.} / \mathbf{sec.}^2\ ]$, or **dyne**, the **CGS** standard unit of spatially-directed force. Each metrical unit «***arché***» assigned to $\hat{\underline{\underline{m}}}_1$ in ${}_{\underline{N}}\underline{\underline{M}}$, is portrayed as if it formed a universe-of-discourse all to itself, all by itself; a separate universe, without any other metrical unit[ie]s co-existing in parallel with it.

These are some of the limits on the analogy relation [$\asymp$] between ${}_{\underline{N}}\underline{\underline{M}}$ & ${}_{\underline{N}}\underline{\underline{Q}}$. Subject to such restrictions, we say that '***metrico-dynamasis***' is a kind of '***onto-dynamasis***', the '***onto-dynamasis***' of a '***metrical*** onto', and that the '***metrico-dynamaseŝ***' expressible in the ${}_{\underline{N}}\underline{\underline{M}}$ arithmetic *are* ***analogous to*** the '***onto-dynamaseŝ***' expressible in ${}_{\underline{N}}\underline{\underline{Q}}$: $[\,\hat{\underline{\underline{m}}}_1\,]^k \asymp [\,\hat{\underline{\underline{q}}}_1\,]^\tau$.

Transition. Thus, ${}_{\underline{N}}\underline{\underline{M}}$ is just the "tip" of a new "iceberg" now taking shape up ahead of us, in front of our minds' inner eyes. To get beyond its bare irruption as $\underline{\Delta}[\,{}_{\underline{N}}\underline{\underline{Q}}\,] = {}_{\underline{N}}\underline{\underline{M}}$, this new emergence must '"assimilate all the wealth of previous development"'. We need to progress the categorial progression two categories further. We need to entertain [as **Step 5**.] the re-assimilation / subsumption of $\underline{\underline{N}}$ by ${}_{\underline{N}}\underline{\underline{M}}$, and [as **Step 6**.] the re-assimilation / subsumption of ${}_{\underline{N}}\underline{\underline{Q}}$ by ${}_{\underline{N}}\underline{\underline{M}}$. Too, these two "half-way house" 'partial syntheses' of steps **5** and **6** will help us to envision the next synthesis, the 'grand synthesis' of **Step 7**. They will help bring it conceptually into our view. They will help to make it less remote; to bring it closer to our mental eye and 'I'; to move us closer to it so as to move it within the range of our in[ner-]sight. They will help to make it 'next' ***for us***.

Step 5. First Category of Partial Synthesis: $\uparrow_{\underline{N}}[\hat{\underline{\underline{q}}}_{\underline{MN}}] \leftrightarrow \hat{\underline{\underline{q}}}_5$. We have seen how ${}_{\underline{N}}\underline{\underline{M}} \asymp {}_{\underline{N}}\underline{\underline{Q}}$. Likewise, we find that ${}_{\underline{N}}\hat{\underline{\underline{q}}}_{\underline{MN}}$ is 'helically' analogous to ${}_{\underline{N}}\hat{\underline{\underline{q}}}_{\underline{QN}} \equiv {}_{\underline{N}}\underline{\underline{U}}$. If ${}_{\underline{N}}\underline{\underline{M}} = {}_{\underline{N}}\hat{\underline{\underline{q}}}_{\underline{QQ}} \asymp {}_{\underline{N}}\underline{\underline{Q}} = {}_{\underline{N}}\hat{\underline{\underline{q}}}_{\underline{Q}}$ and ${}_{\underline{N}}\underline{\underline{M}}$ interprets to an arithmetic of ***additively-idempotent***, '*un*quantifiable *metrical qualifiers*', then ${}_{\underline{N}}\hat{\underline{\underline{q}}}_{\underline{MN}} \asymp {}_{\underline{N}}\hat{\underline{\underline{q}}}_{\underline{QN}} \equiv {}_{\underline{N}}\underline{\underline{U}}$ interprets to one of *additively* ***non***-*idempotent*, '***quantifiable*** *metrical qualifiers*': $[\,\hat{\underline{\underline{o}}}\,\underline{\underline{m}}_k + \hat{\underline{\underline{o}}}\,\underline{\underline{m}}_k = 2\hat{\underline{\underline{o}}}\,\underline{\underline{m}}_k\,]$, and $[\,[2\hat{\underline{\underline{o}}}\,\underline{\underline{m}}_k]^2 = 2\hat{\underline{\underline{o}}}\,\underline{\underline{m}}_k \times 2\hat{\underline{\underline{o}}}\,\underline{\underline{m}}_k = 4\hat{\underline{\underline{o}}}\,\underline{\underline{m}}_{2k} \not\asymp 2\hat{\underline{\underline{o}}}\,\underline{\underline{m}}_k^{\,1}$ & $4\hat{\underline{\underline{o}}}\,\underline{\underline{m}}_{2k} \perp 2\hat{\underline{\underline{o}}}\,\underline{\underline{m}}_k^{\,1}\,]$. Note that *this* multiplication of *metrical* qualifiers must* follow a '***convolute***' rule, not the «***aufheben***» '***evolute***' product rule of the $\hat{\underline{\underline{q}}}$ & $\hat{\underline{\underline{u}}}$ *ontological* qualifiers -- viz.: **inch** × **inch** ≠ **inch** + $\underline{\Delta}$**inch** = **inch** + **sq. inch**, but rather just: **inch** × **inch** = $\underline{\Delta}$**inch** = **sq. inch** $\not\asymp$ **inch** = **linear inch** = **inch**^{1}. [*Otherwise it will fail to capture the known multiplicative behavior of "dimensional units" in "Dimensional Analysis". Notice also that this rule nevertheless still implies 'Contra-Boolean' self-multiplication].

In summary, The 'collision' of the $\underline{\underline{M}}$ principle of *unquantifiable* [$\underline{\underline{Q}}$-like] *metrical qualification* with the $\underline{\underline{N}}$ principle of *unqualified quantification* emerges / elicits the category / conception of a system of arithmetic characterizing ***quantifiable metrical qualification***, denoted by ${}_{\underline{N}}\hat{\underline{\underline{q}}}_{\underline{MN}}$, and we have -- ${}_{\underline{N}}\hat{\underline{\underline{q}}}_{\underline{MN}} \equiv \{\,\hat{\underline{\underline{o}}}\,\underline{\underline{m}}_1, \hat{\underline{\underline{o}}}\,\underline{\underline{m}}_2, \hat{\underline{\underline{o}}}\,\underline{\underline{m}}_3, \ldots\,\}$. Note that $\hat{\underline{\underline{o}}}\,\underline{\underline{m}}_k \not\asymp \hat{\underline{\underline{m}}}_k$. We have here adopted aspects of the circa 250 C.E. notation of Diophantus' «*Arithmetica*», denoting '***quantifiable***' metrical '***unit***-qualifiers' via the '$\hat{\underline{\underline{o}}}$' 'overscore' atop their 'meta-numerals'. The key new principle 'outered' or 'outed' in the form of this **Step 5** 'ideo-onto' of arithmetic is that of the conceptual possibility of a *quantifiable* ***metrical*** *arithmetic*, «***à la***» the *quantifiable* ***ontological*** *arithmetic* of ${}_{\underline{N}}\underline{\underline{U}}$.

This new arithmetical system of rules, denoted ${}_{\underline{\underline{N}}}\hat{\underline{\underline{q}}}_{\underline{\underline{MN}}}$, still retains the other noted limitations of ${}_{\underline{\underline{N}}}\underline{\underline{M}}$, namely -- **(1)** the 'ahistorical', essentially 'synchronic-only' interpretability of its '***metrico-dynamases***'; **(2)** its restriction to the «***arché***» expression of but one unit of measurement at a time, and **(3)** no capacity for explicit expression of 'compounds' of "fundamental" metrical units / 'hybrid' units. The ***unit***-qualifiers of ${}_{\underline{\underline{N}}}\hat{\underline{\underline{q}}}_{\underline{\underline{MN}}}$ are 'Peanic' when considered in their unity, but not so when considered in their multiplicity. In unity or in multiplicity, they are 'Contra-Boolean'.

The generic ***irreducible*** ${}_{\underline{\underline{N}}}\hat{\underline{\underline{q}}}_{\underline{\underline{MN}}}$ meta-numeral is thus $\mathbf{n}\cdot\hat{\underline{\underline{m}}}_{\mathbf{k}}$ or just $\mathbf{n}\hat{\underline{\underline{m}}}_{\mathbf{k}}$, where **n** denotes a quantifier [$\mathbf{n}, \mathbf{k} \in \mathbf{N}$], and where $\hat{\underline{\underline{m}}}_{\mathbf{k}}$ denotes, of course, a [metrical] ***unit-qualifier*** [i.e., analytic-geometrically, a 'finite ray', 'meta-vector', or 'dialector'; a directed line segment of unit axial length, or "modulus 1 axial unit"], whereas one can very well write $\mathbf{n}\cdot\hat{\underline{\underline{m}}}_{\mathbf{k}}$ or $\mathbf{n}\hat{\underline{\underline{m}}}_{\mathbf{k}}$, but, by the additive idempotency rule of ${}_{\underline{\underline{N}}}\underline{\underline{M}}$, this ***reduces immediately*** to just $\hat{\underline{\underline{m}}}_{\mathbf{k}}$.

Sub-Transition. The next arithmetical 'ideo-onto' shows us a way beyond limitations **(2)** and **(3)**, also supplying a new 'twist' on the self-multiplication rules for 'qualifier meta-numbers', an innovation that will be crucial to the 'grand synthesis' of **Step 7**. It presages a capability of the ${}_{\underline{\underline{N}}}\hat{\underline{\underline{q}}}_{\underline{\underline{MQN}}} \equiv {}_{\underline{\underline{N}}}\hat{\underline{\underline{q}}}_{\underline{\underline{MU}}} \equiv {}_{\underline{\underline{N}}}{}_{\alpha}\underline{\underline{\mu}}$ arithmetic to encompass "dimensionless" dimensions, i.e., 'zero degree', 'self-divided', 'dimension zero', 'sub-linear' / 'point-like' / 'unextended', and self-multiplicatively 'Boolean' metrical qualifiers, via a similar 'twist' which arises there immanently, but in a higher [richer, more determinate] form. Inside this progression of '***sub-script arithmetics***', the semantic '*subsumption*' or '*assimilation*' of one system of arithmetic by another, the new arithmetic that is the 'product' of the operation of a later-emergent arithmetic upon an earlier-emergent one, may be mirrored, syntactically, by the '*subordination*' via '***subscriptization***' of the 'meta-numerals' of that older arithmetic to those of the newer. This is, in a sense, what is happening in the formation of ${}_{\underline{\underline{N}}}\underline{\underline{Q}}$; in a sense of '***self***-subsumption', or of '***self-containment***'. There, the subsum***ing*** arithmetic and the subsum***ed*** arithmetic are the same, namely, $\underline{\underline{\mathbf{N}}}$. Recall that ${}_{\underline{\underline{N}}}\underline{\underline{Q}} \equiv \hat{\underline{\underline{q}}}_{\underline{\underline{NN}}}$, & that $\underline{\underline{\mathbf{NN}}} - \underline{\underline{\mathbf{N}}} = {}_{\underline{\underline{N}}}\underline{\underline{Q}}$ or $\hat{\underline{\underline{q}}}_{\underline{\underline{N}}} \times \hat{\underline{\underline{q}}}_{\underline{\underline{N}}} = \hat{\underline{\underline{q}}}_{\underline{\underline{N}}} + \hat{\underline{\underline{q}}}_{\underline{\underline{NN}}} = \hat{\underline{\underline{q}}}_{\underline{\underline{N}}} + \hat{\underline{\underline{q}}}_{\underline{\underline{Q}}} \equiv \underline{\underline{\mathbf{N}}} + {}_{\underline{\underline{N}}}\underline{\underline{Q}}$.

Therefore, the generic ${}_{\underline{\underline{N}}}\underline{\underline{Q}}$ meta-numeral, $\hat{\underline{\underline{q}}}_{\mathbf{N}}$ or $\hat{\underline{\underline{q}}}_{\mathbf{n}}$ for any **n** in **N**, can be viewed as syntactically constituted via '$\underline{\underline{\mathbf{N}}}_{\mathbf{N}}$', or '$\underline{\underline{\mathbf{n}}}_{\mathbf{n}}$', or '$\underline{\underline{\mathbf{N}}}_{\mathbf{n}}$', or '$\underline{\underline{\mathbf{n}}}_{\mathbf{N}}$', where 'script-level' $\underline{\underline{\mathbf{n}}}$ or $\underline{\underline{\mathbf{N}}}$ denotes a universal, general, or ***generic*** form/essence of $\underline{\underline{\mathbf{N}}}$ overall, whereas the 'subscript-level' or 'subscripted' **n** or **N** denotes any ***specific*** element of itself [of non-subscripted **N**], or **n** as an individual element of/from within **N**. Thus the ${}_{\underline{\underline{N}}}\underline{\underline{Q}}$ 'meta-numbers' are, in this sense, meta-**N**-numbers, made up out of many **N**-numbers.

We first bring this view into play in **Step 6** of the argument, not in this form of the '***auto***-subsumption' of $\underline{\underline{\mathbf{N}}}$ by $\underline{\underline{\mathbf{N}}}$, but in the form of an '***other***-', or '***allo***-subsumption' of ${}_{\underline{\underline{N}}}\underline{\underline{Q}}$ by ${}_{\underline{\underline{N}}}\underline{\underline{M}}$, and later, in **Step 7**, in the form of the '***allo***-subsumption' of ${}_{\underline{\underline{N}}}\underline{\underline{U}}$ by ${}_{\underline{\underline{N}}}\underline{\underline{M}}$.

Step 6. **Second Category of Partial Synthesis:** $\uparrow[\hat{\mathbf{q}}_{\underline{MQ}}] \leftrightarrow \hat{\mathbf{q}}_6$. We have, during $\tau = \mathbf{2} \rightarrow \tau = \mathbf{3}$, *within* the $[\,\underline{\mathbf{N}}\,]^{2^\tau}$ *process* $[\,\underline{\mathbf{N}}+\underline{\mathbf{Q}}+\underline{\mathbf{U}}+\underline{\mathrm{M}}\,] \rightarrow [\,\underline{\mathbf{N}} + \underline{\mathbf{Q}} + \underline{\mathbf{U}} + \underline{\mathrm{M}} + \hat{\mathbf{q}}_{\underline{MN}} + \hat{\mathbf{q}}_{\underline{MQ}} + \hat{\mathbf{q}}_{\underline{MQN}} + \underline{\Delta\mathrm{M}}\,]$, the *sub*-process ${}_{\underline{\mathbf{N}}}\underline{\mathrm{M}}[{}_{\underline{\mathbf{N}}}\underline{\mathbf{Q}}] = {}_{\underline{\mathbf{N}}}\underline{\mathrm{M}}\cdot{}_{\underline{\mathbf{N}}}\underline{\mathbf{Q}} = {}_{\underline{\mathbf{N}}}\underline{\mathbf{Q}} + {}_{\underline{\mathbf{N}}}\hat{\mathbf{q}}_{\underline{MQ}} \leftrightarrow \hat{\mathbf{q}}_4[\hat{\mathbf{q}}_2] = \hat{\mathbf{q}}_2 + \hat{\mathbf{q}}_{4+2} = \hat{\mathbf{q}}_2 + \hat{\mathbf{q}}_6$, i.e., *within*:

$$[\,\underline{\mathbf{N}}\,]^{2^2} = [\,\underline{\mathbf{N}}+\underline{\mathbf{Q}}+\underline{\mathbf{U}}+\underline{\mathrm{M}}\,] \;\rightarrow\; [\,\underline{\mathbf{N}}\,]^{2^3} = [\,\underline{\mathbf{N}}+\underline{\mathbf{Q}}+\underline{\mathbf{U}}+\underline{\mathrm{M}}\,]^2 = [\,\underline{\mathbf{N}}+\underline{\mathbf{Q}}+\underline{\mathbf{U}}+\underline{\mathrm{M}}\,]\cdot[\,\underline{\mathbf{N}}+\underline{\mathbf{Q}}+\underline{\mathbf{U}}+\underline{\mathrm{M}}\,]$$

$$= [\,[\,\underline{\mathbf{N}}+\underline{\mathbf{Q}}+\underline{\mathbf{U}}+\underline{\mathrm{M}}\,] + \underline{\mathrm{M}}[\underline{\mathbf{N}}] + \underline{\mathrm{M}}[\underline{\mathbf{Q}}] + \underline{\mathrm{M}}[\underline{\mathbf{U}}] + \underline{\mathrm{M}}[\underline{\mathrm{M}}]\,] = [\,\underline{\mathbf{N}}+\underline{\mathbf{Q}}+\underline{\mathbf{U}}+\underline{\mathrm{M}}+\hat{\mathbf{q}}_{\underline{MN}}+\hat{\mathbf{q}}_{\underline{MQ}}+\hat{\mathbf{q}}_{\underline{MU}}+\underline{\Delta\mathrm{M}}\,]$$

$$\leftrightarrow [\hat{\mathbf{q}}_1+\hat{\mathbf{q}}_2+\hat{\mathbf{q}}_3+\hat{\mathbf{q}}_4]\cdot[\hat{\mathbf{q}}_1+\hat{\mathbf{q}}_2+\hat{\mathbf{q}}_3+\hat{\mathbf{q}}_4] = [\,[\hat{\mathbf{q}}_1+\hat{\mathbf{q}}_2+\hat{\mathbf{q}}_3+\hat{\mathbf{q}}_4] + \hat{\mathbf{q}}_4[\hat{\mathbf{q}}_1] + \hat{\mathbf{q}}_4[\hat{\mathbf{q}}_2] + \hat{\mathbf{q}}_4[\hat{\mathbf{q}}_3] + \hat{\mathbf{q}}_4[\hat{\mathbf{q}}_4]\,]$$

$$= [\,\hat{\mathbf{q}}_1 + \hat{\mathbf{q}}_2 + \hat{\mathbf{q}}_3 + \hat{\mathbf{q}}_4 + \hat{\mathbf{q}}_5 + \hat{\mathbf{q}}_6 + \hat{\mathbf{q}}_7 + \hat{\mathbf{q}}_8\,].$$

Thus, in this model, $\hat{\mathbf{q}}_6 \leftrightarrow {}_{\underline{\mathbf{N}}}\hat{\mathbf{q}}_{\underline{MQ}}$. We will not address the possible meanings of $\underline{\Delta}[{}_{\underline{\mathbf{N}}}\underline{\mathrm{M}}]$ within the scope of this 'Fractment'. If we hold that, in ${}_{\underline{\mathbf{N}}}\underline{\mathrm{M}}[{}_{\underline{\mathbf{N}}}\underline{\mathbf{Q}}]$, the rules-system «***aufheben***»' operator, ${}_{\underline{\mathbf{N}}}\underline{\mathrm{M}}$ 'subsumes' or 'assimilates' the ${}_{\underline{\mathbf{N}}}\underline{\mathbf{Q}}$ rules-system operator, and posits the result externally, ***explicitly***, as the new rules-system ${}_{\underline{\mathbf{N}}}\hat{\mathbf{q}}_{\underline{MQ}}$, then ${}_{\underline{\mathbf{N}}}\hat{\mathbf{q}}_{\underline{MQ}}$ must be a kind of *qualitative, ideo-ontological* ***synthesis*** of ${}_{\underline{\mathbf{N}}}\underline{\mathrm{M}}$ and ${}_{\underline{\mathbf{N}}}\underline{\mathbf{Q}}$. Our ideogram ${}_{\underline{\mathbf{N}}}\underline{\mathrm{M}}$ denotes a system of arithmetic interpretable as one of 'pure, *unquantifiable* ***metrical*** *qualifiers*'. Our ideogram ${}_{\underline{\mathbf{N}}}\underline{\mathbf{Q}}$ denotes a system of arithmetic interpretable as one of 'pure, *unquantifiable* ***ontological*** *qualifiers*'. We thus hold that ${}_{\underline{\mathbf{N}}}\hat{\mathbf{q}}_{\underline{MQ}}$ should denote a system of arithmetic interpretable as one of 'pure, *unquantifiable* ***metrical*** *qualifiers*', but, this time, one involving subscripts from ${}_{\underline{\mathbf{N}}}\underline{\mathbf{Q}}$ rather than from $\underline{\mathbf{N}}$. It should denote an ${}_{\underline{\mathbf{N}}}\underline{\mathrm{M}}^{\underline{\mathbf{Q}}}$ version of $\underline{\mathrm{M}}$, arising from, and thus 'inheriting' from, ${}_{\underline{\mathbf{N}}}\underline{\mathrm{M}}$ and ${}_{\underline{\mathbf{N}}}\underline{\mathbf{Q}}$, after the manner which we term '***ideo-metagenealogical***':

$${}_{\underline{\mathbf{N}}}\hat{\mathbf{q}}_{\underline{MQ}} \equiv \{\, \hat{\underline{\mathbf{m}}}^{\mathbf{o}}_{\hat{\mathbf{q}}_1},\ \hat{\underline{\mathbf{m}}}^{\mathbf{o}}_{\hat{\mathbf{q}}_2},\ \hat{\underline{\mathbf{m}}}^{\mathbf{o}}_{\hat{\mathbf{q}}_3},\ \ldots \,\}.$$

This time, it is *not*, as it *was* in ${}_{\underline{\mathbf{N}}}\underline{\mathrm{M}}\cdot\underline{\mathbf{N}} - \underline{\mathbf{N}} = {}_{\underline{\mathbf{N}}}\hat{\mathbf{q}}_{\underline{MN}}$, the 'collision-fusion' of the principle & meaning of ***quantifier*** *ideography* [and of quantifier *non*-qualification] with that of ***metrical qualifier*** *ideography* [and of metrical qualifier *non*-quantification] that is to be 'ideo-externalized' and 'ideo-objectified'. This time it is the 'collision-fusion' of the principle of ***unquantifiable metrical qualification*** with that of ***unquantifiable ontological qualification***, that comes to the fore in ${}_{\underline{\mathbf{N}}}\underline{\mathrm{M}}\cdot{}_{\underline{\mathbf{N}}}\underline{\mathbf{Q}} - {}_{\underline{\mathbf{N}}}\underline{\mathbf{Q}} = {}_{\underline{\mathbf{N}}}\hat{\mathbf{q}}_{\underline{MQ}}$. The marriage of these principles births a surprise: ***immanent evocation*** of 'Boolean' self-multiplication within a sequence of arithmetics whose fundamental units have been consistently 'Contra-Boolean' ever since their «***arché***»-only meta-state in the $\tau = \mathbf{0}$ 'epoch' of $[\,\underline{\mathbf{N}}\,]^{2^\tau}$. The '*subscriptization*' subsumption of ${}_{\underline{\mathbf{N}}}\underline{\mathbf{Q}}$ meta-numerals to the $\hat{\underline{\mathbf{m}}}^{\mathbf{o}}$ generic meta-numeral script-element, in the formation of the ${}_{\underline{\mathbf{N}}}\hat{\mathbf{q}}_{\underline{MQ}}$ meta-numerals, *because of the* ***additive idempotency*** *of* ${}_{\underline{\mathbf{N}}}\underline{\mathbf{Q}}$ *meta-numbers*, yields this 'sudden' irruption of conformance to the 'Boolean' self-multiplication "law", out of its contrarieties.

The ***same*** subscript-addition rule of multiplication that has been common to all of the dialectical ideographies, to all of the terms of the categorial expansion / rules-systems progression since its «***arché***», $[\ \underline{\underline{\mathbf{N}}}\ ]^{2^0} = \underline{\underline{\mathbf{N}}}$, when applied ***normally***, yields: $[\ \hat{\hat{\mathbf{o}}}\ \mathbf{m}_{\hat{\mathbf{A}}\ \underline{\mathbf{q}}_n}\ ]^2 = \hat{\hat{\mathbf{o}}}\ \mathbf{m}_{\hat{\mathbf{A}}\ \underline{\mathbf{q}}_n + \hat{\mathbf{A}}\ \underline{\mathbf{q}}_n} = [\ \hat{\hat{\mathbf{o}}}\ \mathbf{m}_{\hat{\mathbf{A}}\ \underline{\mathbf{q}}_n}\ ]^1 = \hat{\hat{\mathbf{o}}}\ \mathbf{m}_{\hat{\mathbf{A}}\ \underline{\mathbf{q}}}$, an *immanently-arising reversal* of the previously prevailing 'Contra-Boolean' character of ontological and metrical qualifier self-multiplication. This surprising "twist" surfaces a clue crucial to the '***grand synthesis***' of **Step 7**.

A key determination, and desideratum, of the 'grand synthesis' dialectical ideography is the comprehensive 'character-ization' or 'ideograph-ization' of metrical qualification, and, more specifically, of the ***metrical*** qualification of already ***ontologically***-qualified quantifiers. This requires a fully ideographic 'arithmetico-algebraicization' of classical "Dimensional Analysis", which has heretofore been left at the 'syncopated / rhetorical' stage of arithmetico-algebraic development. Generally, the "Dimensions" of "Dimensional Analysis", and, in particular, "Dimensional Units", "units of measure", i.e., '***metrical unit-qualifiers***', are 'Contra-Boolean' in their self-multiplication behavior. However, a most murky corner of the theory of "Dimensional Analysis" concerns the oxymoron of "dimensionless" dimensions, which arise via the 'self-division', and the resulting 'zero degree' or 'zero power', of a dimensional unit. For example, the "dimension" of U.S. Gross Domestic Product is "U.S. Dollars over U. S. Dollars", or $[\mathbf{dollars}^{+1}/\mathbf{dollars}^{+1}] = [\$^{+1} \times \$^{-1}] = [\$^{+1-1}] = [\$^0] = \mathbf{100\%}$. Such "dimensionless" or 'degree zero' dimensional units are '***Boolean***' in their self-multiplicative behavior: $\%^2 = \%^1 = \%$. The 'Boolean' $_{\underline{\mathbf{N}}}\hat{\underline{\mathbf{q}}}_{\underline{\underline{\mathbf{MQ}}}}$ meta-numbers, of the form $\hat{\hat{\mathbf{o}}}\ \mathbf{m}_{\hat{\mathbf{A}}\ \underline{\mathbf{q}}_n}$, $\mathbf{n} \in \mathbf{N}$, immanently propound a new -- new to this categorial progression -- possibility of arithmetico-algebraic conception, a new principle for a 'sub-ideography' of "dimensionless" metrical qualifiers. Their remaining shortfall for filling this 'Dimension***less*** Analysis' bill is that they, like their forebears $_{\underline{\mathbf{N}}}\underline{\underline{\mathrm{M}}}$ and $_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{Q}}}$, must be 'unquantifiable' or additively idempotent. However, a shift from the subsumption / '***subscriptization***' of $_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{Q}}}$ by $_{\underline{\mathbf{N}}}\underline{\underline{\mathrm{M}}}$ to that of $_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{U}}}$ by $_{\underline{\mathbf{N}}}\underline{\underline{\mathrm{M}}}$ should do the trick. That is exactly what happens in **Step 7**.

A second surprise also surfaces for the first time, in this progression, within ${}_{\underline{N}}\hat{\underline{q}}_{\underline{MQ}}$. Recall that the two preceding systems of metrical qualification ideography, ${}_{\underline{N}}\underline{\underline{M}}$ and ${}_{\underline{N}}\hat{\underline{q}}_{\underline{MN}}$, respectively, could only encode metrical arithmetic for *one metrical unit at a time*. Thus also, neither could bring together distinct co-eval 'simple' or "fundamental" metrical units to explicitly formulate 'compound' metrical units, like the CGS **dyne**. However, ${}_{\underline{N}}\hat{\underline{q}}_{\underline{MQ}}$ immanently propounds a second new and 'concretizing' possibility of arithmetico-algebraic conception, a new principle of 'higher arithmetic' by which both of the limitations / inadequacies of the earlier-emergent ideographies can be transcended. Firstly, ${}_{\underline{N}}\hat{\underline{q}}_{\underline{MQ}}$ can *potentially* co-model the '***metrico-stasis***' of a different "dimensionless" metrical unit for 'up to' each **n** in **N**, viz. --

$$\mathbf{sec.}^{+1}/\mathbf{sec.}^{+1} = \mathbf{sec.}^{+1} \times \mathbf{sec.}^{-1} = \mathbf{sec.}^{+1+(-1)} = \mathbf{sec.}^{0} \leftrightarrow \hat{\mathbf{o}}_{\hat{\underline{q}}_1}\mathbf{m} \; ; [\hat{\mathbf{o}}_{\hat{\underline{q}}_1}\mathbf{m}]^1 = [\hat{\mathbf{o}}_{\hat{\underline{q}}_1}\mathbf{m}]^2 = [\hat{\mathbf{o}}_{\hat{\underline{q}}_1}\mathbf{m}]^3 = \ldots;$$

$$\mathbf{gm.}^{+1}/\mathbf{gm.}^{+1} = \mathbf{gm.}^{+1} \times \mathbf{gm.}^{-1} = \mathbf{gm.}^{+1+(-1)} = \mathbf{gm.}^{0} \leftrightarrow \hat{\mathbf{o}}_{\hat{\underline{q}}_2}\mathbf{m} \; ; [\hat{\mathbf{o}}_{\hat{\underline{q}}_2}\mathbf{m}]^1 = [\hat{\mathbf{o}}_{\hat{\underline{q}}_2}\mathbf{m}]^2 = [\hat{\mathbf{o}}_{\hat{\underline{q}}_2}\mathbf{m}]^3 = \ldots;$$

$$\mathbf{cm.}^{+1}/\mathbf{cm.}^{+1} = \mathbf{cm.}^{+1} \times \mathbf{cm.}^{-1} = \mathbf{cm.}^{+1+(-1)} = \mathbf{cm.}^{0} \leftrightarrow \hat{\mathbf{o}}_{\hat{\underline{q}}_3}\mathbf{m} \; ; [\hat{\mathbf{o}}_{\hat{\underline{q}}_3}\mathbf{m}]^1 = [\hat{\mathbf{o}}_{\hat{\underline{q}}_3}\mathbf{m}]^2 = [\hat{\mathbf{o}}_{\hat{\underline{q}}_3}\mathbf{m}]^3 = \ldots;$$

Secondly, ${}_{\underline{N}}\hat{\underline{q}}_{\underline{MQ}}$ can readily render 'compound "dimensionless" units' made up out of different "dimensionless" metrical units, such as the CGS **dyne** [though differences among "dimensionless" units, 'compound' or 'simple', are often ignored in today's Dimensional Analysis], viz. --

$$\mathbf{dyne/dyne} = \mathbf{dyne}^0 = [\,\mathbf{gm.\cdot cm.\,/\,sec.\cdot sec.}\,]\,/\,[\,\mathbf{gm.\cdot cm.\,/\,sec.}^2\,] \leftrightarrow \hat{\mathbf{o}}_{\hat{\underline{q}}_2 + \hat{\underline{q}}_3 - 2\hat{\underline{q}}_1}\mathbf{m}$$

$$= \hat{\mathbf{o}}_{\hat{\underline{q}}_2 + \hat{\underline{q}}_3 - \hat{\underline{q}}_1}\mathbf{m}.$$

This capability of the ${}_{\underline{N}}\hat{\underline{q}}_{\underline{MQ}}$ language to explicitly express, 'co-eval-ly' or 'contemporaneously', both multiple 'simple' or 'fundamental' "dimensionless" units, and 'compounds' of those units, in a fully-ideographic arithmetic, foreshadows the capability to explicitly formulate a fully-ideographical arithmetic for Dimensional Analysis as a whole that emerges more fully in the **Step 7** system of quanto-qualitative arithmetic.

This includes capacity for algorithmic computation with '***non***-dimensionless' -- "dimensional", or 'dimension-full' -- ***fundamental units***, and 'with compounds' of those fundamental units, as well as with 'compounds' that also involve fully-explicit "dimensionless" units, 'simple' and 'compound', mixing them all together in a seamless, and in a 're-separable' / 'disentangle-able' manner. Encompassing "dimensional" as well as "dimensionless" metrical unit-qualifiers requires the formation of a yet higher rules-system, $_{\mathbf{R}}\hat{\mathbf{q}}_{\underline{\mathbf{MU}}}$, which includes a sub-arithmetic which '***subscriptizes***' to its metrical units the 'ontic' units of the 'fully-quantifiable' arithmetic of $_{\mathbf{R,R,N}}\underline{\mathbf{U}} \equiv \{ \mathbf{R}\hat{\underline{\mathbf{u}}}_{\mathbf{N}}^{\mathbf{R}} \} = \{ \mathbf{R}\hat{\underline{\mathbf{u}}}_1, \mathbf{R}\hat{\underline{\mathbf{u}}}_2, \mathbf{R}\hat{\underline{\mathbf{u}}}_3, \ldots \}$, instead of those of the 'unquantifiable' $_{\mathbf{N}}\underline{\underline{\mathbf{Q}}}$ arithmetic to which the subscripts of the units of the $_{\mathbf{N}}\hat{\mathbf{q}}_{\underline{\mathbf{MQ}}}$ ideographical language are confined.

Sub-Transition. We are seeking a full synthesis, a 'harmonization' / complex unification, of the principles, birthed in inter-mutual opposition, in the previous categories of this $[\ \underline{\underline{\mathbf{N}}}\ ]^{2^{\tau}}$ categorial progression. That is, we seek a category / system of arithmetic which constitutes a higher, complex unity of three ideographical principles: **(1)** 'pure' *quanti*fication, **(2)** 'pure' *ontological quali*fication, and **(3)** 'pure' *metrical quali*fication. The $_{\mathbf{N}}\hat{\mathbf{q}}_{\underline{\mathbf{MQ}}}$ system of arithmetic is but a partial realization of this complex unity. It is confined to metrical qualification, and, indeed, to metrical qualification for "dimensionless" units alone. Moreover, and given its heritage, it is confined also to 'unquantifiability' at both its 'scriptal' and its 'subscriptal' syntactical / 'meta-numeralic' levels. The quest for the category / rules-system of the tripartite complex unity described above must thus move on, beyond $_{\mathbf{N}}\hat{\mathbf{q}}_{\underline{\mathbf{MQ}}}$. But, it does so with the new ideo-ontological possibilities / principles that $_{\mathbf{N}}\hat{\mathbf{q}}_{\underline{\mathbf{MQ}}}$ has heralded in tow, ready for a re-activation and for a fuller realization in the next, new, higher, more adequate context of $_{\mathbf{N}}\hat{\mathbf{q}}_{\underline{\mathbf{MU}}}$.

Step 7. **First Category of 'Grand Synthesis':** $\uparrow[\ _{\mathbf{N}}\hat{\mathbf{q}}_{\underline{\mathbf{MQN}}} \equiv {}_{\mathbf{N}}\hat{\mathbf{q}}_{\underline{\mathbf{MU}}} \equiv {}_{\mathbf{N},\alpha}\underline{\underline{\mu}}\ ]\leftrightarrow\hat{\mathbf{q}}_7$. The putative meaning of the intensional ideogram $_{\mathbf{N}}\hat{\mathbf{q}}_{\underline{\mathbf{MQN}}}$ is that of a rules-system, a system of rules of ideographical arithmetical computation, a [dialectical] ideography that we[l]ds the intensions of '$\underline{\mathrm{M}}$', '$\underline{\underline{\mathbf{Q}}}$', and '$\underline{\underline{\mathbf{N}}}$' together into a new category of intensionality that 'unitarizes', that achieves a 'composite' or 'complex' unity of, all three; that '"assimilates all the wealth of the previous developments"', $[\ \underline{\underline{\mathbf{N}}}\ ]^{\tau}$, $\tau = \mathbf{1}$ through $\tau = \mathbf{6}$. It is also that of one which hails from $\underline{\underline{\mathbf{N}}}$ as its «***arché***», so that its basic units or monads are all 'Peanic'. Since we have implied, at the subscript level, the intensional definition, that 'U ≡ QN', we have equated $_{\mathbf{N}}\hat{\mathbf{q}}_{\underline{\mathbf{MQN}}}$ with $_{\mathbf{N}}\hat{\mathbf{q}}_{\underline{\mathbf{MU}}}$. The latter 'intensionally' defines this seventh arithmetic in our progression as a 'biased' synthesis of $_{\mathbf{N}}\underline{\mathrm{M}}$ and $_{\mathbf{N}}\underline{\underline{\mathbf{U}}}$, a '*subsumption*' of $_{\mathbf{N}}\underline{\underline{\mathbf{U}}}$ by $_{\mathbf{N}}\underline{\mathrm{M}}$. We have also posited a link between the *semantical* '***subsumption***' of one system of arithmetic by another, and the *syntactical* '***subscriptization***' of the [meta-]numerals of the 'subsumed' arithmetic to those of the 'subsuming' arithmetic. That given, we might expect the $_{\mathbf{N}}\hat{\mathbf{q}}_{\underline{\mathbf{MU}}}$ arithmetic to incorporate $_{\mathbf{N}}\underline{\underline{\mathbf{U}}}$ meta-numerals at the 'scriptal' level, &/or to involve new '***meta-numbers***' whose '***meta-numerals***' take their subscripts from the $_{\mathbf{N}}\underline{\mathbf{U}}$ 'meta-numerals'. Thus, the $\hat{\mathbf{q}}_{\underline{\mathbf{MQN}}}$ '***meta-****meta-numbers*' would be ***meta-***$\underline{\mathbf{U}}$ meta-numbers, made up out of the many $\underline{\mathbf{U}}$ '*meta-numbers*'.

We expect ${}_{\underline{\mathbf{N}}}\hat{\mathbf{q}}_{\underline{\mathbf{MU}}} \equiv {}_{\underline{\mathbf{N}},\alpha}\underline{\underline{\mu}}$ to be the most ***concrete***, most ***complex***, most ***determinate*** of the seven systems of arithmetic evoked so far in this categorial progression, in terms of the potential richness of determinations or specifications of its system and meta-system descriptions -- re-constructions of the past, 'pre-constructions', or predictions, of the future -- that its native linguistic capacity supports. However, the definition ${}_{\underline{\mathbf{N}}}\hat{\mathbf{q}}_{\underline{\mathbf{MU}}} \equiv {}_{\underline{\mathbf{N}},\alpha}\underline{\underline{\mu}}$ may suggest that we do not see this meta-systematic dialectical progression ending with '***alpha-mu***'; that we expect further progression, emerging ideographic languages of even greater richness of expressive power -- of even greater concreteness and specificity -- to be dubbed ${}_{\beta}\underline{\underline{\mu}}$ or '***beta-mu***', ${}_{\gamma}\underline{\underline{\mu}}$ or '***gamma-mu***', ${}_{\delta}\underline{\underline{\mu}}$ or '***delta-mu***', etc. Indeed, all of these considerations pertain. The notations for '***quantifiers***', '***metrical qualifiers***', and '***ontological qualifiers***' are unified in ${}_{\underline{\mathbf{N}},\alpha}\underline{\underline{\mu}}$, and are all based upon the same core ideogram, μ, the Greek letter or phonogram '***mu***' used as an ideogram, akin to the English **m**, and connoting "**m**onad", as well as '**m**agnitude', '**m**ultiplicity', '**m**any-ness', '**m**anifold', **m**erates", '**m**ass', '**m**etrical', etc. '***Quantifiers***' are denoted by numerals of the form $\mu_{a,k}(\tau) \in \mathbf{N}$, with *neither* underscores, nor caret 'overscores', '^', nor 'o' 'overscores', and wherein the **a** subscript ties the '***quantifier***' to the '***metrical qualifier***', the **k** subscript to the '***ontological qualifier***'. The '***Metrical qualifiers***' are denoted by 'meta-numerals' of the form $[\,{}_{a}\hat{\hat{\underline{\underline{\mu}}}}_{\underline{\underline{\sum_j \alpha_j \hat{\underline{\underline{u}}}_j}}}\,]$, with underscores, caret 'overscores', and 'o' 'overscores', and whose subscript 'meta-numeral' components, are taken from the ${}_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{U}}}$ arithmetic: $\alpha_j \in \mathbf{N}$; $\hat{\underline{\underline{u}}}_j \in {}_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{U}}}$. '***Ontological qualifiers***' are denoted by 'meta-numerals' of the form $\hat{\underline{\underline{\mu}}}_k$, with both underscores and caret 'overscores', but without 'o' 'overscores', and whose subscripts are taken from **N**. Thus, the generic 'meta-number' of the ${}_{\underline{\mathbf{N}},\alpha}\underline{\underline{\mu}}$ arithmetic is: $(\mu_{a,k}(\tau))\cdot[\,{}_{a}\hat{\hat{\underline{\underline{\mu}}}}_{\underline{\underline{\sum_j \alpha_j \hat{\underline{\underline{u}}}_j}}}\,]\cdot\hat{\underline{\underline{\mu}}}_k$, wherein the subscript-elements **j**, **k**, & α_j are all $\in \mathbf{N}$, and where **a** denotes a label-variable, and wherein the 'space' of the ${}_{\underline{\mathbf{N}},\alpha}\underline{\underline{\mu}}$ arithmetic's 'meta-numbers' is: ${}_{\mathbf{N},\alpha}\underline{\mu} \equiv \{\,(\mathbf{N})\cdot[\,\hat{\hat{\underline{\underline{\mu}}}}_{\underline{\underline{\partial}}_{\mathbf{N}}\underline{\underline{\mathbf{U}}}}\,]\cdot\hat{\underline{\underline{\mu}}}_{\mathbf{N}}\,\}$, where $\underline{\underline{\partial}}$ takes all 'part[-ition]s' of ${}_{\mathbf{N}}\underline{\mathbf{U}}$, or $\{\,(\mathbf{N})\cdot\{\,\hat{\hat{\underline{\underline{\mu}}}}_{\underline{\underline{\partial}}_{\mathbf{N}}\underline{\underline{\mathbf{U}}}}\,\}\cdot\hat{\underline{\underline{\mu}}}_1,\ (\mathbf{N})\cdot\{\,\hat{\hat{\underline{\underline{\mu}}}}_{\underline{\underline{\partial}}_{\mathbf{N}}\underline{\underline{\mathbf{U}}}}\,\}\cdot\hat{\underline{\underline{\mu}}}_2,\ (\mathbf{N})\cdot\{\,\hat{\hat{\underline{\underline{\mu}}}}_{\underline{\underline{\partial}}_{\mathbf{N}}\underline{\underline{\mathbf{U}}}}\,\}\cdot\hat{\underline{\underline{\mu}}}_3,\ldots\,\}$. The ${}_{\underline{\mathbf{N}},\alpha}\underline{\underline{\mu}}$ 'grand synthesis' arithmetic is a full regalia '***quanto-qualitative*** ***ideography*** in a deeper, richer sense than is the earlier, 'initial synthesis' of the ${}_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{U}}}$ ideography. The ${}_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{U}}}$ ideography is one of '***quantified ontological qualifiers***', or of '***ontologically-qualified quantifiers***'. It excludes all ***metrical qualification***, except insofar as it ***implicitly*** involves "dimensionless" metrical qualifiers, by which it counts the numbers of individuals of each $\hat{\underline{\underline{u}}}_k$ onto-type forming the typical level of the type $\hat{\underline{\underline{u}}}_k$ population during a given epoch τ: $\ldots + \mathbf{m}_k(\tau)\cdot\hat{\underline{\underline{u}}}_k + \ldots$. The ${}_{\underline{\mathbf{N}},\alpha}\underline{\underline{\mu}}$ ideography, on the other hand, ***explicitly*** involves '***metrical qualification***' as well as '***ontological qualification***'.

The '***alpha-mu***' arithmetic is one of 'ontologically & metrically' or '*onto-metrically*' qualified quantifiers, or of '*quantified metrico-ontological qualifications*', or of '*metrically quanto-qualified ontological qualifications*'. The $_{\underline{\mathbf{N}},\alpha}\underline{\underline{\mu}}$ arithmetic is a 'unitation', a "complex unity", or a 'product' of three 'component' or 'factor' 'sub-arithmetics'. It is a 'meta-ideography' or 'ideography of second degree', made up out of this multiplicity of three, potentially-separate / independent ideographies taken to be of the first or linear degree. Namely, it 'factorizes' into: **(1)** the $\underline{\mathbf{N}}$ sub-arithmetic of the $\mu_{a,k}(\tau)$, for the '***quantifiers***', **(2)** the $\hat{\hat{\underline{\underline{\mu}}}}$ arithmetic of "Dimensional Analysis" for the '***metrical qualifiers***', and **(3)** the $\hat{\underline{\underline{\mu}}}$ arithmetic of '***ontological qualifiers***'. We are, in effect, '***semantifying***' the syntactical expression of the operation of $_{\underline{\mathbf{N}}}\underline{\underline{\mathrm{M}}}$ upon $_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{U}}}$ in the formation of the $_{\underline{\mathbf{N}},\alpha}\underline{\underline{\mu}}$ arithmetic, $_{\underline{\mathbf{N}}}\underline{\underline{\mathrm{M}}}\,[\,_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{U}}}\,] = {}_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{U}}} + {}_{\underline{\mathbf{N}}}\hat{\underline{\underline{\mathbf{q}}}}_{\underline{\mathbf{MQ}}}$, as a two-component process: **(a.)** a taking over and conversion / translation of the $\mathbf{m}_k(\tau)$ and $\hat{\underline{\mathbf{u}}}_k$ components of the generic $_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{U}}}$ meta-number, e.g., the generic term of a $_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{U}}}$ 'onto-dynamical ***polyqualinomial***' -- namely, the term $\mathbf{m}_k(\tau)\hat{\underline{\mathbf{u}}}_k$ -- by the $_{\underline{\mathbf{N}}}\underline{\underline{\mathrm{M}}}$ or $\hat{\hat{\underline{\underline{\mathbf{m}}}}}$ arithmetic, mapping the $\mathbf{m}_k(\tau)$ to the $\mu_{a,k}(\tau)$, and the $\hat{\underline{\mathbf{u}}}_k$ to the $\hat{\underline{\underline{\mu}}}_k$, with a related shift in the meaning and role of $\hat{\underline{\underline{\mu}}}$ vis-à-vis that of $\hat{\underline{\mathbf{u}}}$, and; **(b.)** a '***subscriptization*** subsumption' of the $_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{U}}}$ arithmetic 'under' the $_{\underline{\mathbf{N}}}\underline{\underline{\mathrm{M}}}$ or $\hat{\hat{\underline{\underline{\mathbf{m}}}}}$ arithmetic, creating the new, $\hat{\hat{\underline{\underline{\mu}}}}$ component-arithmetic within $_{\underline{\mathbf{N}},\alpha}\underline{\underline{\mu}} \leftrightarrow \hat{\underline{\mathbf{q}}}_7$. Via the latter, $_{\underline{\mathbf{N}},\alpha}\underline{\underline{\mu}}$ finally attains the seed-principle of a fully-ideographic arithmetic of "metrical units", of 'metrical qualifiers', i.e., of "Dimensional Analysis", such as was only *nascent* and partially-fulfilled in $_{\underline{\mathbf{N}}}\underline{\underline{\mathrm{M}}} \leftrightarrow \hat{\underline{\mathbf{q}}}_4$, in $_{\underline{\mathbf{N}}}\hat{\underline{\mathbf{q}}}_{\underline{\mathbf{MN}}} \leftrightarrow \hat{\underline{\mathbf{q}}}_5$, and even in $_{\underline{\mathbf{N}}}\hat{\underline{\mathbf{q}}}_{\underline{\mathbf{MQ}}} \leftrightarrow \hat{\underline{\mathbf{q}}}_6$. The later, $\hat{\hat{\underline{\underline{\mu}}}}$ sub-arithmetic of $_{\underline{\mathbf{N}},\alpha}\underline{\underline{\mu}}$ 'comes between', and mediates, the $\mathbf{m}_k(\tau)$ carried over from $_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{U}}}$ as $\mu_{a,k}(\tau)$, and the $\hat{\underline{\mathbf{u}}}_k$ carried over from $_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{U}}}$ as $\hat{\underline{\underline{\mu}}}_k$, in a way which also requires an adjustment in the meaning of $\hat{\underline{\underline{\mu}}}_k$ versus that of $\hat{\underline{\mathbf{u}}}_k$. *The emphasis of the quantification*, by the $\mathbf{m}_k(\tau)$, in $_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{U}}}$, is clearly and immediately upon the $\hat{\underline{\mathbf{u}}}_k$, which are therefore differently construed than their forerunner $\hat{\underline{\mathbf{q}}}_k$ of $_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{Q}}}$. Each $\hat{\underline{\mathbf{q}}}_k$ of $_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{Q}}}$ stands for a particular ontological category ***as a totality***. Each $\hat{\underline{\mathbf{u}}}_k$ of $_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{U}}}$ stands for one individual unit, or 'unit-individual', of the 'population' of individuals "belonging to" that ontological category. On the contrary, *the emphasis of the quantification*, by $\mu_{a,k}(\tau)$, in $_{\underline{\mathbf{N}},\alpha}\underline{\underline{\mu}}$, is on the *metrical* qualifiers, $_{a}\hat{\hat{\underline{\underline{\mu}}}}_{\sum_j \alpha_j \hat{\underline{\mathbf{u}}}_j}$, to which the $\mu_{a,k}(\tau)$ are directly juxtaposed, and which they directly semantically 'modify'. That emphasis does *not* fall directly upon the *ontological* qualifiers, $\hat{\underline{\underline{\mu}}}_k$. The effect of the $\mu_{a,k}(\tau)$ quantification operators falls only indirectly upon the $\hat{\underline{\underline{\mu}}}_k$, mediated by the $_{a}\hat{\hat{\underline{\underline{\mu}}}}_{\sum_j \alpha_j \hat{\underline{\mathbf{u}}}_j}$.

The $\hat{\underline{\mu}}_k$ become mere ontological tags, or labels, 'meta-system identifiers', and the objects of a mediate, mediated 'quanto-metrical quanto-qualification'; no longer the direct objects of the semantic 'quantifier-modifiers'. Thus the $\hat{\underline{\mu}}_k$ denote, again, helically, at a higher level of specificity or determinateness, something similar to what the $\hat{\underline{q}}_k$ of ${}_{\underline{N}}\underline{\underline{Q}}$ denote: a kind of whole ontological category or 'meta-system' symbol, no longer denoting individual units of the populations of such 'meta-systems'; no longer denoting units of onto-population individuation / individuality, as do the $\hat{\underline{u}}_k$ of ${}_{\underline{N}}\underline{\underline{U}}$. The seed-principle of a fully-operational, fully-ideographic arithmetic of "metrical units", 'metrical qualifiers' -- of "Dimensional Analysis" -- is instantiated in this 'Peanic'-units, $\underline{\mathbf{N}}$-restricted version of the $\hat{\hat{\underline{\mu}}}^{0}$ component-arithmetic within ${}_{\underline{N},\alpha}\underline{\underline{\mu}}$. The fully-operational instantiation of this principle awaits a further conceptual movement, having ${}_{\underline{N},\alpha}\underline{\underline{\mu}}$ as its «***arché***», which replicates, at a higher level, the standard pedagogical version of the 'ideo-metadynamic' of Standard $\underline{\mathbf{N}}$ itself, namely the following sequence of systems of arithmetic: $\underline{\mathbf{N}} \rightarrow \underline{\mathbf{W}} \rightarrow \underline{\mathbf{Z}} \rightarrow \underline{\mathbf{Q}} \rightarrow \underline{\mathbf{R}}$, [note: $\underline{\mathbf{Q}} \not\rightarrow \underline{\underline{\mathbf{Q}}}$], yielding, when applied to ${}_{\alpha}\underline{\underline{\mu}}$, ${}_{\underline{N},\alpha}\underline{\underline{\mu}} \rightarrow {}_{\underline{W},\alpha}\underline{\underline{\mu}} \rightarrow {}_{\underline{Z},\alpha}\underline{\underline{\mu}} \rightarrow {}_{\underline{Q},\alpha}\underline{\underline{\mu}} \rightarrow {}_{\underline{R},\alpha}\underline{\underline{\mu}}$, arriving at a "**R**eal" numbers version of ${}_{\alpha}\underline{\underline{\mu}}$ --

$$ {}_{\mathbf{R},\mathbf{R},\mathbf{N},\alpha}\underline{\underline{\mu}} \equiv \{(\mathbf{R})\cdot[\hat{\hat{\underline{\mu}}}^{0}_{\underline{\underline{\partial}}_{\mathbf{R}}\underline{\mathbf{U}}}]^{\mathbf{R}}\cdot\hat{\underline{\mu}}_{\mathbf{N}}{}^{\mathbf{R}}\} \equiv \{\mathbf{R}\cdot\{\hat{\hat{\underline{\mu}}}^{0}_{\underline{\underline{\partial}}_{\mathbf{R}}\underline{\mathbf{U}}}{}^{\mathbf{R}}\}\cdot\hat{\underline{\mu}}_1{}^{\mathbf{R}}, \mathbf{R}\cdot\{\hat{\hat{\underline{\mu}}}^{0}_{\underline{\underline{\partial}}_{\mathbf{R}}\underline{\mathbf{U}}}{}^{\mathbf{R}}\}\cdot\hat{\underline{\mu}}_2{}^{\mathbf{R}}, \mathbf{R}\cdot\{\hat{\hat{\underline{\mu}}}^{0}_{\underline{\underline{\partial}}_{\mathbf{R}}\underline{\mathbf{U}}}{}^{\mathbf{R}}\}\cdot\hat{\underline{\mu}}_3{}^{\mathbf{R}}, \ldots\}. $$

The accession to ${}_{\mathbf{R},\mathbf{R},\mathbf{N},\alpha}\underline{\underline{\mu}}$, or ${}_{\mathbf{R},\alpha}\underline{\underline{\mu}}$ for short, furnishes us with those higher versions of $\hat{\hat{\underline{\mu}}}^{0}_{\underline{\underline{\sum}}_j \alpha_j \hat{\underline{u}}_j}$ wherein $\alpha_j \in \mathbf{R}$, and $\mathbf{j} \in \mathbf{N}$, whose capability to model ideographically the 'Contra-Boolean' arithmetic of "Dimensional Analysis" can be illustrated as follows --

$$ \mathbf{sec.}^1 \leftrightarrow \hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_1} ;\ [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_1}]^1 \not\rightarrow [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_1}]^2 \not\rightarrow [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_1}]^3 \not\rightarrow [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_1}]^4 \not\rightarrow [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_1}]^5 \not\rightarrow [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_1}]^6 \not\rightarrow [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_1}]^7 \not\rightarrow \ldots; $$

$$ \mathbf{gm.}^1 \leftrightarrow \hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_2} ;\ [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_2}]^1 \not\rightarrow [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_2}]^2 \not\rightarrow [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_2}]^3 \not\rightarrow [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_2}]^4 \not\rightarrow [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_2}]^5 \not\rightarrow [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_2}]^6 \not\rightarrow [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_2}]^7 \not\rightarrow \ldots; $$

$$ \mathbf{cm.}^1 \leftrightarrow \hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_3} ;\ [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_3}]^1 \not\rightarrow [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_3}]^2 \not\rightarrow [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_3}]^3 \not\rightarrow [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_3}]^4 \not\rightarrow [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_3}]^5 \not\rightarrow [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_3}]^6 \not\rightarrow [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_3}]^7 \not\rightarrow \ldots; $$

and $$ [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_1}]^1 \perp [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_1}]^2 \perp [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_1}]^3 \perp [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_1}]^4 \perp [\hat{\hat{\underline{\mu}}}^{0}_{\hat{\underline{u}}_1}]^5 \perp \ldots, \text{ etc.} $$

The arithmetical rules-system of ${}_{\underline{\mathbf{R}},\alpha}\underline{\underline{\mu}}$ can readily render the also '***Contra***-Boolean', and '***convolute***' self-multiplicative behavior of "***dimensioned***" or '*dimension-ful*' and '***compound*** metrical units', made up out of distinct "fundamental metrical units", such as the CGS **dyne**, viz. --

$$\mathbf{dyne}^{+1} = [\mathbf{gm.}^{+1}\cdot\mathbf{cm.}^{+1} / \mathbf{sec.}^{+2}]^{+1} \quad \leftrightarrow \quad \hat{\hat{\underline{\mu}}}_{[+1\cdot\hat{\underline{u}}_2]+[+1\cdot\hat{\underline{u}}_3]+[-2\cdot\hat{\underline{u}}_1]} = \hat{\hat{\underline{\mu}}}_{\hat{\underline{u}}_2+\hat{\underline{u}}_3-2\hat{\underline{u}}_1}$$

$$\not\lessgtr,\ \perp\ \mathbf{dyne}^{+2} = [\mathbf{gm.}^{+1}\cdot\mathbf{cm.}^{+1}\cdot\mathbf{sec.}^{-2}]^{+2}$$

$$= [\mathbf{gm.}^{+2}\cdot\mathbf{cm.}^{+2}\cdot\mathbf{sec.}^{-4}]^{+1} \quad \leftrightarrow \quad \hat{\hat{\underline{\mu}}}_{[+2\cdot\hat{\underline{u}}_2]+[+2\cdot\hat{\underline{u}}_3]+[-4\cdot\hat{\underline{u}}_1]} = \hat{\hat{\underline{\mu}}}_{2\hat{\underline{u}}_2+2\hat{\underline{u}}_3-4\hat{\underline{u}}_1}.$$

The arithmetical rules-system of ${}_{\underline{\mathbf{R}},\alpha}\underline{\underline{\mu}}$ can also render the also '***Contra-Contra***-Boolean' and '***convolute***' self-multiplicative behavior of "***dimensionless***" metrical units, viz. --

$$\mathbf{sec./sec.} = \mathbf{sec.}^{0=+1-1} \leftrightarrow \hat{\hat{\underline{\mu}}}_{+\hat{\underline{u}}_1-\hat{\underline{u}}_1} = \hat{\hat{\underline{\mu}}}_{0\cdot\hat{\underline{u}}_1} = \hat{\hat{\underline{\mu}}}_{u_0};\quad [\hat{\hat{\underline{\mu}}}_{u_0}]^1 = [\hat{\hat{\underline{\mu}}}_{u_0}]^2 = [\hat{\hat{\underline{\mu}}}_{u_0}]^3 = \ldots;$$

$$\mathbf{gm./gm.} = \mathbf{gm.}^{0=+1-1} \leftrightarrow \hat{\hat{\underline{\mu}}}_{+\hat{\underline{u}}_2-\hat{\underline{u}}_2} = \hat{\hat{\underline{\mu}}}_{0\cdot\hat{\underline{u}}_2} = \hat{\hat{\underline{\mu}}}_{u_0};\quad [\hat{\hat{\underline{\mu}}}_{u_0}]^1 = [\hat{\hat{\underline{\mu}}}_{u_0}]^2 = [\hat{\hat{\underline{\mu}}}_{u_0}]^3 = \ldots;$$

$$\mathbf{cm./cm.} = \mathbf{cm.}^{0=+1-1} \leftrightarrow \hat{\hat{\underline{\mu}}}_{+\hat{\underline{u}}_3-\hat{\underline{u}}_3} = \hat{\hat{\underline{\mu}}}_{0\cdot\hat{\underline{u}}_3} = \hat{\hat{\underline{\mu}}}_{u_0};\quad [\hat{\hat{\underline{\mu}}}_{u_0}]^1 = [\hat{\hat{\underline{\mu}}}_{u_0}]^2 = [\hat{\hat{\underline{\mu}}}_{u_0}]^3 = \ldots;$$

$$\mathbf{dyne}^0 = [\mathbf{gm.cm.sec.}^{-2}] / [\mathbf{gm.cm.sec.}^{-2}] \leftrightarrow \hat{\hat{\underline{\mu}}}_{+\hat{\underline{u}}_2+\hat{\underline{u}}_3-2\hat{\underline{u}}_1-\hat{\underline{u}}_2-\hat{\underline{u}}_3+2\hat{\underline{u}}_1} = \hat{\hat{\underline{\mu}}}_{u_0}.$$

Note also that $[\sum_j \alpha_j\hat{\underline{u}}_j \not\lessgtr \sum_j \beta_j\hat{\underline{u}}_j]$ implies $[\hat{\hat{\underline{\mu}}}_{\sum_j \alpha_j\hat{\underline{u}}_j}\cdot\hat{\underline{\mu}}_k \not\lessgtr \hat{\hat{\underline{\mu}}}_{\sum_j \beta_j\hat{\underline{u}}_j}\cdot\hat{\underline{\mu}}_k]$, and that

$$[\hat{\hat{\underline{\mu}}}_{\sum_j \alpha_j\hat{\underline{u}}_j}\cdot\hat{\underline{\mu}}_k \equiv \hat{\underline{\mu}}_{k+\sum_j \alpha_j\hat{\underline{u}}_j} \perp \hat{\hat{\underline{\mu}}}_{\sum_j \beta_j\hat{\underline{u}}_j}\cdot\hat{\underline{\mu}}_k \equiv \hat{\underline{\mu}}_{k+\sum_j \beta_j\hat{\underline{u}}_j}],\text{ and}$$

$$[\hat{\hat{\underline{\mu}}}_{\sum_j \alpha_j\hat{\underline{u}}_j} \perp \hat{\hat{\underline{\mu}}}_{\sum_j \alpha_j\hat{\underline{u}}_j}\cdot\hat{\underline{\mu}}_k \perp \hat{\hat{\underline{\mu}}}_{\sum_j \beta_j\hat{\underline{u}}_j};\quad \hat{\hat{\underline{\mu}}}_{\sum_j \alpha_j\hat{\underline{u}}_j}\cdot\hat{\underline{\mu}}_k \perp \hat{\underline{\mu}}_k],\text{ and}$$

$$[\hat{\hat{\underline{\mu}}}_{\sum_j \beta_j\hat{\underline{u}}_j} \perp \hat{\hat{\underline{\mu}}}_{\sum_j \beta_j\hat{\underline{u}}_j}\cdot\hat{\underline{\mu}}_k \perp \hat{\hat{\underline{\mu}}}_{\sum_j \alpha_j\hat{\underline{u}}_j};\quad \hat{\hat{\underline{\mu}}}_{\sum_j \beta_j\hat{\underline{u}}_j}\cdot\hat{\underline{\mu}}_k \perp \hat{\underline{\mu}}_k].$$

Relative Descriptive Specificity / Concreteness of Descriptive Power: ${}_{\underline{\underline{N}}}\underline{\underline{Q}}$ vs. ${}_{\underline{\underline{N}}}\underline{\underline{U}}$ vs. ${}_{\underline{\underline{N}},\alpha}\underline{\underline{\mu}}$. The 'comparative linguistics' of the **3** major ideographic languages -- or 'dialectical ideographies' -- built-up via $[\,\underline{\underline{\mathbf{N}}}\,]^{2^{\tau}}$, $\tau = \mathbf{1}$ through $\tau = \mathbf{3}$, namely, ${}_{\underline{\underline{N}}}\underline{\underline{Q}}$, ${}_{\underline{\underline{N}}}\underline{\underline{U}}$, and ${}_{\underline{\underline{N}},\alpha}\underline{\underline{\mu}}$, reveal a gradient of increasing complexity/concreteness -- or 'determinacy'/'determinateness'/specificity -- of modeling power.

I. When we interpret the raw ${}_{\underline{\underline{N}}}\underline{\underline{Q}}$ arithmetic for a given universe of discourse $\mathbf{u}$, and for a given taxonomic level $\mathbf{n}$, the resulting model algorithm-symbol, $\{\,[\,{}^{\mathbf{n}}_{\mathbf{u}}\underline{Q}_0\,]^{2^{\tau}}\,\}$, instructs us to crank out ***a sequence of expanding series*** of *intensional* symbols for ***possible*** ontological categories or 'ontological increments', in lock-step fashion, and in standard order as the epochs-index or revolutions-index τ advances. It describes the '*Possibility-Spaces*' of ontological '***Possibility-Theory***'.

II. The ${}_{\underline{\underline{W}}}\underline{\underline{U}}$ ideography, wherein $\mathbf{W} \equiv \{\,\mathbf{0, 1, 2, 3,...}\}$, can model a 'meta-evolving', 'onto-dynamical' ***universe of discourse*** as a '*population of populations*', with one population head-count quantifier-function-value, $\mathbf{m}_k(\tau)$, for each 'onto' or ontological category, denoted $\hat{\underline{\mathbf{u}}}_k$, interpreted as denoting an individual 'capita' unit of the population of individuals inhering in that 'onto'. Moving on to the ${}_{\underline{\underline{R}}}\underline{\underline{U}}$ ideography, we can have 'partial values', **R**eal-values, of the quantifier-function-values, $\mathbf{m}_k(\mathbf{t}_\tau)$, with **R**eal, "continuous time" $\mathbf{t}_\tau$ as the argument [the $\underline{\underline{\sum}}$ symbol signifies ***quanto-****qualitative summation*]:

$$ {}^{\mathbf{n}}_{\mathbf{u}}\underline{U}_{t_\tau} \;=\; \mathbf{u}_0 + \underline{\underline{\sum}}_{k=1,N(t_\tau)}\mathbf{m}_k(\mathbf{t}_\tau)\cdot[{}^{\mathbf{n}}_{\mathbf{u}}\hat{\underline{\mathbf{u}}}_k] = \mathbf{u}_0 + \mathbf{m}_1(\mathbf{t}_\tau)\cdot[{}^{\mathbf{n}}_{\mathbf{u}}\hat{\underline{\mathbf{u}}}_1] + \mathbf{m}_2(\mathbf{t}_\tau)\cdot[{}^{\mathbf{n}}_{\mathbf{u}}\hat{\underline{\mathbf{u}}}_2] + \mathbf{m}_3(\mathbf{t}_\tau)\cdot[{}^{\mathbf{n}}_{\mathbf{u}}\hat{\underline{\mathbf{u}}}_3] + \mathbf{...}, $$

where $\mathbf{N}(\tau)$ denotes the maximum subscript 'extant', the subscript of the ontological qualifier assigned to the most advanced onto emerged as of τ. Thus, the $\{\,{}^{\mathbf{n}}_{\mathbf{u}}\underline{U}_\tau\,\}$ models use 'meta-dynamical poly-qualinomials' to describe these '***non-amalgamative sums***' or '***cumula***' *of different populations* as '***multi-population meta-distributions***', 'picto-graphable' as discrete 'meta-histograms' with a vertical bar for each onto, whose population-count-denoting height changes as τ advances. We have, in effect, a 'multi-system', 'multi-state-space' model, but one confined to a single state-variable per 'system', and, thus, to a one-dimensional state-space for each 'system'. Alternately, the entire ${}^{\mathbf{n}}_{\mathbf{u}}\underline{U}_{t_\tau}$ '***poly-qualinomial***' can be viewed as describing a single, ***but*** [***meta-***]***dynamically-evolving*** state-space/control-space combination, with $\mathbf{N}(\mathbf{t}_\tau)$ total axes / dimensions. As in ecological dynamical systems models of predator-prey systems, the changing population head counts of each 'species' provide the sole state-variables. There is an advance, in the $\{\,{}^{\mathbf{n}}_{\mathbf{u}}\underline{U}_\tau\,\}$ models, over the lock-step characteristics of the $\{\,[\,{}^{\mathbf{n}}_{\mathbf{u}}\underline{Q}_0\,]^{2^{\tau}}\,\}$ models, and over their 'possibilistic' nature. Actualized population levels can be expressed via the quantifier functions, $\mathbf{m}_k(\tau)$, which, in ${}_{\underline{\underline{W}}}\underline{\underline{U}}$, unlike in ${}_{\underline{\underline{N}}}\underline{\underline{U}}$, can take on ***zero values***. Thereby, the 'extinction' of formerly extant ontos or systems can be formulated. Even the 'non-arisal' of a certain inherently unviable $\{\,{}^{\mathbf{n}}_{\mathbf{u}}\underline{Q}_\tau\,\}$ possibility, e.g., $\hat{\underline{\mathbf{q}}}_j$, or of one which is not meaningful within a specific model, can be formulated via the $\{\,{}^{\mathbf{n}}_{\mathbf{u}}\underline{U}_\tau\,\}$ modeling language, via the assignment $[\,\forall\tau \in \mathbf{N}\,][\,\mathbf{m}_k(\tau) = \mathbf{0}\,]$, whereas this is ***not*** expressible via $\{\,{}^{\mathbf{n}}_{\mathbf{u}}\underline{Q}_\tau\,\} \sqsubset {}_{\underline{\underline{N}}}\underline{\underline{Q}}$.

Also, reversals or other re-orderings of the standard, 'lock-step' ${}^{n}_{u}\underline{Q}_{\tau}$ historical order of emergence of ontos can be orchestrated via the τ-order in which the $\mathbf{m}_{k}(\tau)$ quantifier functions emerge into positive values from their pre-emergence zero-values. But 'non-capita' state-variables and 'control-parameter-variables', requiring metrics beyond mere 'head-count' cardinality, are not expressible in ${}_{N}\underline{\underline{U}}$, or ${}_{W}\underline{\underline{U}}$, or ${}_{Z}\underline{\underline{U}}$, or ${}_{Q}\underline{\underline{U}}$, or even in ${}_{R}\underline{\underline{U}}$. For that, we must move on to ${}_{\alpha}\underline{\underline{\mu}}$.

In summary, a $\{ {}^{n}_{u}\underline{U}_{\tau} \}$ model describes an evolving 'cumulum of systems', a growing, self-extending, non-amalgamative 'sum of systems' or 'sum of state-spaces' as a 'sum of dynamical, "time-varying" state-variable-values', 'sum of state-vector-values' or 'sum of states', each state-space being one-dimensional only, and orthogonal to all of the others. Each $\hat{\underline{u}}_{k}$ denotes a distinct dynamical system as a separate 'onto' or 'ontological category'. Thus, overall, $\{ {}^{n}_{u}\underline{U}_{\tau} \}$ models '*a meta-system made up out of multiple systems*' in two senses:

(**i.**) ***synchronically***, for any given τ value, ${}^{n}_{u}\underline{U}_{\tau}$ denotes a poly-qualinomial or "***series***" of co-existent, contemporaneous systems, via their states [state-vectors], embedded in the 1-D, population-dynamics [state-]spaces to which those state-values or state-vector-values belong;

(**ii.**) ***diachronically***, for successive values of τ, $\{ {}^{n}_{u}\underline{U}_{\tau} \}$ denotes a temporal / historical, sequence of such ***series***, normally net-expanding in 'number of terms' terms; a temporal succession of such 'states-sums', 'systems-sums', or '[state-]spaces-sums'. These include progressions of system-ontos in which, e.g., a successor system-onto assigned to $\hat{\underline{u}}_{2^{\tau}}$ denotes a 'neo-onto' population, a population of '***meta-fractal meta***^{n}***-units***', that is, of '***meta***^{n}***-monads***', made of multiplicities of the '***meta***$^{n-1}$***-units***' or '***meta***$^{n-1}$***-monads***' from that older or earlier-arising onto / 'system', assigned to $\hat{\underline{\mu}}_{2^{\tau-1}}$. The $\{ {}^{n}_{u}\underline{U}_{\tau} \}$ models can describe 'a [meta-]population of systems' in 'parallel' "time evolutions" in a universe[-of-discourse] consisting of multiple distinct but 'co-extant', "co-evolving" dynamical systems, by means of an 'inter-argument-ation' of the $\mathbf{m}_{k}(\tau)$ quantifier functions. This interlocking of arguments via this inter-mutual argument-inclusion or domain-inclusion gives these models advantages vis-à-vis typical models of dynamical systems theory, which describe single systems in isolation from any explicit context of other systems and other state-spaces; in isolation from both their synchronic 'neighboring systems' and their diachronic 'successor systems'. The $\{ {}^{n}_{u}\underline{U}_{\tau} \}$ models can also describe diachronic '***meta***-dynamical ***meta***-evolutions', to even larger such multiples of [evolute] 'parallel and successor' systems, as well as the giving birth to a new system / system-ontology by an old system / system-ontology, in an ideographical linguistic context which can also accommodate: **(a.)** ***systems-extinctions***, **(b.)** ***non-realizations*** of ***possible*** successor-systems and of ***possible*** hybrid-systems, and; **(c.)** ***re-orderings of system-succession***, again, via 'inter-functionality' or 'inter-argument-ation' of the $\mathbf{m}_{k}(\tau)$ quantifier functions, including inter-determination of their zeros. However, $\{ {}^{n}_{u}\underline{U}_{\tau} \}$ multi-population models remain confined to single state-variable, population state-variable, one-dimensional state-space descriptions. Not so the $\{ {}^{n}_{u}, {}_{\alpha}\underline{\mu}_{\tau} \}$ models.

III. The $_{\mathbf{W},\alpha}\underline{\underline{\mu}}$ ideography can model a 'meta-evolving', 'onto-dynamical' ***universe of discourse*** as a '*population of meta-systems*', with multiple metrical-qualifier quantifier-functions, $\{\mu_{\ell_k,k}(\tau)\}$ quantifying state-variable metrical qualifiers or control-parameter metrical qualifiers, and with multiple such quantifier-functions for each 'metasystem' -- one quantifier-function for each metrical dimension of each meta-system's state/control meta-space -- with each $\hat{\underline{\mu}}_k$ interpreted as denoting a qualitatively, ontologically distinct 'meta-system'. Moving on to the $_{\mathbf{R},\alpha}\underline{\underline{\mu}}$ ideography, we can have **R**eal-values of the quantifier-functions, $\{\mu_{\ell_k,k}(\mathbf{t}_\tau)\}$, with **R**eal, "continuous time" $\mathbf{t}_\tau$ as argument or domain variable, and using a different time-scale for each epoch, τ, to encompass the '***temporal acceleration***' aspect of the diachronic dimension of '***meta-fractal***' scaling, that is, of scale-analogous, '***quanto-qualitative self-similarity structured***', 'quanto-***ontological self-similarity*** structured' 'self-subsumption' scaling:

$$\overset{\mathbf{n}}{\mathbf{u}},_{\alpha}\underline{\mu}_{\mathbf{t}_\tau} = \boldsymbol{\mu}_0 + \underline{\underline{\sum}}_{k=1,N(\mathbf{t}_\tau)}\Big[\underline{\underline{\sum}}_{\ell_k=1,M_k(\mathbf{t}_\tau)}\Big[\overset{\mathbf{n}}{\mathbf{u}}\mu_{\ell_k,k}(\mathbf{t}_\tau)\cdot\Big[{}_{\ell_k}\hat{\underline{\mu}}^{\hat{\mathbf{o}}}_{\underline{\underline{\sum}}_j \boldsymbol{\alpha}_j\hat{\underline{\mathbf{u}}}_j}\Big]\Big]\Big]\cdot\Big[\Big[\overset{\mathbf{n}}{\mathbf{u}}\hat{\underline{\mu}}_k\Big]\Big] = \boldsymbol{\mu}_0 + \ldots$$

$$\underline{\underline{\sum}}_{\ell_3=1,M_3(\mathbf{t}_\tau)}\Big[\overset{\mathbf{n}}{\mathbf{u}}\mu_{\ell_3,3}(\mathbf{t}_\tau)\cdot{}_{\ell_3}\hat{\underline{\mu}}^{\hat{\mathbf{o}}}_{\underline{\underline{\sum}}_j \boldsymbol{\alpha}_j\hat{\underline{\mathbf{u}}}_j}\cdot\overset{\mathbf{n}}{\mathbf{u}}\hat{\underline{\mu}}_3\Big]\ldots + \ldots\ \underline{\underline{\sum}}_{\ell_7=1,M_7(\mathbf{t}_\tau)}\Big[\overset{\mathbf{n}}{\mathbf{u}}\mu_{\ell_7,7}(\mathbf{t}_\tau)\cdot{}_{\ell_7}\hat{\underline{\mu}}^{\hat{\mathbf{o}}}_{\underline{\underline{\sum}}_j \boldsymbol{\alpha}_j\hat{\underline{\mathbf{u}}}_j}\cdot\overset{\mathbf{n}}{\mathbf{u}}\hat{\underline{\mu}}_7\Big] + \ldots$$

$$\underline{\underline{\sum}}_{\ell_{15}=1,M_{15}(\mathbf{t}_\tau)}\Big[\overset{\mathbf{n}}{\mathbf{u}}\mu_{\ell_{15},15}(\mathbf{t}_\tau)\cdot{}_{\ell_{15}}\hat{\underline{\mu}}^{\hat{\mathbf{o}}}_{\underline{\underline{\sum}}_j \boldsymbol{\alpha}_j\hat{\underline{\mathbf{u}}}_j}\cdot\overset{\mathbf{n}}{\mathbf{u}}\hat{\underline{\mu}}_{15}\Big]\ldots + \ldots\ \underline{\underline{\sum}}_{\ell_{31}=1,M_{31}(\mathbf{t}_\tau)}\Big[\overset{\mathbf{n}}{\mathbf{u}}\mu_{\ell_{31},31}(\mathbf{t}_\tau)\cdot{}_{\ell_{31}}\hat{\underline{\mu}}^{\hat{\mathbf{o}}}_{\underline{\underline{\sum}}_j \boldsymbol{\alpha}_j\hat{\underline{\mathbf{u}}}_j}\cdot\overset{\mathbf{n}}{\mathbf{u}}\hat{\underline{\mu}}_{31}\Big] + \ldots,$$

wherein: (α) each term $\underline{\underline{\sum}}_{\ell_k=1,M_k(\mathbf{t}_\tau)}\Big[\overset{\mathbf{n}}{\mathbf{u}}\mu_{\ell_k,k}(\mathbf{t}_\tau)\cdot{}_{\ell_k}\hat{\underline{\mu}}^{\hat{\mathbf{o}}}_{\underline{\underline{\sum}}_j \boldsymbol{\alpha}_j\hat{\underline{\mathbf{u}}}_j}\cdot\overset{\mathbf{n}}{\mathbf{u}}\hat{\underline{\mu}}_k\Big]$, describes a *sum of axes* [coordinates; meta-states], i.e., a separate meta-system, via a separate 'meta-space', or 'meta-dynamical', ***time-varying space***; a unified state-space/control-parameter-space, with ***time-varying dimensionality***; time-varying axial, metrical [state-variable & control-parameter] content; (β) the function $N(\mathbf{t}_\tau)$ determines the maximum subscript 'extant', the subscript of the ontological qualifier assigned to the most advanced onto/meta-system emerged as of time $\mathbf{t}_\tau$; (γ) the function $M_k(\mathbf{t}_\tau)$ determines the maximum count of dimensions, i.e., of state-variable or control-parameter metrics / "axes", extant in the state/control meta-space of the $\hat{\underline{\mu}}_k$ meta-system as of $\mathbf{t}_\tau$, and; (δ) the value $\boldsymbol{\mu}_0$ denotes the [self-multiplicatively 'Boolean'] repository of all ***not-yet***-'finitely'-manifest meta-systems, and of all ***formerly***-'finitely'/discernibly-manifest, ***but no longer*** manifest meta-systems ontology. The value $\boldsymbol{\mu}_0$ is key to the '***semantification***' of the finite-time infinities or "***singularities***" besetting especially the ***nonlinear*** differential equation models of dynamics, a '[re-]semantification' which occurs when those equations are '***re-qualified***' using the metrical & ontological qualifiers of the $_{\mathbf{R},\alpha}\underline{\underline{\mu}}$ arithmetics. If the value $\mathbf{0}^{-1}$ augurs escalation to "infinite" quantifier magnitudes, $+\infty$, «***au contraire***»:

$$\Big[\mathbf{0}\cdot\hat{\underline{\mu}}^{\hat{\mathbf{o}}}_{\underline{\underline{\sum}}_j \boldsymbol{\alpha}_j\hat{\underline{\mathbf{u}}}_j}\cdot\hat{\underline{\mu}}_k\Big]^{-1} = \big[\overset{\circ}{\mu}_0\cdot\hat{\underline{\mu}}_k\big]^{-1} = \big[\boldsymbol{\mu}_0\big]^{-1} = \big[\boldsymbol{\mu}_0^{+1}/\boldsymbol{\mu}_0^{+1}\big] = \big[\boldsymbol{\mu}_0^{+1}\cdot\boldsymbol{\mu}_0^{-1}\big] = \mu_{+0-0} = \mu_{+0} = \mu_{-0} = -\boldsymbol{\mu}_0 = +\boldsymbol{\mu}_0 = \boldsymbol{\mu}_0.$$

The $\overset{\circ}{\mu}_0$ & μ_0 values signify 'existential', 'qualitative', 'ontological' "***missing-ness***", e.g., the ***sudden absence*** of an onto / meta-system when 'ontological conversion' becomes complete within the locus modeled. Thereby, the ontological commitments of a model-specification are locally self-violated by the temporal out-working of the very temporal and contental logic implicit in that model-specification itself, a kind of 'meta-*dynamical*', '*temporal* ***self-«reductio ad absurdum»***' of the model. Thus, consider the context of collision-singularities in the self-building-up of a "solar system"; of two large planets in a nebular disk together with myriads of smaller planets, micro-planets, & dust, where the two collide & coalesce/fragment, forming many '***meta^{n+1}-planets made up out of*** *parts of* ***the two meta^{n}-planets***'. The planetary ontology of the problem changes at the ***moment of collision***, which is also the moment of a ***zero-division singularity*** in the Newtonian formulation. That formulation idealizes the **3**-D bodies of planets **1** & **2** to mere centers-of-mass, or mass-***points***. The Newtonian gravitic force formula's denominator is the square of the distance, changing as a function of time, between these two mass-points, $\mathbf{r}_{1,2}(\mathbf{t})^2$. "Collision" of these two planets means, in this Newtonian idiom, that the two mass-points converge until, at $\mathbf{t} = \mathbf{t}^*$, they co-inhabit the same point of physical space. So $\mathbf{r}_{1,2}(\mathbf{t}^*)^2 = \mathbf{0}^2 = \mathbf{0}$ at collision time, $\mathbf{t}^*$. At that moment, the Newtonian force formula attains a zero denominator. This division by zero implies an *infinite* gravitic *force* "between" mass-points that no longer have "anything" "between" them. The original 'planet-ontos' pre-supposed in the model specification ***cease to exist*** at that moment. A new planetary ontology comes into being. The original ontology has thus self-invalidated & self-negated at and as that moment. That original ontology there 'converts' itself into a new, qualitatively-different ontology, and, in the process, the dynamics of the many-planet system, its ***particular*** laws of motion, its vector flow-field in its phase space, ***and the very dimensionality and axial content of that phase-space***, as of its ***planets-masses control-parameter-space***, change as a result. If we limit our modeling to ***pure quantifier*** dynamics, ***eliding*** any metrical or ontological qualifications, then this moment of collision-singularity of these two planets, of masses $\mathbf{m}_1$ & $\mathbf{m}_2$, computes to: $\mathbf{f}_{1,2}(\mathbf{t}^*) = \mathbf{G}\cdot\mathbf{m}_1\cdot\mathbf{m}_2/\mathbf{0}$ "$= \infty$", *an* ***infinitely wrong*** *answer empirically*, since all actual forces remain finite throughout. 'Metrically [***re-***]***qualifying***' that equation via $[\,\underline{\mathbf{G}}\cdot\underline{\mathbf{M}}^2\,] / [\,\underline{\mathbf{L}}^2\,] = [\,\underline{\hat{\overset{\circ}{\mu}}}_{+\underline{u}_2+\,3\underline{u}_3-\,2\underline{u}_1}\,] / [\,\underline{\hat{\overset{\circ}{\mu}}}_{+2\underline{u}_3}\,]$, we obtain instead [dropping some of the 'caret overscores' for convenience]:

$$(dp(t^*)/dt)\cdot[\,\underline{\hat{\overset{\circ}{\mu}}}_{+\underline{u}_2+\,\underline{u}_3-\,2\underline{u}_1}\,] = (f_{1,2}(t^*))\cdot[\,\underline{G}\cdot\underline{M}^2\,] / [\,\underline{L}^2\,] = (G\cdot m_1\cdot m_2)\cdot[\,\underline{\hat{\overset{\circ}{\mu}}}_{+\underline{u}_2+\,3\underline{u}_3-\,2\underline{u}_1}\,] / (0)\cdot[\,\underline{\hat{\overset{\circ}{\mu}}}_{+2\underline{u}_3}\,]$$

$$= (G\cdot m_1\cdot m_2)\cdot[\,\underline{\hat{\overset{\circ}{\mu}}}_{+\underline{u}_2+\,3\underline{u}_3-\,2\underline{u}_1}\,] / \{0\cdot\underline{\hat{\overset{\circ}{\mu}}}_{+2\underline{u}_3}\} = (G\cdot m_1\cdot m_2)\cdot[\,\underline{\hat{\overset{\circ}{\mu}}}_{+\underline{u}_2+\,3\underline{u}_3-\,2\underline{u}_1}\,] / \overset{\circ}{\mu}_0 =$$

$$= (G\cdot m_1\cdot m_2)\cdot[\,\underline{\hat{\overset{\circ}{\mu}}}_{+\underline{u}_2+\,3\underline{u}_3-\,2\underline{u}_1}\,]\cdot\overset{\circ}{\mu}_{-0} = (G\cdot m_1\cdot m_2)\cdot\overset{\circ}{\mu}_{-0} = \overset{\circ}{\mu}_{-0} = \overset{\circ}{\mu}_0 \quad [\text{as } -\mathbf{0} = +\mathbf{0} = \mathbf{0}].$$

The $\overset{\circ}{\mu}_0$ 'placeholder' value signifies, not an "infinite" force magnitude, but, on the contrary, a '***dis-extension***', that is, a '***dis-existentiation***' of that force, as of the former planets which emanated it. One potential problem here is that even recurring *numerator*-zeros, due to repeated fluctuations back and forth across arbitrary or merely conventional scalar zeros of certain metrics, e.g., temperature, current, voltage, etc., would lead to momentaneous $\overset{\circ}{\mu}_0$ values, appearing to suggest a 'winking into and out of and again back into' existence of, e.g., nonlinear, "self-oscillator" dynamical systems. How would one distinguish between true existential zeros and mere quantifier zeros in ${}_{\underline{\mathbf{w}},\alpha}\underline{\mu}$? It appears, on the face of it, that the ${}_{\alpha}\underline{\mu}$ language has no inherent capacity to distinguish between 'pure-quantitative', 'scalar' zeros, and 'quanto-qualitative' zeros like $\overset{\circ}{\mu}_0$ and μ_0. As an illustration of the model above, $\underline{\hat{\mu}}_3$ might be assigned to the ontological category of the '***meta-system***' of [proto-] ***galaxies***, 'converting' ***sub-nuclear particles*** to sub-nuclear "particles" + ***sub-atomic particles***; $\underline{\hat{\mu}}_7$ to that of $\underline{\hat{\mu}}_3$'s successor 'meta-system' of ***stars***, 'converting' sub-atomic "particles" [ionized, 'plasmic' Hydrogen Ur-atoms, i.e., protons] to sub-atomics + ***atoms***; $\underline{\hat{\mu}}_{15}$ to that of $\underline{\hat{\mu}}_7$'s successor 'meta-system' of [proto-] ***planetary nebulae***, 'converting' atoms to atoms + ***molecules***, & $\underline{\hat{\mu}}_{31}$ to that of $\underline{\hat{\mu}}_{15}$'s successor 'meta-system' of planetary ***deep-lithospheric*** [proto-]***biospheres***, 'converting' molecules to molecules + anaerobic, chemosynthetic and hyper-thermophilic / chemotrophic ***prokaryotic cells*** [archaea and archeo-bacteria].

By virtue of the descriptive facilities inherent in the $_{\underline{\mathbf{N}},\alpha}\underline{\mu}$ 'epoch' of our meta-systematic dialectical exposition of these dialectical ideographies, the "***state-spaces***" & "***control-parameter-spaces***" of contemporary "***dynamical systems theory***" can be unified in the concept of an integrated state/control '***meta-space***' for '***meta-system meta-dynamics***'. This unification provides a natural venue for modeling the many nonlinear, 'self-reflexive', 'self-refluxive' systems that exhibit the phenomena of **(1)** a deep form of '***self-bifurcation***', tied to **(2)** '***meta-finite***, ***complete-conversion singularity***'. These linked phenomena involve a coupling of control-space to state-space, because the behaviors described by some state-variables of the state-space are causal drivers of the values of one or more control-space control-parameters, whose shifts in turn induce "bifurcations", profound breaks in the 'dynamical laws' or 'trajectory-geometries' of the co-varying state-variables within the state-space. This coupling of state-space & control-space can be captured via 'arguments interlock'; implicit 'inter-inclusions' of the quantifier functions, $\{\mu_{\ell_k,k}(\mathbf{t}_\tau)\}$, in each other's domains, whereas their domains appear, explicitly, to contain or depend upon the "time parameter", $\mathbf{t}_\tau$, alone. When the control parameters at issue are involved in time-varying functions that measure the amount remaining of a store of ontological resource material that the state-variable-measured behavior is converting to material of a different ontological category, functions that appear *in the denominator* of the differential equation-function, &/or of the solution-function, then ***locally complete depletion***, or ***complete conversion***, of that old ontology into the new may coincide with a zero-division singularity in the model equations. This joins phenomena **(1)** & **(2)** together in a '***metafinite, self-bifurcation conversion-singularity***'. For example, a chemical-combustion rocket is an 'autokinetic' or self-moving, 'self-refluxively' self-propelling, 'self-forced' projectile, converting chemical fuel mass to exhaust mass and kinetic energy, and 'nonlinearized' because its mass parameter is no longer constant & no longer independent of its [unknown, to-be-solved-for] velocity-function state-variable. Rather, it is an [unknown] function of that function-unknown. Thus its momentum & [self-]force are 'entangled' functions of both its mass & its celerities of spatial motion. The closed-form solution of the "***constant-throttle***" version of the resulting nonlinear differential ***rocket equation*** for an idealized rocket harbors a '***metafinite, self-bifurcation complete-conversion singularity***' if the initial, $\mathbf{t} = \mathbf{0}$, total rocket mass equals its initial fuel-mass. Via this unification & coupling-together of state- & control-space, we may also see that the resulting 'meta-space' is no static backdrop, but a '[***meta-***]***dynamical object***' in its own right. Therein, finite quantitative change [still] signifies 'mere' "evolution". The movements along state-space "trajectories" & control-space "paths" that reflect 'purely quantitative' changes in the magnitudes of state-variables & control-parameters, as the "time parameter" grows, [still] signify "***dynamical***" or "time" "***evolution***". But singularity is the sign of '***meta-***evolution'. Singularity signifies '***meta-dynamical***' movement, i.e., '***change of*** [dynamical] ***laws***', '***change of space***'; changes in the ***dimensionality*** & ***metrical content*** of the state-space & the control-space aspects of this now-unified '***meta-space***', as well as '***change of epoch***'. Singularity is the signpost of that deeper form of '***self-***bifurcation'. Therein, some of the state-space state-variables tie to some of the control-space control-parameters, so that the conversion-driven movement of those state-variables drives movements of those control-parameters, until the moment of locally complete conversion. That moment finally, fully 'absents' the old ontology locally, within the 'conversion-locus' modeled. This moment of complete conversion / complete 'absenting' is also the moment of singularity. It is the point at which the meta-space, in effect, changes itself, its metrical, axial content; its dimensionality. The old ontology, the one enshrined in the "ontological commitments" of the model specification, has ceased to exist locally, &, in that locus, its old metrics "zero-out" in a permanent way. That, in effect, collapses their dimensions, their meta-space axes, back into the origin, $\mathbf{\mu}_{\underline{0}}$. Likewise, from the inception of the conversion process, the new ontology output by that process requires new metrics, new meta-space dimensions, new axes to irrupt from the origin, & sometimes for old axes to shift from state-space to control-space or vice versa, also signifying '***change of space***'↔'***change of laws***'.

There is advance, in the $\{ {}^{\mathbf{n}}_{\mathbf{u}}{}_{,\alpha}\underline{\mu}_{\mathbf{t}_\tau} \}$ models, as in the $\{ {}^{\mathbf{n}}_{\mathbf{u}}\underline{\mathbf{U}}_\tau \}$ models, over the lock-step characteristics of the $\{ [{}^{\mathbf{n}}_{\mathbf{u}}\underline{\mathbf{Q}}_0]^{2^\tau} \}$ models, as over their 'possibilistic' nature. Actualized state-variable and control-parameter dimensions / metrics / axes can be expressed via the quantifier functions, $\mu_{\ell_k,k}(\mathbf{t}_\tau)$, which, in ${}_{\underline{\mathbf{R}},\alpha}\underline{\underline{\mu}}$, unlike in ${}_{\underline{\mathbf{N}},\alpha}\underline{\underline{\mu}}$, can take on ***zero values***, so that the 'extinction' or 'collapse back into the origin', $\boldsymbol{\mu}_{\underline{0}}$, of formerly extant axes can be formulated. The 'non-arisal' of certain inherently unviable $\hat{\underline{\boldsymbol{\mu}}}_k$ possibilities, or of $\hat{\underline{\boldsymbol{\mu}}}_k$ possibilities that are not meaningful for a specific model, something not expressible in $\{ {}^{\mathbf{n}}_{\mathbf{u}}\underline{\mathbf{Q}}_\tau \}$, can also be formulated in $\{ {}^{\mathbf{n}}_{\mathbf{u}}{}_{,\alpha}\underline{\mu}_{\mathbf{t}_\tau} \}$. Reversals or other re-orderings of the standard, 'lock-step' ${}^{\mathbf{n}}_{\mathbf{u}}\underline{\mathbf{Q}}_\tau$ historical order of emergence of ontos can be orchestrated via the τ-order in which the $\mu_{\ell_k,k}(\mathbf{t}_\tau)$ metrical-qualifier quantifier-functions emerge into positive values from their initial, pre-emergence $\mathbf{0}$ values. Both population-head-count, or 'capita', and 'non-capita' state-variables and 'control-parameter-variables', are expressible in ${}_{\underline{\mathbf{R}},\alpha}\underline{\underline{\mu}}$. In summary, a $\{ {}^{\mathbf{n}}_{\mathbf{u}}{}_{,\alpha}\underline{\mu}_{\mathbf{t}_\tau} \}$ model describes an evolving & 'meta-evolving' 'cumulum of meta-systems', a self-growing, self-extending, non-amalgamative '***sum of meta-spaces***' as a '***sum of*** dynamical, "time-varying" '***meta-states***', of meta-number values, i.e., of 'meta-vector' '*magnituded directions*', residing in those '***meta-spaces***'. Each $\hat{\underline{\boldsymbol{\mu}}}_k$'s '***meta-space***', $\underline{\underline{\Sigma}}_{\ell_k=1,M_k(t_\tau)}[{}^{\mathbf{n}}_{\mathbf{u}}\mu_{\ell_k,k}(\mathbf{t}_\tau)\cdot {}_{\ell_k}\hat{\underline{\boldsymbol{\mu}}}^{\hat{\mathbf{0}}}_{\underline{\underline{\Sigma}}_j \boldsymbol{\sigma}_j \hat{\underline{\mathbf{u}}}_j}\cdot {}^{\mathbf{n}}_{\mathbf{u}}\hat{\underline{\boldsymbol{\mu}}}_k]$, will usually be a ***multi-***dimensional, multi-metrical, multi-axial space, with each axis orthogonal to all of the others. Each $\hat{\underline{\boldsymbol{\mu}}}_k$ denotes a distinct 'meta-evolving, meta-dynamical meta-system' as a separate 'onto' or 'ontological category'. Thus, overall, $\{ {}^{\mathbf{n}}_{\mathbf{u}}{}_{,\alpha}\underline{\mu}_{\mathbf{t}_\tau} \}$ models '***a meta-meta-system made up out of multiple meta-systems***', such that: (**i.**) ***synchronically***, for any given $\mathbf{t}_\tau$ value, ${}^{\mathbf{n}}_{\mathbf{u}}{}_{,\alpha}\underline{\mu}_{\mathbf{t}_\tau}$ denotes a '***poly-qualinomial***' or "***series***" of contemporaneous meta-systems, via their meta-states, residing in the typically multi-dimensional, unified state / control 'meta-spaces' by which those meta-systems are described; (**ii.**) ***diachronically***, with growth of $\mathbf{t}_\tau$, $\{ {}^{\mathbf{n}}_{\mathbf{u}}{}_{,\alpha}\underline{\mu}_{\mathbf{t}_\tau} \}$ denotes a temporal/historical, usually net-expanding sequence of ***series*** of 'meta-spaces', a temporal succession of such 'metastates-sums', 'metasystems-sums', or '[meta-] spaces-sums', which include progressions of metasystem-ontos in which, e.g., a successor metasystem-onto assigned to $\hat{\underline{\boldsymbol{\mu}}}_{2^\tau}$ denotes a 'neo-onto' population, a newer population of '***meta-fractal meta$^{\mathbf{n}}$-monads***', which are 'meta-units', composed of multiplicities of the '***meta$^{\mathbf{n-1}}$-monads***' from the older or earlier-arising onto / 'meta-system' assigned to $\hat{\underline{\boldsymbol{\mu}}}_{2^{\tau-1}}$. By a '***meta-meta-system***' or '***meta$^{\mathbf{2}}$-system***', we mean an 'evolute', diachronic, temporal, historical ***sequence of meta-systems***, which, because 'evolute', also manifests as a synchronic, contemporaneous co-existence, and 'hybrid' mutual 'eco-systemasis', of multiple 'meta-systems'. [Two examples: **(1)** a star as a meta-system of its succession of conversion-singularity-separated 'meta-evolutionary' epochs, burning **H**, then **He**, then **C**, etc., with each epoch categorized as that of a separate system with distinct, e.g., "[non-]main sequence" dynamics/"dynamical laws"; **(2)** human society as a meta-system of successive, social-singularity-separated social systems, e.g., tribalism;...multi-city-state empire, commercial-agricultural 'slave-ocracy'/'chattelism'; feudalism; capitalism;...].

The $\{ {}^{\mathbf{n}}_{\mathbf{u},\alpha}\underline{\mu}_{\mathbf{t}_\tau} \}$ models can describe 'a population of meta-systems' self-deploying in 'parallel' "time evolutions" in a universe[-of-discourse] consisting of multiple distinct but 'co-extant', "co-evolving" 'meta-dynamical meta-systems', via 'inter-argument-ation' of the $\{\mu_{\ell_k,k}(\mathbf{t}_\tau)\}$ metrical-qualifier quantifier-functions. This gives them advantages vis-à-vis typical current models of dynamical systems theory, which describe single systems in isolation from any explicit representation of both their synchronic 'neighboring systems' and their diachronic 'successor systems'. The $\{ {}^{\mathbf{n}}_{\mathbf{u},\alpha}\underline{\mu}_{\mathbf{t}_\tau} \}$ models can also describe diachronic '***meta-meta***-dynamical ***meta-meta***-evolutions', to even larger such multiplicities of [evolute] 'parallel and successor' meta-systems, as well as the giving birth to a new meta-system/meta-system-ontology by an old meta-system/meta-system-ontology, in a context which can also encompass (**a**.) the ***self***-extinction, or ***other-meta-system-induced***-extinction, of entire meta-systems [e.g., stellar self-explosion "novae" phenomena], (**b**.) the ***non-realization*** of ***possible*** successor meta-systems and of ***possible*** hybrid-meta-systems, and; (**c**.) re-orderings of meta-system-succession, again, via the 'inter-functionality' or 'inter-argument-ation' of the $\mu_{\ell_k,k}(\mathbf{t}_\tau)$ metrical-qualifier quantifier-functions, including the mutual determination or 'inter-determination' of their zeros, as well as of their irruptions from zero into positive manifestation. Finally, some further words should be said about '***temporal acceleration***', and the capacity of the $\{ {}^{\mathbf{n}}_{\mathbf{u},\alpha}\underline{\mu}_{\mathbf{t}_\tau} \}$ models to capture it. By '***temporal acceleration***', as briefly indicated above, we mean, e.g., the trend of the speeding-up of successor 'whorls' of 'monadic self-incorporation', 'self-containment', or 'self-involution' -- i.e., of 'sub-atoms', or "subatomic particles", forming 'meta-subatomic' "atoms", atoms forming 'meta-atomic' molecules, molecules forming 'meta-molecular' cells,..., cells forming 'meta-cellular', or "multi-cellular", "metazoa" and "metaphyta", etc. -- relative to predecessor 'whorls'. We mean the tendentially declining duration from one 'epoch' of self-incorporation to the next, as measured relative to an external, standardized physical process or 'clock-process', i.e., to the rhythm of flow, the 'process-celerity' or 'local / concrete temporality' of another process, one held to exhibit a relatively lessor, or even, in principle, a "zero", rate of change in its local / concrete '*temporal celerity*'. This phenomenon might be modeled as a '***dialector force***'; a '***temporal force***', via a '***temporal***[ized***] vector***', 'pointing' not in any combination of the $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, or $\hat{\mathbf{z}}$ directions of physical space, but in the 'past-to-future' sense of the $\hat{\mathbf{t}}$ direction, or in a 'state-vector' '***direction of state***' within a state-space. Such a 'force' can also be seen a '***self-force***', a 'self-refluxive, self-refluxive 'causator' of change', directed 'self-ward', directed from the 'self' back to the 'self' of the '[ev]entity' described as emanating it, and arising from the '*self-duality*' or '*intra-duality*' of that '[ev]entity'. Defining "time" as '***change-in-general***', the '$\hat{\mathbf{t}}$-direction' of this '***self-force***' or '***force-of-***[***self***-]***change***[-in-general]' can be conceived, not as pointing into a somehow "pre-existent", Parmenidean-static future-time "dimension", as per the Minkowski four-space paradigm of Einstein's Special Theory of Relativity, but as a '***force of chrono-poiesis***', a force for '***the*** [***re-***]***production / continuation of time***', for *the on-going construction of the time dimension*, when time is concretely defined, i.e., identified with change. The quantifier or magnitude of such a force might be measured as proportional to the product of the magnitude of the temporal acceleration with the magnitude of a measure of the '[meta-] evolutionary resistance', '[meta-]evolutionary reluctance', or '[***meta-***]***evolutionary inertia***' of the '[ev]entity' generating that 'self-force' / 'force-upon-itself'. Such a force might thus increase in magnitude as 'cosmological self-[meta-]evolution' proceeds, if the '[meta-]evolutionary inertia' of more complex/more self-involuted meta-monads, and of their population-formations, is measured as greater than that of less-self-involuted meta-monads.

The $\{ \overset{\mathbf{n}}{\mathbf{u}}_{,\alpha} \underline{\mu}_{\mathbf{t}_\tau} \}$ models so far capture these concepts of '***variable temporal celerity***' -- namely, of '***temporal velocity***', '***temporal momentum***', '***temporal acceleration***', and '***temporal***[ized] ***force***' -- only via the differential scaling of the $\mathbf{t}_\tau$ arguments of the $\mu_{\ell_k,k}(\mathbf{t}_\tau)$ quantifier-functions of ${}_{\mathbf{R},\mathbf{R},\mathbf{N},\alpha}\underline{\underline{\mu}}$, as a function of τ.

Summary of the Meta-Systematic Dialectical Progression from $\underline{\underline{\mathbf{N}}}$ to $_{\underline{\mathbf{N}},\alpha}\underline{\underline{\mu}}$. This 'meta-systematic' dialectical argument, this categorial progression path of exposition of the 'Peanic' dialectical ideographies, began with $\underline{\underline{\mathbf{N}}}$, the ideography of the abstraction of 'pure quantity', and of maximal abstractness overall, i.e., of otherwise minimal explicit concreteness.

It next progressed to the 'other' of $\underline{\underline{\mathbf{N}}}$; to $_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{Q}}}$, an ideography of the abstraction of 'pure ontology', of 'pure ontological qualities'; also an 'intensional heuristic calculus', and one, again, of otherwise minimal explicit concreteness.

The argument then progressed to a first synthesis of ideographical quantification with ideographical qualification, known herein as $_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{U}}}$.

This progression has come to an end, so far and herein, with a second, higher synthesis of quantification and qualification, dubbed $_{\underline{\mathbf{N}},\alpha}\underline{\underline{\mu}}$.

The latter is an ideographic language of far greater intrinsic, explicit concreteness; of far greater capability for 'richness' or 'specificity' of representation, than are $\underline{\underline{\mathbf{N}}}$, $_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{Q}}}$, or $_{\underline{\mathbf{N}}}\underline{\underline{\mathbf{U}}}$. It encompasses determinateness regarding both units of measure and units of ontology -- that is, of identity -- as well as determinateness of magnitude. It is an ideography of '***quantified and metrically-qualified ontological qualification***', or of '***ontologically and metrically qualified quantification***'.

The 'meta-systematics' so far thus arrives at the arithmetical foundations of an analytical and descriptive linguistic tool that goes one step beyond the separate state-spaces and control-spaces of conventional [linear and nonlinear] dynamical systems theory. It arrives at a conception of '***via singularity self-bifurcation***' in the context of a unified, mutually-coupled state-space and control parameter-space. It arrives, thereby at a capability to describe, 'quanto-qualitatively', the 'revolutions', the 'meta-evolutions' of ***qualitative change***, of ***change of ontology***, that grow out of apparently ***purely quantitative change***; out of "dynamical, time evolution". It arrives at a capability to describe the 'meta-dynamics' that grows out of dynamics -- change of ontology, change of space, change of dynamical "laws", change of evolutionary 'epoch'; the 'self-transcendance', or '*dynamical, temporal* ***self-«reductio ad absurdum»***', of initial model specifications. All of this new capability arises in the context of a unitary, 'state/control meta-space' which is itself become a dynamical object, one whose 'dynamification' enables a '***metafinite semantification***' of the "meaningless" finite-time infinities, or zero-division "singularities", which otherwise beset, and help defeat solvability, interpretability, and tractability for, especially the ***nonlinear*** [integro-]differential equations formulation of dynamical "laws" under the conventional state-space paradigm. Finally, the $_{\mathbf{R},\mathbf{R},\mathbf{N},\alpha}\underline{\underline{\mu}}$ language is no longer confined to 'narcissistic', ***single-system universes*** descriptions. It is able to describe, ideographically, the dynamical and 'meta-dynamical' "***environment***" of each system, one that includes other systems and their meta-systems. This "***milieu***" or '***medium***' is that of the 'multi-system' context of this 'system dynamics', and the 'multi-meta-system' context of this 'meta-system meta-dynamics'. This "environment" is that of the co-existence, 'inter-determinism', "co-evolution", and 'co-meta-evolution' of these contemporaneous 'meta-systems', and of the prior meta-systems' [meta^{n-1}-]monads, contemporaneous ***and*** 'successive-consecutive', out of which the 'meta-monads' of those successor meta$^{[n]}$-monads' meta-systems are composed.

In overview, the foci of the 'native' modeling capabilities of this arithmetics' meta-system, this progression of systems of arithmetic, exhibits the following gradient of descriptive confinements --

N: ***Sub-System*** focus -- single state-variable or single control-parameter ["scalar"] models, barring intervention of other languages [e.g., phonogramic narrative];

Q: The 'ontos' or 'ontological categories' that constitute the 'self-expanding ontological cumula' described by **Q** models can be drawn from sub-system, system, or meta-system levels, or otherwise, based on the modeler's definitions of these partitionings, and the modeler's choices;

U: ***Uni-System*** or ***Multi-System***, Meta-System focus, with *synchronic **and** diachronic system multiplicity*; describing multi-population dynamics and '***meta***-dynamics' [emergence of new populations, belonging to newly-emergent ontological categories or 'ontos', from those of earlier-extant 'ontos'], in the form of '*multi-population meta-histogramic, meta-dynamical meta-distributions*'; ['meta'-]populations [made up out] of populations;

$_{\alpha}\underline{\underline{\mu}}$: ***Multi-Meta-System***, *Meta-Meta-System* focus, *synchronic and diachronic*, instantiating the '*via singularity self-bifurcating meta-systems paradigm of quanto-qualitative meta-dynamics*'.

Our pre-$\underline{\underline{\mu}}$ 'epochs' of arithmetic need not be 'convolutely' discarded, but, rather, may, with gain, be 'evolutely' conserved. For example, in the main body of ***Encyclopedia Dialectica***, each entry will provide its ideographical definition of the [ev]entity to be described in that entry by means of, not just a single model of that [ev]entity, but in terms of an orchestrated system of **R**⊕, **Q**, **U**, and $_{\alpha}\underline{\underline{\mu}}$ models. No doubt coordinated $_{\beta}\underline{\underline{\mu}}$ and $_{\gamma}\underline{\underline{\mu}}$ models, etc., will also be added, later, as they become operational.

Beyond $_{\alpha}\underline{\underline{\mu}}$. Even the $_{\mathbf{R},\alpha}\underline{\underline{\mu}}$ ideographical language still falls short of the full concreteness, specificity, and determinateness that we need and seek. For example, the $_{\alpha}\underline{\underline{\mu}}$ '***dialectical ideography***' is not yet a '***locus-specific***', '***locational***' or '***locationally determinate***' ideography, i.e., it is not yet either a 'spatio-temporal' ideography, or a '***functional*** [sub-system specific or 'meta-organ' specific] ideography. For example, suppose $\hat{\underline{\underline{\mu}}}_7$ denotes the ***star*** as a meta-system, meta-evolving from its Hydrogen-burning, Helium-accumulating epoch, to its Helium-burning, Carbon/Oxygen-accumulating epoch, to its Carbon-burning epoch, and beyond. Then does $\underline{\underline{\Sigma}}_{\ell_7=1,M_7(t_\tau)}[{}^{n}_{u}\mu_{\ell_7,7}(t_\tau)\cdot {}_{\ell_7}\hat{\hat{\underline{\underline{\mu}}}}_{\underline{\underline{\Sigma}}_j \sigma_j \hat{\underline{\underline{u}}}_j}\cdot {}^{n}_{u}\hat{\underline{\underline{\mu}}}_7]$ denote an average meta-dynamics for all of the stars in the cosmos?; for all of the stars in a particular galaxy?; for a particular, individual star? Does this expression model the '***meta-dynamics***' of the stellar-core region, the functional '*conversion-locus*' for stellar conversion of sub-atomic particles into atoms, and of "lower [atomic number] species" into "higher [atomic number] species" of atoms, or does it, instead, model the entire '*conversion-formation*', the star as a whole? Also, how would one distinguish state-variable dimensions from control-parameter dimensions in $_{\alpha}\underline{\underline{\mu}}$? Beyond the state-space 'analytic geometry of *total* differential equations', how would one express solutions for *partial* differential equations/"field theories" in $_{\alpha}\underline{\underline{\mu}}$? We expect that this 'locational specificity' belongs to $_{\beta}\underline{\underline{\mu}}$, or $_{\gamma}\underline{\underline{\mu}}$, and beyond, i.e., to the continuation of the meta-systematic dialectic of $[\,\underline{\underline{\mathbf{N}}}\,]^{\tau}$ beyond $\tau = 7$, and, before that, to the 'ideo-historical ideo-dialectic' of continuing new languages-discovery, of ongoing new languages-design research, and of 'linguistic engineering'.

The Principle of 'Onto-Qualitative' Innovation. The whole pattern of the generic dialectical interpretations of the $\hat{\underline{\mathbf{q}}}_{\kappa}$ implements a unifying principle regarding the sources of ontological 'novelty' in the [cosmological] universe as a whole, and in all of the other diverse 'universes of discourse' definable within it. This principle provides a '***Genetic Logic***', not in the sense of a 'DNA logic' of biological genotypes, but of a "Logic of Genesis', a 'generative logic', or 'logic of generation', which we term '***The Logic of Meta-Genealogy***'. It describes the 'descent' of ontological categories, e.g., the results of sub-nuclear particles begetting sub-atomic particles begetting atoms begetting molecules begetting cells, etc. It thus includes those 'genealogies' ***not*** effected by biological reproduction. This 'Genetic Logic', '***Creative Logic***', or '***Logic of Creation***' is a '***Logic of Onto-Dynamics***', that is, it is a '***Logic of Onto-Dynamasis***'. We state this '***Principle Of Innovation***' via the following four-part proposition --

• ↑[**antithesis**] < §α >: 'Metafractalogenic', «***aufheben***» '***self-containment***', '***self-subsumption***', or '***self-meta-monadization***' is the principal source of ontological innovation, of new ontology as *new difference*. [Other terms that describe aspects of this process include '***self-reflexion***', '***self-refluxion***', '***self-involution***', '***self-internalization***', '***self-incorporation***', '***self-re-entry***', '***self-hybridization***' or 'auto-hybridization', as well as '***self-entanglement***'].

[[syn]***thesis*** → [syn]***thesis*** + [higher-]***anti-***[syn]***thesis***]

• ↑[**synthesis**] < §β >: 'Ontological [allo-]***hybridization***', partial and full/'grand' ***dialectical synthesis***, '***hetero-monadization***', '***entanglement***', '***complex*** [***re-***]***unification***', or '***flexion***' is the other, co-principal source of innovation in ontology, as new unification, *new unity*.

[[syn]***thesis*** + [higher-]***anti-***[syn]***thesis*** →

[syn]***thesis*** + [higher-]***anti-***[syn]***thesis*** + [higher-]***synthesis***]

• **Model** < §γ >: The [meta-]*finite* ***Set Of All Sets***: $\underline{\mathbf{S}}_0^{2^\tau}$. *This set* ***is*** *a definition/self-driven* ***process*** *by which this totality of sets continually reincarnates inside itself, as a new part / new individual member of itself.* Its formula is: $\underline{\mathbf{S}}_{\tau+1} = \underline{\mathbf{S}}_\tau \cdot \underline{\mathbf{S}}_\tau = \underline{\mathbf{S}}_\tau^{\,2}$, or $\underline{\mathbf{S}}_\tau = \underline{\mathbf{S}}_0^{\,2^\tau}$, via what we term the '***Power-Set Evolute Product Rule***', $\underline{\mathbf{S}}_\tau \cdot \underline{\mathbf{S}}_\tau \equiv \underline{\mathbf{S}}_\tau \cup \mathbf{2}^{\underline{\mathbf{S}}_\tau} \equiv \underline{\mathbf{S}}_\tau \oplus \underline{\Delta}^1[\,\underline{\mathbf{S}}_\tau\,] \ddagger \underline{\mathbf{S}}_\tau$, wherein $\mathbf{2}^{\underline{\mathbf{S}}_\tau}$ denotes the "Power-Set" [the set of all subsets] of $\underline{\mathbf{S}}_\tau$. Herein, $\underline{\mathbf{S}}_0 \equiv \underline{\mathbf{U}}$, with $\underline{\mathbf{U}}$ denoting the ***finite*** universal set or set of all '***constructed***' "logical individuals" inhering in the universe of discourse in question [per '***The Principle Of Metafinity***', that '***infinity is counter-[f]actual***']. The next, (τ+**1**)st stage of $\underline{\mathbf{S}}_\tau$ always contains ***every subset*** of the τth stage of $\underline{\mathbf{S}}_\tau$, including the "improper" subset of $\underline{\mathbf{S}}_\tau$, $\underline{\mathbf{S}}_\tau$ itself. That is, it contains the '***extensional expression***' of every possible ***predicate*** of this universe of discourse for 'epoch' τ of $\underline{\mathbf{S}}_0^{\,2^\tau}$ self-inclusion. *Thus the* $\underline{\mathbf{S}}_0^{\,2^\tau}$ *cumulum is also an extensional, syntactical model of '**Predico-Dynamasis**'.* It is, as well, a set-theoretical model of '***Onto-Dynamasis***', wherein sets of higher "logical type" model 'meta-ontos' of higher 'meta' degree, or of higher τ-degree in $\underline{\Delta}^\tau$. Thus, if $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \ldots\}$ denotes a set of atoms, then the '***self-inclusion***', ***self-incorporation***', or '***self-containment***' of that set, i.e., $\{\,\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \ldots\}, \mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \ldots\}$, may be interpreted as one of atoms plus 'meta-atoms', i.e., ['self-', 'set-', '{'-&-'}'-]***contained*** atoms', i.e., 'molecules', via incorporation of the "***improper***" sub-set of $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \ldots\}$. Each distinct "***proper***" subset of $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \ldots\}$ then denotes the "extension" of (a) distinct predicate(s)/"intension(s)"/-qualit(y)(ies) shared by each member of that subset. The set $\underline{\mathbf{S}}_\tau$ always excludes itself and its "proper" subsets. But its definition calls for it to include ***all*** sets. It therefore must take all of them, and itself, into itself. But the self-induction of itself and its "proper" subsets into membership within itself is also the qualitative transformation of the set and 'self' that it was prior to that self-inclusion. It thus emerges from each such self/subsets inclusion as a new set, self-excluded once more. The process of self-inclusion thus ensues anew.

• **Result** < §δ >: '***Meta-Fractals***': Unified, diachronic / synchronic, 'quanto-qualitative', 'quanto-ontological' ***self-similarity formations*** with spatial and temporal ***scale-regression structure***; 'metafinite', 'multi-meta-ontic', 'multi-meta-monadic' meta-fractal 'cumula', for example, the 'level one' (**n** = **1**, **u** = total universe) cosmological 'cumulum' represented, in the $_{\underline{\mathbf{N}}}\underline{\mathbf{Q}}$ language, by --

[...atoms +...molecules +...pro-cells +...eu-cells +...multicell[ular]s +...societies...]

=

[...atoms +...meta^1-atoms +... meta^2-atoms +.... meta^4-atoms +... meta^5-atoms...]

=

[...$\underline{\Delta}^0$[atoms] +... $\underline{\Delta}^1$[atoms] +... $\underline{\Delta}^2$[atoms] +.... $\underline{\Delta}^4$[atoms] +... $\underline{\Delta}^5$[atoms] +...];

as well as '***eco-systemasis***', whereby the resulting multi-populations of mutually ontologically, 'quanto-qualitatively' scaled meta-monads self-organize into media and «***milieux***», including 'vessels' which mediate their 'quanto-ontologically' expanding self-reproduction, e.g., ...galaxies, stars, stellar-planetary systems, planetary atmo-hydro-lithospheres, planetary biospheres, planetary economic system 'ecologies' or "noospheres", etc. Using '⧺' to denote the '*ontic* ***hybridization operator***' --

[...+ a +...+ m +...+ ⧺[m; a] + p +...+ ⧺[p; m; a] + e +...+ ⧺[e; p; m; a] +...].

Generic Dialectical Interpretations of $_{N}\underline{Q}$ to $\hat{\underline{q}}_{16}$, & Extensions by '*Ontological Induction*'. Collecting our experiences with the two, disparate '***specific***' models of examples **A** and **B**, above, and distilling the patterns of the phenomenology of their arguments, suggests the following catalog of "***generic***" interpretations of $\hat{\underline{q}}_1$ through $\hat{\underline{q}}_{16}$. Interpretations for $\hat{\underline{q}}_{17}$ and beyond can be inferred by '***qualifier induction***' on the ordinal patterns of the "***generic***" characterizations given below --

Catalog.

$\hat{\underline{q}}_1$ ↔ «***Arché***»; First Unity; Simple Unity; Initial Unity; Primitive Undifferentiated Unity; [stipulated] Primordial Unity; First Thesis; Initial Thesis; Initiating Thesis; Initiating Posit; Initial Monad;

$\hat{\underline{q}}_2$ ↔ First Anti-«***Arché***»; First Difference; Simple Difference; Initial Difference; Initial Self-Differentiation; [Relative] Primordial Difference; First Anti-Thesis; Initial Contra-Thesis; First Contra-Posit; Initial Contra-Posit; Initiating Counter-Posit; First Meta-Monad;

$\hat{\underline{q}}_3$ ↔ Second [Re-]Unity; First Complex Unity; First Higher Unity; First Synthesis; Second Thesis; First Grand Synthesis; First [Grand] Hybrid Monad; First Monads' Hybrid;

$\hat{\underline{q}}_4$ ↔ Second Anti-«***Arché***»; Second Difference; First Complex Difference; Second Self-Differentiation; First Higher Difference; Second Anti-Thesis; Second Contra-Thesis; Second Contra-Posit; Secondary Counter-Posit; Second Meta-Monad;

$\hat{\underline{q}}_5$ ↔ First Partial Unity; First Partial Complex; First Partial Composite; First Partial Synthesis; Second Hybrid Monad; First Partial Hybrid;

$\hat{\underline{q}}_6$ ↔ Second Partial Unity; Second Partial Complex; First Partial Composite; Second Partial Synthesis; Third Hybrid Monad; Second Partial Hybrid;

$\hat{\underline{q}}_7$ ↔ Third [Re-]Unity; Second Complex Unity; Second Higher Unity; Second Synthesis; Third Thesis; Second Grand Synthesis; Second Grand Hybrid Monad;

$\hat{\underline{q}}_8$ ↔ Third Anti-«***Arché***»; Third Difference; Second Complex Difference; Third Self-Differentiation; Second Higher Difference; Third Anti-Thesis; Third Contra-Thesis; Third Contra-Posit; Tertiary Counter-Posit; Third Meta-Monad;

$\hat{\underline{q}}_9$ -- $\hat{\underline{q}}_{14}$ ↔ Third through Eighth Partial Syntheses or Partial Hybrids;

$\hat{\underline{q}}_{15}$ ↔ Fourth [Re-]Unity; Third Complex Unity; Third Higher Unity; Third Synthesis; Fourth Thesis; Third Grand Synthesis; Third Grand Hybrid Monad;

$\hat{\underline{q}}_{16}$ ↔ Fourth Anti-«***Arché***»; Fourth Difference; Third Complex Difference; Fourth Self-Differentiation; Third Higher Difference; Fourth Anti-Thesis; Fourth Contra-Thesis; Fourth Contra-Posit; Fourth Counter-Posit; Fourth Meta-Monad. . . .

Tabulation.

Uninterpreted $\underline{\underline{Q}}$ Meta-Numeral	Hypothesis Type	Category Type	Principle Type	'Meta-Monadology' Type
$\hat{\underline{\mathfrak{q}}}_1$	First Thesis	Simple Unity	Principle 1	«***Arché***» Monad
$\hat{\underline{\mathfrak{q}}}_2$	First Anti-Thesis	Simple Difference; First Difference	Contra-Principle 1	First Meta-Monad
$\hat{\underline{\mathfrak{q}}}_3$	First Synthesis; Second Thesis	First Complex Unity	Principles' Reconciliation 1	First Full-Hybrid Monad
$\hat{\underline{\mathfrak{q}}}_4$	Second Anti-Thesis	Second Difference	Contra-Principle 2	Second Meta-Monad
$\hat{\underline{\mathfrak{q}}}_5$	First Partial Synthesis	First Partial Unity	Principles' Partial Reconciliation 1	First Partial-Hybrid Monad
$\hat{\underline{\mathfrak{q}}}_6$	Second Partial Synthesis	Second Partial Unity	Principles' Partial Reconciliation 2	Second Partial-Hybrid Monad
$\hat{\underline{\mathfrak{q}}}_7$	Second Synthesis; Third Thesis	Second Complex Unity	Principles' Reconciliation 2	Second Full-Hybrid Monad
$\hat{\underline{\mathfrak{q}}}_8$	Third Antithesis	Third Difference	Contra-Principle 3	Third Meta-Monad
$\hat{\underline{\mathfrak{q}}}_9$	Third Partial Synthesis	Third Partial Unity	Principles' Partial Reconciliation 3	Third Partial-Hybrid Monad
$\hat{\underline{\mathfrak{q}}}_{10}$	Fourth Partial Synthesis	Fourth Partial Unity	Principles' Partial Reconciliation 4	Fourth Partial-Hybrid Monad
$\hat{\underline{\mathfrak{q}}}_{11}$	Fifth Partial Synthesis	Fifth Partial Unity	Principles' Partial Reconciliation 5	Fifth Partial-Hybrid Monad
$\hat{\underline{\mathfrak{q}}}_{12}$	Sixth Partial Synthesis	Sixth Partial Unity	Principles' Partial Reconciliation 6	Sixth Partial-Hybrid Monad
$\hat{\underline{\mathfrak{q}}}_{13}$	Seventh Partial Synthesis	Seventh Partial Unity	Principles' Partial Reconciliation 7	Seventh Partial-Hybrid Monad
$\hat{\underline{\mathfrak{q}}}_{14}$	Eighth Partial Synthesis	Eighth Partial Unity	Principles' Partial Reconciliation 8	Eighth Partial-Hybrid Monad
$\hat{\underline{\mathfrak{q}}}_{15}$	Third Synthesis; Fourth Thesis	Third Complex Unity	Principles' Reconciliation 3	Third Full-Hybrid Monad
$\hat{\underline{\mathfrak{q}}}_{16}$	Fourth Anti-Thesis	Fourth Difference	Contra-Principle 4	Fourth Meta-Monad

Induction for *generic* dialectical interpretation of the $\hat{\underline{\mathfrak{q}}}_k$, including of $\hat{\underline{\mathfrak{q}}}_{17}$ and beyond:

0. The meta-number $\hat{\underline{\mathfrak{q}}}_1$ is assigned to the «***arché***», the beginning '***onto***', the "ever-controlling source" or "ever-present origin" of the dialectical ontological 'lineage' being modeled;

1. That meta-number $\hat{\underline{\mathfrak{q}}}_k \mid \mathbf{k}, \tau \in \mathbf{N}$, whose subscript is of the form $\mathbf{k} = \mathbf{2}^\tau$, models the τth Anti-Thesis, the τth Category of [Complex] Difference, Contra-Principle τ, or the Meta$^\tau$-Monad;

2. That $\hat{\underline{\mathfrak{q}}}_k \mid \mathbf{k}, \tau \in \mathbf{N}, \tau > \mathbf{1}$, whose subscript is of the form $\mathbf{k} = \mathbf{2}^\tau - \mathbf{1}$, models the [$\tau$–**1**]st [Grand] Synthesis, the [τ–**1**]st Complex Unity, Principles' Reconciliation τ–**1**, or the [τ–**1**]st Full-Hybrid Monad;

3. The $\hat{\underline{\mathfrak{q}}}_k$ for all other $\mathbf{k} \in \mathbf{N}$, model Partial Syntheses, Partial Complex Unities, Principles' Partial Reconciliations, or Partial-Hybrid Monads.