F.E.D. Vignette #4, Part I. --

The Gödelian Dialectic

of

the Standard Arithmetics.

A *Dialectical* '*Meta-Model*' of the F.E.D. '*Meta-Systematic Dialectical Method of Presentation*'
of the *Axioms-Systems Progression* of *the Standard Arithmetics*, Using F.E.D.'s '*First Dialectical Algebra*'.

'Habilitation-«*Fest*»' Essay

by **Miguel Detonacciones**

of

Foundation *Encyclopedia Dialectica*

Part I., Version: 04.7:07.07.2014
Last Updated: 07 July 2014 C.E. / B.U.E.
First Distributed: 02 September 2012 C.E. / B.U.E.

B. ***Background***.

B.α. Scope of this Essay. The '***dialectical meta-model***' exposited and narrated herein, using the ideographical language of the $_{N}\underline{Q}$ ***dialectical arithmetic***, the «***archê***» of all of the **F**.***E***.***D***. ***dialectical ideographies***, is, clearly, ***not*** a 'meta-model' of an "objective", past, pre-human-natural -- nor even of a contemporary, but extra-human-natural -- progression, of '*physio*-ontologies', ***un****like* **F**.***E***.***D***.'s *'Dialectical Theory of Everything'* 'meta-model'.

It is, on the contrary -- and in several dimensions of such contrariety at once -- a 'meta-model' of a pedagogical *strategy of presentation* of the human, "inter-subjective" totality, or universe[-of-discourse], of the *"Standard Arithmetics"*, i.e., of the arithmetics built upon that human 'ideo-ontology' of the "kinds" of number standardly denoted today as -- **N**, **W**, **Z**, **Q**, **R**, **C**, etc. -- in the form of *a systematically-ordered progression of higher-order-logic axioms-****systems*** of arithmetic, presenting the totality of these systems as they appear from the vantage point of the present development, and 'self-theorization', by the contemporary consensus of the community of mathematicians, of mathematics as a whole. That **F**.***E***.***D***. pedagogical strategy seeks a best, ***optimal ordering*** for the exposition of these "qualitatively different" -- 'ideo-ontologically different' -- axioms-systems, and for their sub-topics. This strategy thus involves choices among alternatives, by the pedagogue -- in this case, by the **F**.***E***.***D***. research collective. The 'meta-model' presented herein is thus ***not*** one of **F**.***E***.***D***.'s '[***psycho***]***historical***-***dialectical*** meta-models' in **F**.***E***.***D***.'s terms. That is, it does ***not*** address, among other issues ***not*** addressed, the actual, chronological order of the effective birth-dates of related systems of arithmetic in Terran human history to-date. It does ***not*** explain the differences in the semantic 'self-theorization', and in the timing of emergence, of related number-kinds 'ideo-ontologies' in the past-historical, geographically-localized 'sub-phenomes' of the total Terran 'Human Phenome', as key chapters in the story of the «***bildung***» of that total Terran Human Phenome.

Moreover, this essay does ***not*** instantiate the '***Dialectical Meta-Axiomatics***' method of presentation, which is the planned future ***E***.***D***. standard for presentations of such material [link: **definition** & **diagram** for '***Dialectical Meta-Axiomatics***'].

B.β. The '''Driver''' for the Progression of the "Standard Arithmetics": The Solution of "Unsolvable" Equations. Despite those limitations of its scope, the 'meta-model' presented herein does capture '***The Gödelian Dialectic***' of this progression of the axiomatic systems of the Standard Arithmetics. That is, this 'meta-model' captures a pedagogically-selected, pedagogically-optimized progression of "Gödel formulae" analogues[1] for this progression of Arithmetics.

Each such Gödel formula "deformalizes" into a proposition asserting that a certain "diophantine" algebraic equation cannot be solved within the ["incomplete"] *'number-ontology'*, or *'number-kind ideo-ontology'*, of the given arithmetical axiomatic system, *inside* which it arises as a 'well-formed equation' of that system. Also, as a 'Gödel formula proposition', asserting the unsolvability of such an equation in such an axioms-system, such a proposition would be true of the axioms-system within which it arises, but would be neither deductively provable nor deductively dis-provable -- i.e., would be "***un***decidable" -- from the axioms of that axioms-system, i.e., within the logical-language resources, syntactical and semantic, of that axioms-system. Derivation of such a well-formed ***un***solvable equation, and ***un***provable theorem, within such an axioms-system precipitates an immanent formal-logical «***aporia***», ***impasse***, ***quandary***, or ***predicament***, for that axioms-system: ***a logical inadequacy*** -- the '***a***-pore', or '***not***-pore'; the absence of a "passage", of an "opening", beyond the present "***im***passe", which appears, at first, to be a hopeless dead end.

However, such an equation ***would be*** solvable within the consecutive ***next*** arithmetical axioms-system, the "successor system" of that "predecessor" arithmetical axioms-system, by means of the new *'ideo-ontology'* -- by means of the new ***kind*** of *'number ontology'* -- which first fully arrives in that successor system. That same equation would also be solvable in all subsequent arithmetical axioms-systems in this progression, using that new *'ideo-ontology'*, which is *conserved*, as well as qualitatively/'ideo-ontologically' *further extended*, in all subsequent arithmetical axioms-systems in this 'meta-system', or systems-progression. Such a 'Gödel formula proposition', asserting the ***un***solvability of that equation ***in*** that predecessor system, will become ***provably true***, via the expanded logical-language resources of that successor arithmetical axioms-system, as well as ***in*** all of ***its*** "successor systems", in which those new logical-language resources are also both *conserved* **&** *determinately-negated/changed/elevated/further-extended* in a qualitative/'ideo-ontological' sense.

B.γ. **The 'Dyadic Seldon Function Meta-Equational Meta-Model' of the "Standard Arithmetics" Progression**.
The starting-point, beginning, governing source, ever-present origin, or «***archê***», of the ***Encyclopedia Dialectica*** 'meta-model' of the 'Meta-Systematic Dialectic of the Systems of the Standard Arithmetics' is --

$$\left(^{3}_{h}\mathbf{N}_{\#}\right)$$

-- connoting the first-***and-higher-order-logic*** axiomatization of the arithmetic of the "**N**atural Numbers", **N** ≡ {**I**, **II**, **III**, ...}. The central, **N**, symbol in this 'symbol complex' is surrounded by three ***E.D.*** 'dialectical diacritical marks'. They serve to *'locate'* the '''ideo-eventities''' being modeled/defined by their 'dialectical meta-models', in an encyclopedic, systematic, 'ideo-taxonomic' structure. In particular, these three *'locator'* marks consist of a **3** as *'pre-superscript'*, an **h** as *'pre-subscript'*, and a **#** as [*post-*]*subscript*. The '$\left($' & '$\right)$' are the special kinds of "open" **&** "close" parentheses used to enclose symbols denoting '''synchronic''', ***presentational***, [***meta-***]***systematic-dialectical*** ontological constants and variables. The **h** indicates that the 'ideo-ontology' implicit denoted by the $^{3}_{h}\mathbf{N}_{\#}$ 'symbol complex' is sourced in «***genos***» **h**umanity, that is, in the **h**uman[oid] "«***species***»" of 'cosmo-ontology', **&** its '**h**uman[oid] Phenome'. The **3** indicates that this ontology is categorized into taxonomy level **3** of the ***E.D.*** Universal Taxonomy, hence, for the **h** «***genos***», consists primarily of *'**ideo**-ontology'*, unlike **h**'s taxonomy level **2**, consisting mainly of ontological mixtures or hybrids of *'**ideo**-'* [memetic] and *'**physio**-'* components [e.g., objects of "economic value", i.e., of "exchange-value"], and likewise unlike **h**'s taxonomy level **1**, consisting almost entirely of *'**physio**-ontology'*, e.g., of **h**uman *physical* and *physiological* artefacts, 'sociofacts', and 'biofacts' in themselves, as such, e.g., **h**uman physical remains]. The 'post-subscript', **#** -- the "number sign", "count sign", "hash sign", or "pound sign", with single underscore -- signifies the ***gene***ral topic of the ***dialectic*** which $^{3}_{h}\mathbf{N}_{\#}$ ***gene***rates: "number" in the specific sense of "*cardinal* number", or "*counting* number".

The dialectical process of '***ideo***-«***auto-kinesis***»' that is rooted and seeded in $^{3}_{h}\mathbf{N}_{\#}$, will give rise to a series of 'ideo-ontologically' heterogeneous, irreducible, non-amalgamative '''sums''', or 'cumula', generically denoted by $\underline{)H(}$, which will represent sums, or mutual '''super[im]positions''', of the different qualitative determinations, or characteristics, of the different ***kinds*** of number immanently and progressively evoked starting from the initial '***self***-critique' of $^{3}_{h}\mathbf{N}_{\#}$. The ideographical symbolization of these 'cumula' will involve the sign '$\vdash\!\oplus\!\dashv$', signifying 'oppositional addition'. [Note: The operation-signs for generalized 'pure-ideic' addition, '$\vdash\!\oplus\!\dashv$' & '$\oplus$', are formally equivalent. We use '$\vdash\!\oplus\!\dashv$', signing 'mutual-negation ['¬'] addition', when we want to *emphasize* the opposite qualities connoted by a -- typically *ordinally consecutive* -- pair of 'connotograms', 'categorograms', or 'systemograms'].

I.e., an expression of the form $\mathbf{X} \vdash\!\oplus\!\dashv \mathbf{Y}$ signs the external, additive combination, or co-presence, of an **X** & a **Y** which are opposites/'''negations''' of one another in their ***primary*** qualities/characteristics/determinations. The compound symbol '$\mathbf{s}_{\#}\Upsilon$', with no underscores under the '$\mathbf{s}_{\#}$' or under the 'Υ', means that $\mathbf{s}_{\#}$ is a monotonically increasing "purely-quantitative" variable, in this case ∈ **W**. Given those preliminaries, we can now present the 'Dialectical Meta-Equation' which constitutes the ***E.D.*** 'dialectical meta-model' of the ***E.D.*** 'Meta-Systematic Dialectical ***Method of Presentation*** of the Progression of the Systems of ***the Standard Arithmetics***':

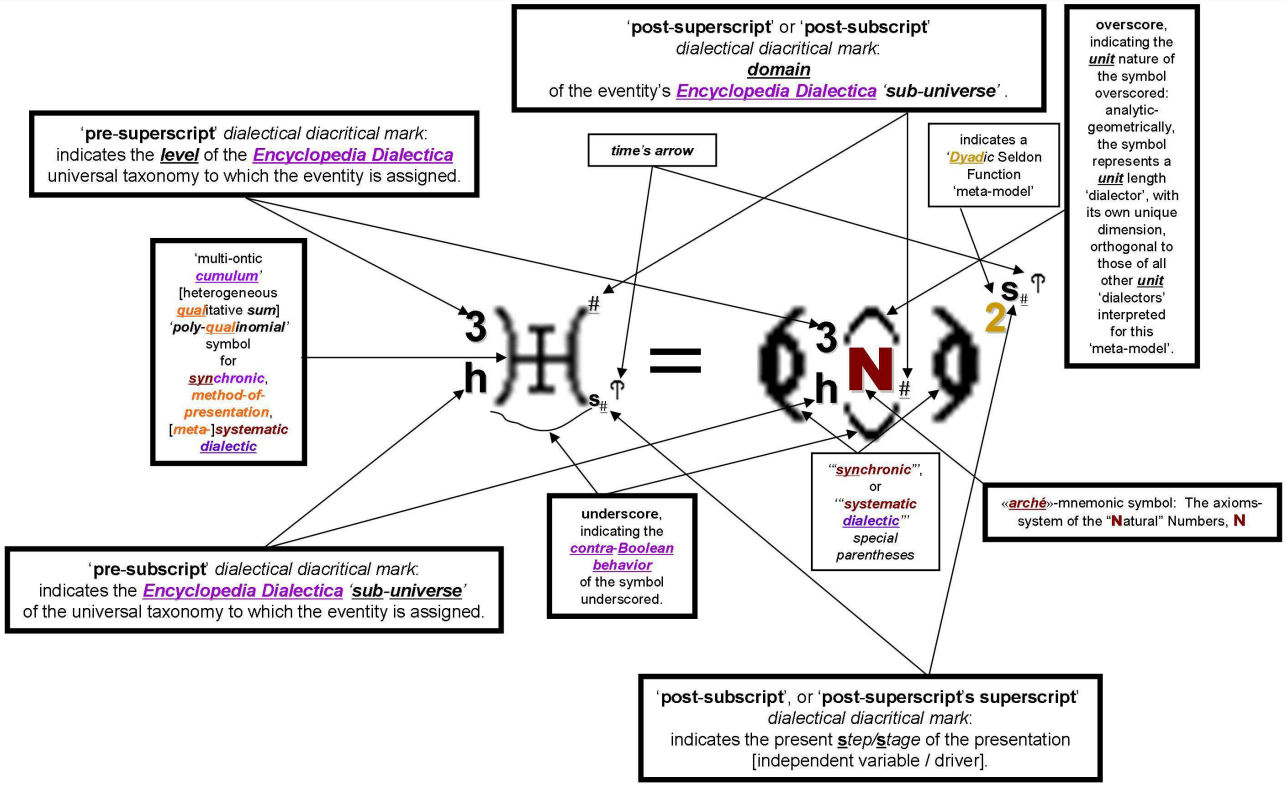
$$\underset{\mathbf{s}_{\#}}{\overset{\#}{\underset{h}{\overset{3}{)H(}}}}\Upsilon \;=\; \left(^{3}_{h}\mathbf{N}_{\#}\right)^{2^{\mathbf{s}_{\#}\Upsilon}}.$$

The following page contains a JPEG graphic of the above-presented 'meta-equation', with its symbolic elements defined via call-outs.

Dialectical 'Meta-Model'

for 'The Gödelian Dialectic of the Standard Arithmetics':

Its Symbolic Elements Defined via Callouts



As the **S**tep/**S**tage of presentation advances, as the *'dialectical **self**-argument'* ensues, this 'dialectical meta-equation' takes as a "given" the sequence of 'qualitative series'/'ideo-cumula' given on the next page. Each also expresses yet a new «***aporia***» or ***quandary*** regarding the nature of "cardinal", or "counting", number, by way of each '''[syn]thesis''' "Standard Arithmetic" Axioms-System symbol being 'oppositionally-added' to an 'anti-thesis'/'contra-thesis' symbol, symbolizing an immanently-arising '''Lakatosian counter-example''', as a 'supplementary-opposite' to the new [syn]thesis's notion/definition of "[counting] number".

By a 'meta-equation', we mean the «***aufheben***» 'meta-***monad***ization', or «***aufheben***» 'meta-***unit***ization', of an equation as «***monad***» or ***unit***: *a 'meta-equation' **unit** is a meta-**unit** of equations as **units**, made up out of a heterogeneous multiplicity of equation-**units***. The 'meta-equation' above '''contains''' a qualitatively different, 'ideo-ontologically different' equation for every value of its independent variable, $\mathbf{S}_{\#}$ -- seven qualitatively distinct equations total for the range $\mathbf{S}_{\#} = 0$ through $\mathbf{S}_{\#} = 6$.

That sequence of «***aporia***» is as follows --

0. $\underline{s}$tage / $\underline{s}$tep **0**, $s_{\#} = 0$: $\;{}^{3}_{h}\underline{)\text{H}(}^{\#}_{0} \;=\; \left({}^{3}_{h}\underline{\mathbf{N}}_{\#}\right)^{2^{0}} \;=\; \left({}^{3}_{h}\underline{\mathbf{N}}_{\#}\right)^{1} \;=\; {}^{3}_{h}\underline{\mathbf{N}}_{\#}.$

1. $\underline{s}$tage / $\underline{s}$tep **1**, $s_{\#} = 1$: $\;{}^{3}_{h}\underline{)\text{H}(}^{\#}_{1} \;=\; \left({}^{3}_{h}\underline{\mathbf{N}}_{\#}\right)^{2^{1}} \;=\; \left({}^{3}_{h}\underline{\mathbf{N}}_{\#}\right)^{2} \;=\; {}^{3}_{h}\underline{\mathbf{N}}_{\#} \oplus {}^{3}_{h}\underline{\mathbf{a}}_{\#};$

2. $\underline{s}$tage / $\underline{s}$tep **2**, $s_{\#} = 2$: $\;{}^{3}_{h}\underline{)\text{H}(}^{\#}_{2} \;=\; \left({}^{3}_{h}\underline{\mathbf{N}}_{\#}\right)^{2^{2}} \;=\; \left({}^{3}_{h}\underline{\mathbf{N}}_{\#}\right)^{4} \;=\; {}^{3}_{h}\underline{\mathbf{W}}_{\#} \oplus {}^{3}_{h}\underline{\mathbf{m}}_{\#};$

3. $\underline{s}$tage / $\underline{s}$tep **3**, $s_{\#} = 3$: $\;{}^{3}_{h}\underline{)\text{H}(}^{\#}_{3} \;=\; \left({}^{3}_{h}\underline{\mathbf{N}}_{\#}\right)^{2^{3}} \;=\; \left({}^{3}_{h}\underline{\mathbf{N}}_{\#}\right)^{8} \;=\; {}^{3}_{h}\underline{\mathbf{Z}}_{\#} \oplus {}^{3}_{h}\underline{\mathbf{f}}_{\#};$

4. $\underline{s}$tage / $\underline{s}$tep **4**, $s_{\#} = 4$: $\;{}^{3}_{h}\underline{)\text{H}(}^{\#}_{4} \;=\; \left({}^{3}_{h}\underline{\mathbf{N}}_{\#}\right)^{2^{4}} \;=\; \left({}^{3}_{h}\underline{\mathbf{N}}_{\#}\right)^{16} \;=\; {}^{3}_{h}\underline{\mathbf{Q}}_{\#} \oplus {}^{3}_{h}\underline{\mathbf{d}}_{\#};$

5. $\underline{s}$tage / $\underline{s}$tep **5**, $s_{\#} = 5$: $\;{}^{3}_{h}\underline{)\text{H}(}^{\#}_{5} \;=\; \left({}^{3}_{h}\underline{\mathbf{N}}_{\#}\right)^{2^{5}} \;=\; \left({}^{3}_{h}\underline{\mathbf{N}}_{\#}\right)^{32} \;=\; {}^{3}_{h}\underline{\mathbf{R}}_{\#} \oplus {}^{3}_{h}\underline{\mathbf{i}}_{\#};$

6. $\underline{s}$tage / $\underline{s}$tep **6**, $s_{\#} = 6$: $\;{}^{3}_{h}\underline{)\text{H}(}^{\#}_{6} \;=\; \left({}^{3}_{h}\underline{\mathbf{N}}_{\#}\right)^{2^{6}} \;=\; \left({}^{3}_{h}\underline{\mathbf{N}}_{\#}\right)^{64} \;=\; {}^{3}_{h}\underline{\mathbf{C}}_{\#} \oplus {}^{3}_{h}\underline{\mathbf{h}}_{\#};$

7. $\underline{s}$tage / $\underline{s}$tep **7**, $s_{\#} = 7$: $\;{}^{3}_{h}\underline{)\text{H}(}^{\#}_{7} \;=\; \left({}^{3}_{h}\underline{\mathbf{N}}_{\#}\right)^{2^{7}} \;=\; \left({}^{3}_{h}\underline{\mathbf{N}}_{\#}\right)^{128} \;=\; {}^{3}_{h}\underline{\mathbf{H}}_{\#} \oplus {}^{3}_{h}\underline{\mathbf{o}}_{\#};$ ...

-- wherein we are now utilizing, as in the core section of this essay, visible-light-spectrum ***ordinal*** color-coding -- ***red***, ***orange***, ***yellow***, ***green***, ***blue***, ***indigo***, and ***violet*** -- to emphasize the ***ordinality*** of the successive dialectical syntheses signed by the 'dialectical formulae' above, viz. -- ${}^{3}_{h}\underline{\mathbf{N}}_{\#}$, ${}^{3}_{h}\underline{\mathbf{W}}_{\#}$, ${}^{3}_{h}\underline{\mathbf{Z}}_{\#}$, ${}^{3}_{h}\underline{\mathbf{Q}}_{\#}$, ${}^{3}_{h}\underline{\mathbf{R}}_{\#}$, ${}^{3}_{h}\underline{\mathbf{C}}_{\#}$, & ${}^{3}_{h}\underline{\mathbf{H}}_{\#}$.... Each increment of number-system 'ideo-ontology', shown 'oppositionally-added' to each arithmetic's axioms-system "dialectical [syn]thesis" symbol in each presentation $\underline{s}$tep above, is also a 'pre-given', as follows --

${}^{3}_{h}\underline{\mathbf{a}}_{\#} \equiv$ axioms-increment for the set of all **N** self-subtractions, named herein the "**a**ught numbers", $\mathbf{a} \equiv \{\, \mathrm{I} - \mathrm{I},\ \mathrm{II} - \mathrm{II},\ \mathrm{III} - \mathrm{III},\ \ldots \,\}$;

${}^{3}_{h}\underline{\mathbf{m}}_{\#} \equiv$ axioms-increment for all **W** "negative result" subtractions, named herein the "**m**inus numbers", $\mathbf{m} \equiv \{\, 0 - \mathrm{I},\ 0 - \mathrm{II},\ 0 - \mathrm{III},\ \ldots \,\}$;

${}^{3}_{h}\underline{\mathbf{f}}_{\#} \equiv$ axioms-increment for **Z**'s [non-zero] divisions, named herein the "**f**ractional numbers", $\mathbf{f} \equiv \{\, \mathbf{f} = \mathbf{n}/\mathbf{d} \mid \mathbf{n},\ \mathbf{d} \in \mathbf{m} \,\}$ ['|' ≡ "such that"];

${}^{3}_{h}\underline{\mathbf{d}}_{\#} \equiv$ axioms-increment for the '''**d**iagonal numbers''', $\mathbf{d} \equiv \{\, \{\, \mathrm{SBT}_{\pi}(x) \mid \mathrm{AP}_{\pi}(x) = 0\ \&\ x \neq \pm\mathbf{f},\ 0\ \&\ x \notin \Delta^{w}\mathbf{C} \,\} \cup \mathrm{SBT}_{\pi}\{e,\ \pi,\ \ldots\} \,\}^{*}$;

${}^{3}_{h}\underline{\mathbf{i}}_{\#} \equiv$ axioms for the "**i**maginary numbers" increment, $\mathbf{i} \equiv \{\, \mathbf{d}\underline{\mathbf{i}} \,\}$, wherein $+\underline{\mathbf{i}} \equiv +\underline{\sqrt{-1}}$, & $\underline{\mathbf{i}}^{2} \nparallel \underline{\mathbf{i}} \equiv [[\, \underline{\mathbf{i}}^{2} \not> \underline{\mathbf{i}} \,]\ \&\ [\, \underline{\mathbf{i}}^{2} \neq \underline{\mathbf{i}} \,]\ \&\ [\, \underline{\mathbf{i}}^{2} \not< \underline{\mathbf{i}} \,]]$;

${}^{3}_{h}\underline{\mathbf{h}}_{\#} \equiv$ axioms for the '''**h**amiltonian quaternionic increment''', $\mathbf{h} \equiv \{\, \mathbf{d}\underline{\mathbf{j}} + \mathbf{d}\underline{\mathbf{k}} \,\} \mid \underline{\mathbf{j}} \nparallel \underline{\mathbf{k}}\ \&\ \underline{\mathbf{j}}^{2} = \underline{\mathbf{k}}^{2} = -1,\ \&\ +\underline{\mathbf{jk}} = -\underline{\mathbf{kj}}$;

${}^{3}_{h}\underline{\mathbf{o}}_{\#} \equiv$ axioms for the '''**o**ctonionic ideo-ontological increment''', $\mathbf{o} \equiv \{\, \mathbf{d}\underline{\ell} + \mathbf{d}\underline{\mathbf{m}} + \mathbf{d}\underline{\mathbf{n}} + \mathbf{d}\underline{\mathbf{o}} \,\} \mid \underline{\ell} \nparallel \underline{\mathbf{m}} \nparallel \underline{\mathbf{n}} \nparallel \underline{\mathbf{o}}$.

Assuming the meanings "given" to us in advance for the first twelve symbols listed above [i.e., not counting ${}^{3}_{h}\underline{\mathbf{H}}_{\#}$ and ${}^{3}_{h}\underline{\mathbf{o}}_{\#}$], we will, in the core section of this essay, solve for the remaining **52** terms of $\left({}^{3}_{h}\underline{\mathbf{N}}_{\#}\right)^{64}$, using the '***Organonic Algebraic Method***', which is defined for you further on in this background section.

The ‘meta-model’ of the ***Gödelian Dialectic*** of the “***Standard Arithmetics***”, described above, is, per **F.E.D.**’s parlance, ***not*** a ***model***/an ideographical/equational description, or ‘character-ization’, of the ‘‘‘***ideic evolution***’’’ of, or ***within***, a ***single axioms-system***, for just one of these “***Standard Arithmetics***”, say the $^{3}_{h}\mathbf{C}_{\#}$ system. It is, on the contrary, a ‘***meta-model***’ of their ‘***ideo-meta-evolution***’; of the first seven **S**tages of their ‘***meta-system***’.

By the ‘‘‘***evolution***’’’ of an ***axioms-system***, **F.E.D.** means the process of proving more and more theorems of/about/-*within* that ***system***, formally and rigorously deduced from the ***axioms*** of that ***system***, and/or from other, previous deductions therein, thereby adding to / expanding / growing the ‘explicitized’, comprehended meaning of, and therefore also the known nature of, that ***axioms-system***, by the ‘qualitative super[im]position’ of the growing ‘cumulum’ of such theorems, and by the human-cognitive perception of the univocal unity, or «***genos***», of all those theorems, taken together.

By the ‘‘‘***meta-evolution***’’’ of/from a single arithmetical axioms-system -- e.g., from the beginning or «***arché***»-system of a ‘diachronic meta-system’, or ‘***arché***onic consecuum’, of such ***axioms-systems*** -- **F.E.D.** means the process of leaping, from an «***aporia***» arrived at ***immanently***, *within* that ***axioms-system***, to the outside of that ***system***, and thence to the inside of a new, higher ***axioms-system***; to a ‘‘‘consecutive’’’/successor ***system***, “higher” in the Gödelian **&** “logical type” senses. Such a ‘Gödelian successor-system’ is a ‘‘‘conservative extension’’’, or «***aufheben***»-negation, of its predecessor ***axioms-system***, retaining, if with some modification, most of the ***axioms*** of its predecessor, but becoming-successor-***system*** principally by adding new “comprehension ***axioms***” to the old/modified axioms-set, to *define*, to ‘*semantify*’ [to establish the arithmetical meaning of] the sets of higher “logical type” added to the previous, predecessor sets, of lower “type”, with the latter representing [‘‘‘modeling’’’] the kinds of numbers that were already comprehended, and ‘semantified’, by the predecessor arithmetical- ***system***’s ***axioms***, and with the former representing *new* kinds of number, *beyond* the latter.

This process of ***axioms-system*** ‘‘‘***meta-evolution***’’’ is ***not*** a process of ***deductive formal logic***, as is the process of ***axioms-system*** ‘‘‘***evolution***’’’-- the process of moving from axiomatically assumed and/or previously proven propositions to newly-proved theorems. Gödel himself noted this ***non***-formal-logical character of ***new-axioms discovery***: **“**...in the systematic establishment of the axioms of mathematics, new axioms, which do not follow by formal logic from those previously established, again and again become evident. ... it is just this becoming evident of more and more new axioms on the basis of the meaning of the primitive notions [**F.E.D.**: on the basis of the very notion of the meaning and nature of “number” itself] that a machine cannot imitate. ...this ***intuitive*** grasping of ever newer axioms that are logically independent from the earlier ones...is necessary for the solvability of all problems even within a very limited domain...**”** [Kurt Gödel, “The Modern Development of the Foundations of Mathematics in the Light of Philosophy (***1961**)”, in S. Feferman, *et. al.*, editors, ***Kurt Gödel***: ***Collected Works*** (*Volume* **III**: *Unpublished Essays and Lectures*), Oxford University Press [NY: **1995 C.E.**], page **385**, ***emphasis*** added]. Plato, in *The Republic*, offered a similar observation, somewhat earlier [*circa* **388 B.C.E.**], *and made that observation the* ***foundation*** *of his notion of* ***dialectic***: **“**These considerations point in particular towards that aspect of dialectic which lay at the forefront of Plato’s concern. He insisted upon two fundamental ideas: (1) that a starting point for rational argumentation cannot be merely assumed or *postulated*, but must itself be *justified*, and (2) that the *modus operandi* of such a justification can be dialectical. He introduced the special device he called *“dialectic”* to overcome this dependence upon unquestioned axioms. It is worthwhile to see how he puts [this] in his own terms: “There remain geometry and those other allied studies which, as we said, do in some measure apprehend reality; but we observe that they cannot yield anything clearer than a dream-like vision of the real so long as they leave the assumptions they employ unquestioned and can give no account of them. If your premiss is something you do not really know and your conclusion and the intermediate steps are a tissue of things you do not really know, your reasoning may be consistent with itself, but how can it ever amount to knowledge? ... So ... the method of dialectic is the only one which takes this course, doing away with assumptions .... Dialectic will stand as the coping-stone of the whole structure; there is no other study that deserves to be put above it.”.**”** [Nicholas Rescher, ***Dialectics***: ***A Controversy-Oriented Approach to the Theory of Knowledge***, SUNY Press [Albany, **1977**], page **48**].

Thus, the ‘‘‘***logic***’’’ -- the ‘***ordinality***-rules-system’ as to *“what follows from what”* -- driving the ‘***meta-model***’ of the ‘‘‘***meta-evolution***’’’ of arithmetic introduced above, and to be explicated in the sequel, cannot be ***deductive formal logic***. The $^{3}_{h}\mathbf{W}_{\#}$ ***axioms-system*** of arithmetic does ***not*** follow from the $^{3}_{h}\mathbf{N}_{\#}$ ***axioms-system*** of arithmetic, ***nor*** the $^{3}_{h}\mathbf{Z}_{\#}$ from the $^{3}_{h}\mathbf{W}_{\#}$, ***nor*** the $^{3}_{h}\mathbf{Q}_{\#}$ from the $^{3}_{h}\mathbf{Z}_{\#}$, ***nor*** the $^{3}_{h}\mathbf{R}_{\#}$ from the $^{3}_{h}\mathbf{Q}_{\#}$, ***nor*** $^{3}_{h}\mathbf{C}_{\#}$ from $^{3}_{h}\mathbf{R}_{\#}$, by ***deductive formal logic***.

Instead, the rules of followership ['$\rightarrow\!\!\exists$'] for the $^{3}_{h}N_{\#} \rightarrow\!\!\exists\, ^{3}_{h}W_{\#} \rightarrow\!\!\exists\, ^{3}_{h}Z_{\#} \rightarrow\!\!\exists\, ^{3}_{h}Q_{\#} \rightarrow\!\!\exists\, ^{3}_{h}R_{\#} \rightarrow\!\!\exists\, ^{3}_{h}C_{\#} \rightarrow\!\!\exists \ldots$ ***Gödelian Dialectic*** are '''***intuitional***''' [cf. Gödel, in quote above], ***heuristic***, ***connotational***, "***in****tension-al*" [versus "***ex****tension-al*"], '***contental***', and fully ***semantic***, ***not*** merely syntactic or '''***form***-al'''. The '''***logic***''' of this 'meta-model' is meant to satisfy the ***intuition*** of those to whom it is presented -- to be "***seen***" [«***eídõlon***», image; «***eídos***», look, form; «***eíde***»; «***ιδεα***», ideo ~ [v]ideo] by them -- not merely to ***compel*** their "***un***knowing" [cf. Plato] assent by rigorous formal proof, ***however opaque*** and uncomprehended/unconceptualized the 'contental' reasons for that merely formal 'followership' may remain.

In summary, our Dyadic Seldon Function 'meta-model' of '***connotational entailment***' --

$$\underline{^{3}_{h})\!\!\mathrm{H}\!\!(^{\#}_{s_{\#}}}\,\Upsilon = \left(^{3}_{h}N_{\#}\right)^{2^{s_{\#}\Upsilon}}$$

-- invokes a rules-system of '***heuristic*** *followership*' or ordering [of ***progressive order*** or '***ordinality***'], a higher '''logic''', as one which appeals to, and directly addresses, human ***intuition***; the '*followership*' of ***connotations***, of "***in***tensions", of ***meanings***. It is ***not*** a logic of ***deductive*** '*followership*', but it is one with strong algorithmic guidance, with strong syntactic and 'qualo-parametric' ['epithetic'] cues and underpinnings. For example, the term $^{3}_{h}\mathcal{q}^{\#}_{aN}$, which arises in **s**tage **2** of our 'meta-model' of ***the Standard Arithmetics***, has two ['post-subscripted'] '''epithets'''/'qualo-parameters', **a** and **N**, in addition to 'pre-subscript', 'pre-superscript' and 'post-superscript' epithets/'qualo-parameters', **3**, **h**, and **#**, respectively. This term corresponds to the **3**rd generic ordinal quality, '[ideo-]ontology', or '[ideo-]ontological category', denoted by П_{3}, in the generic/"uninterpreted"/"unassigned" $_{N}Q_{\#}$ dialectical arithmetic. I.e., the ordinal numeric value of **N**, **(N)**, is **1**, and that of **a**, **(a)**, is **2**, so that $(\,^{3}_{h}\mathcal{q}^{\#}_{aN}) \equiv \,^{3}_{h}\mathcal{q}^{\#}_{(aN)} \equiv \text{П}_{2+1} = \text{П}_{3}$, and is the first "algebraic" [$\equiv$initially of "***un***known" **q**ualitative value: '***un***semantified'] term of this 'meta-model' that would be solved for, in the core section, using the '***Organonic Algebraic Method***', had we not chosen to 'pre-stipulate' its meaning. The term $^{3}_{h}\mathcal{q}^{\#}_{aN}$ signifies a ***combination*** of the **q**ualities/'ideo-ontology' of $^{3}_{h}a_{\#}$ & of $^{3}_{h}N_{\#}$: a "***dialectical synthesis***" of '''***thesis***''' $^{3}_{h}N_{\#}$ &/with '***contra-thesis***' $^{3}_{h}a_{\#}$; a "real subsumption" / conversion of $^{3}_{h}N_{\#}$ by/into $^{3}_{h}a_{\#}$. Our 'pre-solution', making "known" the initially "***un***known" meaning of this therefore initially "algebraic" term, $^{3}_{h}\mathcal{q}^{\#}_{aN}$, is that it represents the 'ideo-ontology' of, e.g., medieval, Indo-Arabic place-value-notation arithmetic, with **0** as "place-holder", e.g., **aN** $\equiv$ **{ 0...01**, **0...02**, **0...03**, **0...04**, **0...05**, **0...06**, **0...07**, **0...08**, **0...09**, **0...010**, **... }**, trumping the ancient 'place-value-less', '[subtractive/]additive/juxtapositional' arithmetics, e.g., the Roman Numerals version, **N** $\equiv$ **{ I**, **II**, **III**, **IV**, **V**, **VI**, **VII**, **VII**, **VIII**, **IX**, **X**, **... }**. That value completes a process whereby the meaning of the '''***thesis***''', $^{3}_{h}N_{\#}$, step **0**, passes ***dialectically*** into the meaning of the '***synthesis****-sum*', $\left(^{3}_{h}N_{\#} \oplus \,^{3}_{h}a_{\#}\right) \oplus \,^{3}_{h}\mathcal{q}^{\#}_{aN}$, of step **2**, via the mediation of the '''***anti-thesis***''', $^{3}_{h}a_{\#}$, and of the '***antithesis****-sum*', $^{3}_{h}N_{\#} \oplus \,^{3}_{h}a_{\#}$, step **1**. This 'trans-formal', 'trans-deductive' ***logic*** is meet for the tasks of such 'meta-models', because the intimation of incremental, new, deductively-independent axioms in the Gödelian movement from predecessor arithmetical axioms-system to successor arithmetical axioms-system, is, as we have seen, in the words of Plato and of Gödel, above, ***not*** a process of deductive logic. Indeed, each **s**tep in this axioms-system progression is a questioning of the «***arché***»-axioms of the «***arché***»-system, as of the added axioms of every subsequent axioms-system, through ***immanent critique***, through the surfacing/-'explicitization' of their immanent, implicit «***aporia***»; thence "conservatively extending" -- or '***aufhebening***' -- their axioms in order to address, and in order to redress, by '''supplementation''', the thus-surfaced deficiencies /-incompletenesses of those axioms, and of the notion / definition of "[counting] number" that they imply.

A 'Dialectical Meta-Equation' is, first and foremost, an equating between multiplicity and unity. In the 'Dialectical Meta-Equation' above, the 'cumulum' of a growing multiplicity of terms, ${}^{3}_{h}\Xi^{\#}_{s_\#}$ -- growing as $s_\#$ grows -- is related, indeed equated, to a source which has unity at its core: ${}^{3}_{h}N_\#$, i.e., to the 'iterated self-re-flex-ion' [the 'bending-back-of-self-upon-self' /-'operating-upon-self-of-/by-self'] of the '«***arché***»-ic' ***unit***[y], ${}^{3}_{h}N_\# \mapsto q_1$ [wherein '$X \mapsto q_n$' means the ***assignment*** of 'synchronic' ***speci***fic category X to the ***gene***ric ${}_{N}Q$ 'meta-numeral' q_n].

A 'Dialectical Meta-Equation' is, furthermore, a ***quali*tative** multiplicity, a heterogeneity of contents, one which "contains" a growing richness of ***opposing*** ***qual***ities; one which includes advancing instances of «***aporia***» and of the ***dialectical resolution*** of those instances; a series of ever more dazzling correctives to the original/originating, '«***arché***»-ic' posit, the 'qualitative ***first*** approximation', '''named''' ${}^{3}_{h}N_\#$.

The 'Dyadic Seldon Function meta-model' explicated herein is a self-iterated succession of 'self-confrontations' of 'ideo-ontologically'-distinct «***aporia***» of arithmetics, each representing a qualitatively different -- progressively-expanding -- oppositional co-positing of two opposing notions/definitions of [counting] number.

This 'Seldon Function' signifies the recurrent self-reflection of a human mind, 'mentally embodying' a given system of arithmetic, X, and reflecting, as ***conscious subject***, '$X(\_)$', upon its 'mental embodiment' of that system, via reflection upon the same system in its ***object*** [objec*tified*], e.g., written, axiomatic '''body''', '$\_(X)$': $X(X) = \hookleftarrow(X) = X \oplus \triangle(X)$. This ***self-reflection*** of X, as it rises to ***full self-reflection***, i.e., as X^1 gradually and '"continuously"' becomes X^2, represents the ***dialectical*** [***self***-«***aufheben***»] ***self-negation*** of X^1, which is equivalent to adding the opposing '***correction term***', $\triangle(X^1)$, to X^1: $X^1 \oplus \triangle(X^1)$. The cognitive ***self-confrontation*** of X denoted by $X(X)$, signifies a process in which implicit flaws, inadequacies, incompletenesses -- e.g., "***un***solvable equations" **&** "***un***provable theorems" -- of the X axioms-system are turned-up, **&** turned-out/'''outered'''/'explicitized' -- located, noticed, discerned, discovered, uncovered, divulged. It signifies a process of ***immanent critique***, i.e., of the ***internal***, ***self-critique***, of that axioms-system, X; of ***self-critique*** of the ***immanent*** 'ideo-ontology' of that 'ideo-system', and of its concomitant calculative 'ideo-technology'. For example, first, in --

$$ {}^{3}_{h}\Xi^{\#}_{s_\#}\Uparrow \quad = \quad \left({}^{3}_{h}N_\# \right)^{2^{s_\#\Uparrow}} $$

-- in its step $s_\# = 1$, the «***arché***»-system, $X = {}^{3}_{h}N_\#$, confronts itself --

$$ {}^{3}_{h}\Xi^{1}_{0} \rightarrow {}^{3}_{h}\Xi^{2}_{0} = \hookleftarrow {}^{3}_{h}\Xi^{1}_{0} = {}^{3}_{h}\Xi^{1}_{1} \Rightarrow \left({}^{3}_{h}N_\#\right)^1 \rightarrow \left({}^{3}_{h}N_\#\right)^2 = {}^{3}_{h}N_\#\left({}^{3}_{h}N_\#\right) = {}^{3}_{h}N_\#\text{ "of" }{}^{3}_{h}N_\# $$

$$ = \hookleftarrow\left({}^{3}_{h}N_\#\right) = \left({}^{3}_{h}N_\#\right) \oplus \triangle\left({}^{3}_{h}N_\#\right) \equiv \left({}^{3}_{h}N_\# \oplus {}^{3}_{h}a_\#\right)^1 $$

, the first «***aporia***». Then, next, cognition rolls on to $\left({}^{3}_{h}N_\# \oplus {}^{3}_{h}a_\#\right)^2$, the $s_\# = 2$ ***self-confrontation***/***self-critique*** of that first «***aporia***» ***itself*** --

$$ {}^{3}_{h}\Xi^{1}_{1} \rightarrow {}^{3}_{h}\Xi^{2}_{1} = \hookleftarrow {}^{3}_{h}\Xi^{1}_{1} = {}^{3}_{h}\Xi^{1}_{2} \Rightarrow \left({}^{3}_{h}N_\# \oplus {}^{3}_{h}a_\#\right)^1 \rightarrow \left({}^{3}_{h}N_\# \oplus {}^{3}_{h}a_\#\right)^2 $$

$$ = \left({}^{3}_{h}N_\# \oplus {}^{3}_{h}a_\#\right)\left(\left({}^{3}_{h}N_\# \oplus {}^{3}_{h}a_\#\right)\right) = \left({}^{3}_{h}N_\# \oplus {}^{3}_{h}a_\#\right)\text{ "of" }\left({}^{3}_{h}N_\# \oplus {}^{3}_{h}a_\#\right) $$

$$ = \hookleftarrow\left({}^{3}_{h}N_\# \oplus {}^{3}_{h}a_\#\right) = \left(\left({}^{3}_{h}N_\# \oplus {}^{3}_{h}a_\#\right) \oplus \triangle\left({}^{3}_{h}N_\# \oplus {}^{3}_{h}a_\#\right)\right) = $$

$$ = \left(\left(\left({}^{3}_{h}N_\# \oplus {}^{3}_{h}a_\#\right) \oplus {}^{3}_{h}\varrho_{aN}^{\#}\right) \oplus {}^{3}_{h}m_\#\right) \equiv \left({}^{3}_{h}W_\# \oplus {}^{3}_{h}m_\#\right)^1 $$

; containing both ${}^{3}_{h}W_\#$, the 'dialectical ***synthesis***-sum'/'first ***uni-thesis***' ***solution*** to the 1st «***aporia***», and the irruption of the 2nd «***aporia***». The ***self-critique*** of the 2nd «***aporia***», $\left({}^{3}_{h}W_\# \oplus {}^{3}_{h}m_\#\right)^2$, leads to $\left({}^{3}_{h}Z_\# \oplus {}^{3}_{h}f_\#\right)$, with ${}^{3}_{h}Z_\#$ as 2nd «***aporia***» ***solution***, but also precipitating the 3rd «***aporia***». ...and so on, as the value of $s_\#$ escalates.

B.δ. ***Dialectic***: **The Journey of the Progression of the «*Aporia*»**. We will endeavor to instill, herein, for readers unversed in dialectic, the minimum background in that cognitive stage requisite for an apt appropriation of our narration of 'The Dyadic Seldon Function Meta-Equational Meta-Model of our Meta-Systematic Dialectical Presentation of the Progression of the Standard Arithmetics' in the core section of this essay -- the same 'Meta-Model' introduced in the background sub-section immediately preceding this one. We will here present, with our commentary, an extended excerpt from an excellent source, recently published, on 'The [Psycho]Historical Dialectic of Modern Science': ***Newton, Maxwell, Marx***, by Thomas K. Simpson, Green Lion Press [Santa Fe: **2012**]. Despite the 'extendedness' of this excerpt, it provides one of the most succinct introductions to **~2500** years of Terran humanity's [psycho]history of conscious dialectic that we have so far found, an introduction which is located toward the end of the final section of the above-cited book, the section on the work of Karl Marx. Here it is: **"**We have spoken earlier of two models of "science" -- that which aims to save the appearances, and was exemplified by Ptolemy, and another, exemplified by Newton and *The Principia*, which seemed the model that Marx follows in *Capital* in its aspect as formal science. Now, however, I am proposing a third, for I believe that the dialectical reading of *Capital* is not less "scientific" than the formal one, but rather that it very much enlarges the sense and scope of "science". We would be right, I believe, in speaking of *Capital* in its larger aspect as a work of dialectical science. Though this thought cannot be adequately developed here, I would like to suggest what it might mean. To do so, however, we must take a moment to reflect on the notion of "dialectic" itself. For though I have often referred to the concept in the course of this study, and made certain claims about it in passing, I have not explained what I understand it to mean.**"** [*op. cit.*, pp. **261**-**262**].

Thus, Dr. Simpson introduces his account of the history and meaning of the word "dialectic", in the context of a concept of "dialectical science". As regular readers of the www.dialectics.org website will be aware, that phrase names the stage of science to which we refer our work as well. Dr. Simpson next notes the [long] history of the concept of "dialectic" in the tradition of the Occidental hemisphere of the Terran Human[oid] Phenome, **&** that this history exhibits a [psycho]historical dialectic of its own, "the dialectic of the dialectic [itself]" -- also a theme which guests of this site will recognize as seminal for **F**.*E*.*D*. [dialectic⟨dialectic⟩ link]: **"**Dialectic has its own history in our Western experience; we might speak of its own "dialectical" development -- or of "the dialectic of the dialectic." It will suffice for the moment if we take just three great benchmarks of dialectic as exemplars of this unfolding concept: the Platonic dialogues, of course; the Hegelian dialectic; and the dialectic of *Capital*.**"** [*op. cit.*, p. **262**]. [Link to Publisher's Page for ***Newton, Maxwell, Marx***: http://www.greenlion.com/nmm.html].

Aside from the writings of Heraclitus -- only shreds of which survived the turmoils of the last Dark Ages -- Dr. Simpson's selection of dialectic's "benchmarks" is one with which **F**.*E*.*D*. heartily agrees. However, the reader should note that it is not clear, in Dr. Simpson's account, that he would agree entirely with **F**.*E*.*D*.'s delineation of four 'historical «***species***»' of dialectic, as constituting "the dialectic of the dialectic[itself]". Dr. Simpson's account continues as follows: **"**... the term "dialectic" suggests ... a certain structure of inquiry, and we need to consider the relation between such a real question on the one hand, and the pattern of dialectic on the other: not only the vividness of the human questions that they ask, but a certain common form links Plato, Hegel, and Marx. As we say this, however, it is very important to avoid possible misunderstanding: though the thread that links these stages of dialectic is very real, and fundamental in particular to our understanding of Marx, to point to this common principle is not at all to assert that dialectic is not very much transformed in its passage from one stage to the next.**"** [*op. cit.*, p. **262**].

In the ***Encyclopedia Dialectica*** shorthand, this pattern of dialectic, common to Plato, Hegel, and Marx, can be rendered generically as: $(x) \multimap x(x) = \multimap(x) = (x^2) = (x \oplus y) \multimap (x \oplus y)^2 = (x \oplus y)((x \oplus y)) = \multimap((x \oplus y)) = ((x \oplus y)^2) = ((x \oplus y) \oplus z)$..., or, in words, as: '"***thesis***" [**x**] ***becomes thesis self-reflected & immanently*** [***self***-]***critiqued*** **=** '***antithesis-sum***' [$(x \oplus y)$], which in turn ***becomes*** '***antithesis-sum***' itself ***self-reflected*** and ***immanently*** [***self***-]***critiqued*** **=** '***synthesis-sum***' $\oplus$... [$((x \oplus y) \oplus z) \oplus$...]'. But, before formulating any such pattern of dialectic in general, Dr. Simpson cites and summarizes Plato's dialogue *The Meno*, as a paradigmatic example thereof:

"The dialogue *Meno* serves well as a paradigm of the dialectical motion in its Socratic mode ... To focus, then, for a moment on the *Meno* as a paradigm of the dialectical form: the opening question is developed in a variety of artful ways that steadily reduce this brash young general, initially confident enough in the world's ways, to a state of what may be serious wonder and concern. He meets unexpected difficulty in defining virtue ("excellence"), though he had evidently never before doubted that he was himself a living model of it. His failure leads him to a certain, perhaps petulant, despair, and it is significant that here the dialogue comes to a dead stop -- at its effective center -- with an outcry from Meno that seems in its own terms unanswerable ... This point of death of the argument -- which appears, I am tempted to say at the virtual center of all dialectic -- is often referred to in Greek as the *aporia* -- the sticking point, the passage of no passage, the point of no return for the argument.**"** [*op. cit.*, pp. **262**-**264**].

What is at stake in the 'self-dialogue meta-model' to be narrated and explicated in the core section of this essay, is the definition of [counting] number, and its stag***é***d *incompletenesses* and *self-antitheses*, starting from the definition of "cardinal number" that is embodied in the **N** axioms-system for the "**N**atural" Numbers as *"**first thesis**"*. The succession of «***aporia***», or of ***quandaries***, that are both evoked and transcended in the course of this 'meta-model' may seem ***muted*** in their conceptual and emotional intensity, relative to those encountered, e.g., in Plato's *Meno*, with regard to what constitutes *"the good life"* for humans *individually*, and, e.g., in Marx's ***Capital***, regarding the good and the bad of the life of the human -- historical -- «***species***» *collectively*. It will remain so muted, at least ***until*** the ***connexion*** between the history of the concept of number -- as a psychohistorical barometer of the initially unconscious historical emergence of the '''Law of [***Capital***-]Value''' in human praxis, and, consequently, of that '''Law's''' shaping of the quality, for good and for ill, of the life of the human «***species***», as an unconsciously-legislated, and continually 're-legislated', *by-collective-action-promulgated*, collective-***un***conscious ***self***-imposition, *by* that life, *upon* that life *itself*, both collective and individual -- ***becomes a connexion that is far more widely grasped within the human*** «***species***» ***than it is at present***. This *e*ffective *a*ffective muting will be amplified by the fact that the readers of this essay, in their majority, ***will already know*** the ways around and beyond these «***aporia***» of the concept of [counting] number that their ancestors have long since already discovered -- sometimes only after long and arduous conceptual struggle. Such readers will thus tend not to feel the ***bite*** of each ***quandary*** personally -- the ***bite*** that at least some of their ancestors *did* once feel, when such a ***quandary*** was still ***un***resolved -- here, in this present-mathematical-viewpoint-based, essentially ***syn***chronic, 'ideo-systematic', 'ideo-taxonomic' exposition. Only narration of a ***dia***chronic, [psycho]***historical***-***dialectical*** 'meta-model', of the actual historical order and content of the successive human-phenomic *memes of number*, with careful exegesis of well-chosen selections from their still-extant 'psycho-artefacts' -- their written controversies, etc. -- can serve to revivify and to re-illuminate the heat and the rancor of past conceptual struggles; can overcome the illusory muting affect induced by our already knowing the historic resolutions of past «***aporia***». Such a historical narration is not the purview of the present essay, but is, in part, that of the multi-volume treatise *Dialectical Ideography*, especially of its volume **2**.

Having so descried, and so described, the «***aporia***» that irrupts in and as the middle of the *Meno*, Dr. Simpson leaves behind the *Meno*'s specificity, and formulates the common pattern of dialectic in its generality, thusly: **"**We cannot here trace that way by which Socrates at once opens a path for Meno -- and, we must add, for us. ... The *aporia* of the dialectic will always lead us into the darkest obscurity; but if all goes well, we will emerge empowered with knowledge we had not known we possessed. To be schematic about this, and as an aid in tracing something of this same pattern in Hegel and Marx, we may say that there are three parts to every dialectical motion.

I. The opening question, a real question, which takes form through an intense searching in the mode of questions and answers, not yet fully articulate;
II. The clarifying argument to the point of contradiction and despair; the question becomes articulate but, at the same time, leads to *aporia*;
III. The passage beyond the *aporia*, through yet more serious questioning, which yields whatever knowledge is humanly possible; not, however, in the mode of syllogistic consequence, but drawing on some larger source of human intuition, in the form of image, myth and mystery.**"** [*op. cit.*, pp. **264**-**265**].

We would characterize the «***genos***» of the items in Dr. Simpson's list of larger sources of human intuition under the category which one of **F**.*E*.*D*.'s early mentors, Dr. Charles Musés, used to call «***Symbólos***», «***Symballein***», or «***Symbolon***» -- that of ***The Symbolic***. This is a category which also includes ***the mathematical*** in its largest sense.

In the core narrative of this essay, it is new ***mathematical symbolizations***, symbolizations of *new number* '***ideo-ontology***'; of ***new kinds*** of number(s) -- and, therefore, of new '***ideo-technology***', making possible useful calculations which could ***not*** previously, on the "pre" side of each such *'ideo-systematic'* «***aporia***», have been carried out -- that objectify our ancestors' subjective escape from the ***dialectical*** «***aporia***» of «***arithmêtikê***»; from the ***dialectical*** «***aporia***» of the concept of "***cardinal*** number". Definitions for the term '''ontology[-in-general]''', for the term 'physio-ontology', and for the term 'ideo-ontology', in their ***Encyclopedia Dialectica*** senses, can be reviewed via the following link --

Link to *F.D*. Definition of the Term "Ontology": http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Ontology/Ontology.htm.

¿But, given the above rendition of the pattern of dialectic in general, what do we mean by '***The Gödelian Dialectic***' specifically -- the term at the root of this essay's title, '***The Gödelian Dialectic of the Standard Arithmetics***' -- and how does this ***speci***fic, ***Gödelian***, ***dialectic***, fit the general pattern outlined by Dr. Simpson**?**

B.ε. '*The Gödelian Dialectic*', or, "The Incompletability of Mathematics".

Kurt Gödel, arguably the contributor of the greatest leaps forward in the science of formal logic since classical antiquity, described an 'axiomatic dialectic' of mathematics, albeit in "['early'-]*Platonic*", '''***a****-psychological*''' and '''***a****-historical*''' terms, hence in '***a-psychohistorical-dialectical***' terms as well, as follows --

"It can be shown that ***any formal system whatsoever*** — whether it is based on ***the theory of types*** or not, if only it is free from contradiction — ***must necessarily be deficient in its methods of proof***. Or to be more exact: For any formal system you can construct a proposition — in fact a proposition ***of the arithmetic of integers*** — which is certainly true if the system is ***free from contradiction*** but ***cannot be proved in the given system*** [F.E.D.: the foregoing summarizes Gödel's "First Incompleteness Theorem"]. Now if the system under consideration (call it ***S***) is ***based on the theory of types***, it turns out that ***exactly the next higher type*** not contained in ***S*** is necessary to prove this ***arithmetic*** proposition, i.e., this proposition ***becomes a provable theorem*** if you add to the system ***the next higher type and the axioms concerning it.*****"** [Kurt Gödel, "The Present Situation of the Foundations of Mathematics (*1933o)", in S. Feferman, *et. al.*, editors, ***Kurt Gödel***: ***Collected Works*** (*Volume* III: *Unpublished Essays and Lectures*), *ibid.*, page **46**; **bold**, *italics*, underline, and color emphasis added by **F.E.D.**]. Again --

"If we imagine that the system **Z** [F.E.D.: a formal, logical, propositional-/predicate-calculus system inclusive of "***Natural***" *Numbers'* Arithmetic, ***not*** the full system of the positive and negative ***Integers***, and zero[which is both, [or neither] positive [n]or negative], standardly also denoted by **ℤ**] is successively enlarged by the introduction of variables for classes of numbers, classes of classes of numbers, and so forth, together with the corresponding ***comprehension axioms***, we obtain a ***sequence***...***of formal systems*** that satisfy the assumptions mentioned above, and it turns out that ***the consistency*** (ω-consistency) ***of any of these systems is provable in all subsequent systems***. Also, ***the undecidable propositions constructed for the proof of Theorem 1*** [F.E.D.: Gödel's "First Incompleteness Theorem"] ***become decidable by the adjunction of higher types and the corresponding axioms***; however, ***in the higher systems we can construct other undecidable propositions*** by the ***same procedure***. ...To be sure, all the propositions thus constructed are ***expressible in*** **Z** (hence are ***number-theoretic propositions***); they are, however, ***not decidable in*** **Z**, but ***only in higher systems***..." [Kurt Gödel; On Completeness and Consistency (1931a), J. van Heijenoort, editor, ***Frege and Gödel***: ***Two Fundamental Texts in Mathematical Logic***, Harvard University Press [Cambridge: **1970**], page **108**, **bold**, *italic*, underline, and color emphasis [and square-brackets-enclosed commentary] added by **F.E.D.**].

The cumulative «***aufheben***»-progression of '''***conservative extensions***''' -- i.e., the advancing *'ideo-cumulum'* of axiomatic systems which Gödel describes above -- was viewed ***a***historically by him. Gödel was an avowed '''neo- Parmenidean''', and a professed "mathematical Platonist" [in the sense of the *earlier* rather than of the *later* Plato]. He didn't intend this '***meta-system***' — this *cumulative micro-diachronic progression of* [*axioms*-] *systems* — to serve as a ***temporal*** or ***psychohistorical-dialectical*** [meta-]model of the stages of human mathematical cognition, as reflective of the stages of the self-development of humanity's collective cognitive powers as a whole; of the ***knowledges*** to which each such epoch of those powers renders access, and of the *"**historically-specific**"* ***ideologies*** [or ***pseudo-knowledges***] to which human thinking is susceptible and vulnerable within each such epoch.

We of **F.E.D.**, however, do wish to explore its efficacy as such. Note how, as Gödel narrates this axioms-system**s**-progression above, each successor system '«***aufheben***»-***contains***' its immediate predecessor system, and, indeed, **all** of its predecessor systems; how each higher logical type '«***aufheben***»-***contains***' **all** predecessor logical types. Can Gödel's theory of this ***cumulative***, '***evolute***', « ***aufheben***» ***progression*** of axioms-systems, which we term '***The Gödelian Dialectic***', or '***The Gödelian*** [Idea-Systems' ***Ideo***-]***Metadynamic***', provide at least an idealized [i.e., a distorted] image of **actual history**, of the actual ***psychohistorical-dialectical*** struggle, process, and progress of mathematical aspects of the self-development of a humanity's ***universal labor*** [Marx]; of its collective cognitive capabilities, hence of its ***knowledges*** and ***ideologies***? We say "yes!". [To clarify our terminology here: we mean, by an '***ideo-dynamic***', a pattern that characterizes the 'int***ra***-systemic' process of, e.g., ***deductively*** proving ever-more theorems ***within*** a single axioms-system, in the context of a diachronically presented '***meta-system***[***atic***]' ***progression*** of axioms-system**s**. We mean, on the contrary, by the neologistical term '***ideo-meta-dynamic***', the pattern characterizing the 'int***er***-systemic', '***non-deductive***-logic-al' process of moving from ***inside*** a given predecessor axioms-system, to outside it, to ***inside*** its successor axioms-system].

Each of Gödel's "undecidable" propositions of arithmetic that plague each '''epoch''' of this formal axiomatic expansion is a proposition asserting, ***of itself*** -- '''***self-reflexively***''' -- that it is a formula which cannot be ***deductively*** demonstrated from the axioms of its axioms-system of arithmetic. That is, each '''Gödel formula''' states that "***I am not a theorem of my axioms-system***." -- ***not*** a formally provable statement of the axioms-system of arithmetic inside which it immanently arises, yet still as a well-formed formula ***within*** that system.

If such a proposition is ***true within*** its axioms-system -- if the "***I am not a theorem of my axioms-system***." proposition actually ***cannot*** be ***deductively*** demonstrated from the axioms of the axioms-system in which it arises -- then things are ***very bad*** from an immanent, formal-logical point-of-view; then the system in which that proposition arises immanently is logically '"***incomplete***"' -- i.e., is incapable of ***deductively*** demonstrating all of the ***true*** propositions which arise as well-formed formulas ***within*** the rules of that system, because its Gödel proposition exhibits that there is ***at least*** this one ***true*** proposition ***within*** it that it cannot ***deductively*** prove.

If such a proposition is ***false within*** its axioms-system -- if the "***I am not a theorem of my axioms-system***." proposition actually ***can*** be ***deductively*** demonstrated as a theorem from the axioms of the axioms-system in which it arises immanently -- then matters are ***even worse***, immanently, from a formal-logical point-of-view. For then the axioms-system of arithmetic in which this proposition arises immanently is even ***contradictory***; '''***self-inconsistent***''', i.e., is capable of ***deductively*** demonstrating at least one ***false*** proposition, as well as the [***true***] negation of that proposition, and thus of ***deductively*** "demonstrating" two [or more] mutually ***formally contradictory*** propositions, because its Gödel proposition exhibits that there is at least this one ***false*** proposition ***within*** it that it can, erroneously, ***deductively*** "prove". That is, the axioms-system can derive, ***as a theorem***, a proposition which, in effect, asserts, of itself, that "***I am not a theorem of my axioms-system***.", and which, thus ***deductively*** provable as a theorem, is thereby shown to be a ***false*** proposition.

That is the "meta-mathematical" content of a '''Gödel Formula''', its meaning at the level of the formal-logical "meta-language", which talks about axioms-systems, provability from axioms, etc. But the real secret of the meaning of such '''Gödel Formulae''', seldom mentioned in the standard accounts of Gödel's "First Incompleteness Theorem", is their "deformalized" content, their "mathematical", i.e. arithmetical and algebraic content; their meaning in the "object language", the language which talks about numbers, algebraic ["diophantine"] equations, and their solvability, etc. At that deeper, more concrete level, a '''Gödel Formula''' is a proposition asserting the ***un*solvability** of a specific "***diophantine***" [F.E.D.: referring to the famous, foundational, *circa* **250** C.E. 'proto-ideographic-algebraic' work by ***Diophant***us of Alexandria, entitled the «***Arithmetica***»] algebraic **equation**. I.e., each "Gödel formula", or "Gödel sentence", which, at the "meta-mathematical" level, asserts the ***either***-self-incompleteness-***or***-self-inconsistency of its axioms-system, "deformalizes" to one which asserts the ***un*solvability** -- ***within*** that system of arithmetic, and given the *limitations* of the *kinds* of *numbers* that are at that axioms-system's disposal -- of a specific, algebraic, "***diophantine* equation**" --

"... The Gödel sentence **φ...** asserts its own ***un****deducibility* from the postulates....Deformalizing **φ**... we see that under the standard interpretation it expresses a fact of the form [F.E.D.: for every **n**-ary list of number-components of ***x*** such that each number-component is a member of the set of 'diophantine numbers', e.g., of "**N**atural" numbers] ...$fx \neq gx$... , where f and g are **n**-ary polynomials [F.E.D.: I.e., more accurately, are **n**-ary *polynomial* ***functions***, whereas fx and gx denote their *function-****values***]....An equation $fx = gx$, where f and g are two such polynomial[-***function***]s, is called *diophantine* .... By a solution of the equation we mean an **n**-tuple α of natural numbers such that $f\alpha = g\alpha$... . So **φ...** asserts the ***un***solvability of the...equation $fx = gx$, and the proof of [F.E.D.: Gödel's "First Incompleteness Theorem"] *produces... a* ***particular*** *diophantine equation that is really* ***un****solvable, but whose* ***un****solvability cannot be* ***deduced*** *from the postulates...***"** [Moshé Machover, ***Set Theory, Logic, and their Limitations***, Cambridge University Press [Cambridge: **1996**], pages **268**-**269**, ***emphases*** ***and*** square-brackets-enclosed commentary **added** by F.E.D.].

Each of the ['"locally"'] ***un****solvable* algebraic equations that we will evoke, in the core section of this essay, in our narration of the Dyadic Seldon Function 'meta-model' of ***the dialectic of the Standard Arithmetics***, will be seen to formulate, and to '***ex***plicitize', a ***paradox*** for that concept of [counting] number which is ***im***plicit in the "[syn]thesis" arithmetical axioms-system term $^{3}_{h}\mathbf{X}_{\#}$, of the '***aporial*** *antithesis-sum'*, $^{3}_{h}\mathbf{X}_{\#} \oplus \triangleq {}^{3}_{h}\mathbf{x}_{\#}$.

Per the ***modern*** definition, a "diophantine equation" is an equation whose *parameters* [e.g., *coefficients*] and whose *solutions* are restricted to the "integers", or, sometimes, to "rational" numbers [in the case of Diophantus himself, positive rational numbers only, excluding zero, etc.]. Each "Gödel sentence"-encoded equation truly is ***un***solvable ***within*** the given axioms-system. However, the proposition that it is so, ***cannot*** be ***deductively*** proven ***within*** that axioms-system. But that proposition ***can*** be so proven within the ***next*** axioms-system, its immediate successor — the ***latter*** being created through the «***aufheben***» '***self-internalization***' of the '''vanguard''', 'meristemal', highest [in "***logical type***"] set '''idea-objects''' of the universe of discourse of the ***predecessor*** axioms-system, a '***self-internalization***' which produces sets of the ***next higher*** "***logical type***". That proposition can also be so ***proven*** within all subsequent successor-systems, created by yet-further such «***aufheben***» '***self-internalizations***'. "***Logical type***" works like this. If we say that the '"universal set"', or '"universal class"', containing all of the finite number of ***non***-set, "logical individuals", e.g., all of the individual numbers, that constitute a given "universe of discourse [of arithmetic]", e.g., **{0, 1}**, for the "universe of discourse" of the arithmetic of Boolean algebra, is defined as being of "***logical type***" **one**, then the set of all of its subsets, **{ {0}, {1}, {0, 1}, {_} }**, with '**{_}**'denoting the "empty set", is of "***logical type***" **two**, and the set of subsets of that set of subsets --

{ {{0}}, {{1}}, {{0, 1}}, {{_}}, { {0}, {1}, {0, 1}, {_} }, {{0}, {1}}, {{0}, {0, 1}}, {{0}, {_}}, ... }

-- is of "***logical type***" **three**. That is, we are, in effect, counting the '"depth"' of the braces of a set -- including counting the main, outer, braces -- to assess the "***logical type***" of that set, i.e., to measure the '''depth''' of the 'sets-***as***-***elements***' content of the set in question. If the "*logical individuals*", or '*arithmetical idea-objects*', "existing" per the "*comprehension axioms*" of a given axioms-system, are limited to "**N**atural" Numbers, *classes* of "**N**atural" Numbers,..., all the way up to *classes of classes of*... of "**N**atural" Numbers, e.g., to 'class-objects' up to a given "***logical type***", then the next system will ***cumulatively*** expand those '''existential''' limits by one step, to include also *classes of classes of classes*... of "**N**atural" Numbers, i.e., 'class-objects' of next-higher "***logical type***". Starting from the "universal class", each second and higher class-inclusion of previous 'class-objects' can model [including via *adjunction* of those object's corresponding "*comprehension axioms*", defining the '*computative behavior*' of these new entities] -- e.g., via the new, higher "***logical type***"-level of those special kinds of sets called "ordered pairs", that arise, for the first time, in the second step of universal class self-inclusion -- a *new* ***kind*** of arithmetical '*idea-object*'; *indeed, a new,* ***higher kind of number***. Thereby, this *qualitative expansion of each* ***predecessor axioms-system***, in the formation of its ***successor axioms-system***, together with the adjunction of the additional, "*comprehension axioms*" to the previous, **predecessor** axioms, corresponds to a *qualitative* expansion of the '*idea-ontology*' -- of the '*arithmetical ontology*', i.e., of the '***number****-ontology*' -- of that ***predecessor axioms-system***, thereby transforming it into its ***successor axioms-system***.

Hypothesis: ▮ Specifically, *the diophantine equation that was **unsolvable*** as such *within the **predecessor axioms-system** itself becomes **solvable**, albeit in a **non**-diophantine sense, i.e., by a **non**-diophantine type of number -- a number beyond the '"**N**atural"' Numbers -- within the **next** [as well as within all subsequent] **successor axioms-systems** in this **cumulative** sequence of **axioms-systems**, precisely by means of these **next higher** kinds / logical types of "ordered pair" sets, and by means of the **new kinds of numbers** which they ground, which will **not** be 'diophantine numbers', e.g., which will **not** be '"**N**atural"' Numbers.* ▮

The result is a progression of qualitatively-distinct, 'ideo-ontologically' distinct, 'number-space distinct', axioms-systems of arithmetic. Each successor arithmetical axioms-system "contains" the kinds of numbers, and at least some of the axioms, of all of its predecessor arithmetical axioms-systems. It also contains a new kind of number, "absorbing" and "converting" into itself -- into its kind -- all of the previous-systems' kinds of number that it "contains", and, with this new kind of number, solving a kind of algebraic equation that was ***un***solvable in its immediate predecessor arithmetical axioms-system, and proving a kind of proposition that was "***un***decidable" -- neither provable nor dis-provable -- in its immediate predecessor arithmetical axioms-system. But this successor arithmetical axioms-system also has its own, new kind of ***un***decidable theorem, and its own, corresponding, new kind of ***un***solvable algebraic equation, thus exhibiting a 'self-***in***completeness' which leads on to its own successor axioms-system of arithmetic. We term such a progression a '***Gödelian Dialectic***'. What we present herein, in the core section of this essay, is, precisely, '***The Gödelian Dialectic of the Standard Arithmetics***'[1], as encoded in a '***dialectical meta-model***' of its systematically-ordered method-of-presentation, expressed via the '***Dyadic Seldon Function Dialectical Meta-Equation***' introduced in the third sub-section of this background section. Given the [***potentially***-]infinite character of this ***dialectical***, «***aufheben***» systems-progression of axioms-systems of arithmetic, driven by the provability-***in***completeness and the equational-***un***solvability that characterizes every possible axioms-system of arithmetic in this progression of axioms-systems of arithmetic, Gödel himself describes this progression as manifesting "***the incompletability or inexhaustibility of mathematics***", which, from our point of view, represents also the ***potential interminability*** of '[***The Gödelian***] ***Dialectic***' --

"The metamathematical results I have in mind are all centered around, or, one may even say, are only different aspects of, one basic fact, which might be called ***the incompletability or inexhaustibility of mathematics***. ... The phenomenon of the inexhaustibility of mathematics, however, [is] always present in some form, no matter what standpoint is taken. So I might as well explain it for the simplest and most natural standpoint, which takes mathematics as it is, without curtailing it by any criticism. From this standpoint all of mathematics is reducible to abstract set theory. ... So the problem at stake is that of axiomatizing set theory. Now, if one attacks this problem, the result is quite different from what one would have expected. Instead of ending up with a finite series of axioms, as in geometry, one is faced with [F.E.D.: a ***potentially***] infinite series of axioms, which can be extended further and further, without any end being visible and, apparently, without any possibility of comprising all of these axioms in a finite rule producing them. This comes about through the circumstance that, if one wants to avoid the paradoxes of set theory ... the concept of set must be axiomatized in a stepwise manner. If, for example, we begin with the integers, that is, the finite sets of a special kind, we have at first the integers and the axioms referring to them (axioms of the first level), then the sets of sets of integers with their axioms (axioms of the second level), and so on for any finite iteration of the operation "set of". ...**"** [Kurt Gödel, "Some Basic Theorems on the Foundations of Mathematics and Their Implications (*1951)", in S. Feferman, *et. al.*, editors, ***Kurt Gödel***: ***Collected Works*** (*Volume* III: *Unpublished Essays and Lectures*), *ibid.*, pages **305-306**].

¿Is this ***Gödelian*** '***Dialectic of Arithmetics***' -- this "inexhaustibility" and "incompletability" of arithmetics -- still alive and in evidence today**?** **¿**Are new, 'ideo-*ontologically* expanded' -- *kinds*-of-numbers-expanded -- arithmetics still being discovered**?** **¿**I.e, do we today ***need*** to search for a Gödelian "next" arithmetic, with its new kinds of numbers: an arithmetic that has eluded us so far**?** **¿**Do we ***need*** that next, new arithmetic, and its new kind of numbers, in order to solve equations that have remained "***un***solvable" ever since their discovery, so far, and right up until to today**?** **¿**Do we need it to enable ***calculations*** that we cannot perform even with today's most advanced mathematics, hence to enable ***predictions*** that we cannot, even with today's most advanced *'ideo-technology'*, discern, hence to enable the rise of new 'physio-technologies', new technologies that are crucial to the very survival of the Terran human «***species***», e.g., to mitigate external hazards -- 'exolithic bombardments', solar coronal mass ejections; solar and other-stellar mega-flares; magnetars; gamma ray bursts/hypernovae -- and to mitigate internal hazards, e.g., of a New/Final Ice Age, and of a New/Final Dark Age, due to ***dialectical***, ***internal***, ***immanent***, ***ontological*** [***self***-]***contradictions*** of the descendant phase of our present «***species***» of global [proto-]human civilization**?** **¿**I.e., do we need this new math. to grow ***the human-societal self-force of self-expanding human-societal self-re-production*** to '''the next level'''**?**

Consider the greatest scandal of modern science, which, despite its vast magnitude, is mainly mentioned as such only in whispers: the "***un***solvability" of "most" ***nonlinear*** integrodifferential equations. This problem has fettered the advance of modern science since the very inception of modern science, since the very discovery of such equations, over **300** years ago, and it encompasses the "***un***solvability" of the most important of such equations, those that constitute Terran humanity's most advanced formulations of its "laws" of nature to date. These include the "many-body" Newtonian '''gravitic''' equations, the Einsteinian General-Relativistic '''gravitic''' equations, the Navier-Stokes equations of '"***rhe***id"' dynamics [the dynamics of "***rhe***ological", or flowing, matter, in the form of electrically nearly-neutralized liquids and gases, thus of weather, etc.], the Maxwell-Boltzmann-Vlasov equation [for electrically ***non***-neutral, plasma '''***rhe***ids'''], etc. This ***central failure of modern science*** is not some merely esoteric, ethereal, rarefied, merely conceptual failure, of concern only to specialists. ***It is a failure also in deeply practical terms***. Were we to discover how to analytically solve the Navier-Stokes equations, we would probably, given the disproportionalities of cause/effect inherent in ***nonlinearity***, be enabled to stop hurricanes in their tracks, by applying a presently-'harnessable' amount of energy to their "Achilles-heals"; and to nip tornados in the bud. Were we to learn how to analytically solve the Vlasov equation, we could design a global grid of zero pollution ***fusion power reactors***, harnessing radioactivity-free advanced fuel regimes, emitting only electrons. The irruption of human capability to analytically solve "most" ***nonlinear*** integrodifferential equations would represent an enormous leap forward in "***universal labor***" [Marx], that could quickly translate into an enormous upsurge in the level of development of *the human* ***social*** [***self***-]***forces of*** [human society's self-expanding self-re-]***production***, ***if*** the core -- the ***evil***, craven, *technodepreciation-terrified*, and *technologically-educated*-middle-working-class-terrified ruling faction -- of the global ruling plutocracy could be prevailed upon, ***by popular insistence***, to desist from their strategy of global ***enfetterment*** and ***reversal*** of those productive, creative forces [i.e., from their strategy of "***negative*** *growth*"]. *Sans* that irruption, and that persuasion, Terran humanity is headed downwards, into a new, global, and, this time, likely ***final*** **Dark Age**. We have mentioned that the «***aporia***» of the "***un***solvable equations" that motivate our [meta-]models' ***dialectic*** *movement* from arithmetic to higher arithmetic will seem "muted" -- because we already know ***that*** they were eventually solved, and because we already know ***how*** they were eventually solved, and because we already know the ***new kinds of numbers*** which made those solutions possible. Many scientists and mathematicians are wont to say, today -- and without proof***!*** -- that most ***nonlinear*** differential equations must remain '''***forever*** unsolvable'''. The «***aporia***» of the "***un***solvability" of most such ***nonlinear*** equations is ***drastically alive today***, but the perception of that 'drasticity' is muted ***by despair***, by ***an unfounded ideology of eternal unsolvability***. Still, new "closed-form", "exact", "analytical" solutions to vast classes of '''minimally ***nonlinear***''' partial-differential "evolution equations", *including to the* ***nonlinear*** [*cubic*] *Schrödinger equation*, keep being discovered -- e.g., those called "***nonlinear*** wave", or "solitary wave" solutions; the "soliton" solutions, for waves that act like "particles". Many such "exact solutions" don't even require *new* "transcendental functions". *Old* "transcendental functions", e.g., the hyperbolic-tangent **&** hyperbolic-secant functions, provide their "exact", "analytical", "closed-from" solutions.

Are ***nonlinear*** differential equations possibly a kind of Gödelian "diophantine equation", such as could be the subject of a "Gödel Formula" for the *de facto* axioms-set of today's most advanced system of mathematics? Yuri Matiyasevich, on the way to his solution of Hilbert's Tenth Problem, his proof that arbitrary diophantine equations are "semidecidable" as to their solvability [in terms of 'diophantine numbers'] -- including the proof of the 'diophantinicity' of the prime numbers, and the derivation of diophantine algebraic [***finite***] polynomial equations with only integer coefficients, whose positive solution sets, the sets of all positive values that they yield for integer values of their variables, is exactly the set of all prime numbers -- found that, indeed, "the problem of the existence of the solutions of a Diophantine equation in natural numbers can be reduced [*sic*] to the problem of the existence of a solution of a system of polynomial ***differential equations*** of first order." [Yuri Matiyasevich, ***Hilbert's Tenth Problem***, MIT Press [Cambridge: **1994**], pages **xix-xx**, **46**, **54-56**, **85**, **176**, ***emphasis added***].

But let us consider an "***un***solvable" ***diophantine equation*** that is, apparently, a little less "esoteric", and "much simpler", than a ***nonlinear*** differential equation -- that is a little "closer to home": $x = z/0$, for any z in $\mathbb{Z}$ [including for $z = 0 \in \mathbb{Z}$: $x = 0/0$]. That kind of ***diophantine equation*** is ***un***solvable among the $\mathbb{N}$, the $\mathbb{W}$, the $\mathbb{Z}$, the $\mathbb{Q}$, the $\mathbb{R}$, the $\mathbb{C}$, the $\mathbb{H}$ -- is still, to this very day, ***un***solvable in any of ***the*** "***Standard Arithmetics***".

This «***aporia***», the ***quandary*** of ***division by zero***, is still very much alive in our own times, and, again, its intensity is muted ***only by despair***, only by the ***hopelessness*** that most of us are taught to feel about ever "fixing" this locus where our ***Standard Arithmetic breaks down***, where it ***fails to work***; by the official, consensus denial that there can exist any axioms-system of arithmetic in which ***zero division*** could "***make sense***"; in which this kind of diophantine equation could be ***usefully solved***.

Actually, our ***nonlinear*** differential equation example is not so remote from our "more homey" ***zero division*** example after all: ***zero division*** is the proximate cause of ***the problem of singularity***, of the "meaningless" or "***un***defined" values that plague, especially *and* ***essentially***, the ***nonlinear*** differential equations, which are so inherently prone to ***singularities***, helping to thwart their solvability/"integrability" under their present-day "***standard***" arithmetical undergirdings.

If you want to feel the "***bite***", the ***intensity***, the ***fierceness***, ***the searing mental mood***, of a *real* ***dialectical*** «***aporia***» -- still alive and still burning -- ***un***muted, then simply let yourself ***let go of your despair*** about ever finding an arithmetic in whose context, e.g., $r/0 = x$, $r \in \mathbb{R}$ is rendered intelligible. Simply imagine believing that a solution is "out there", ready to be found, to that "diophantine equation". Simply imagine that finding that arithmetic will solve ***the singularity problem***, and, even more, help to unlock the door to the secrets that will habilitate to a full breach of the '''***Nonlinearity Barrier***''' -- including its aspect of the 'patterned-ness' of the never-repeating -- a breakthrough to beyond the world-historical «***aporia***» that presently blockades Terran humanity from access to the higher theories, and to the higher technologies, that reside beyond that ***Barrier***, and upon whose acquisition the matriculation of this humanity, from its looming '***Meta-Darwinian Planetary Selection Test***', and from its "***prehistory***", in Marx's sense, so vitally depends.

The "solution" that $r/0 = \infty = x$, $r \in \mathbb{R}$ is known to incur devastating problems in physical models -- indeed, it leads to '***infinity residuals***', to '***infinite error***', to '***infinitely wrong answers***'. There is almost equal despair, in the consensus view of today's physics and applied mathematics communities, that the ***singularities*** of the extant ***nonlinear*** partial differential equation models of the "***laws*** ***of nature*** can ever be "solved". ***¿***Can ***the Gödelian Dialectic***, pursued to a sufficiently advanced **s**tage, overcome this '''incompleteness''' too***?***

B.ζ. The Inherent, Ineluctable '*Self*-Problematicity' of 'Ideo-Formations' [as of 'Physio-Formations'].

We assume herein, throughout, that, as with the 'physio-formations' of extra-human/pre-human ***Nature***, so with the 'ideo-formations' of human[oid] ***Nature***: they all inescapably carry inside themselves the seeds of their own '***self***-«***aufheben***»' ***self-supersession***.

We assume further that the primary impetus for *their* [[self-]be]*coming* [as] ***self-supersession*** arises in response to conditions that are inescapably present -- if each such formation is to manifestly ***be*** what it ***is*** -- "***in***side" them; conditions that they 'essence-ially' and necessarily "contain". These conditions induce or constitute a process of progress, of 'perfect-tion' within them, which can never eventuate in any final state of absolute "perfection", but which does eventuate, beyond a critical threshold in this ***in***ternal process of '''***self***-development''', in taking them, in part, definitively, ***ontologically*** outside of and beyond themselves.

We assume that such 'ideo-systems' are, in particular, '«***aporia***»-prone'. They are, in this sense, "***un***stable" -- surpassable, and, indeed, tend to be, over sufficient time, surpassed in fact, owing to the agency ***of their own internal causes***. That is, they fail, in the course of their development, even ***in***ternally, by their own native criteria, based upon their own ["internally self-ravaged"] ground, their own '''***in***side agitators''', and hence, must 'ideo-ontologically' '''***ex***tend''' that ground in order to overcome their own, ***im***manent failing(s), or ***self***-failing(s). But that very '***ex***tent-tion', as redress of ***self***-failing(s), in turn engenders a ***new***, ***unprecedented kind*** of ***in***ternal inadequacy, or ***self***-failing, requiring a further, ***new kind*** of '***self***-extent-tion' of ground for its rectification. And so on. Human-phenomic 'ideo-systems' are inherently '***self***-problematical'. That fact expresses the very *essence* of their '***dialecticality***'.

The Gödel incompleteness theorems represent a formal and predictable way in which formal-logical systems encompassing at least '"**N**atural Numbers"' arithmetic will be '***self***-problematic', and demanding of ***im***manent critique, i.e., of ***self***-critique, in the sense of being '''formal-logically, immanently ***incomplete***''' [or, far worse, in the sense of being formal-logically ***self-inconsistent!***]. But there are other ways as well in which the '***self-problematicity***' of Terran human 'ideo-formations', as components of the Terran 'Human Phenome', will manifest. Our assumption and principle of the ***dialectical*** '***self-problematicity***' of human 'ideo-formations' is applied by us -- ***most decidedly and without the slightest exception or hesitation*** -- to all [psycho]historical human[oid]-phenomic 'ideo-formations', ***including*** to those 'ideo-formations' developed by we of **F.E.D.** ourselves.

Every "intension" harbors '***in***-tension' -- '***in****ternal tension*' -- '***intra***-duality'; ineluctable '***self***-duality'.

B.η. The Pedagogical Strategy Guiding System Order Choices for Our Presentational 'Meta-Model'.

F.E.D. ***could have*** chosen to present the **6** kinds of *"unsolvable diophantine equations"* listed below in another possible order than the order in which they are listed below, which is also that in which our *'meta-model'* actually presents them --

1. $[n + x_1 = n]$, for **n** in **N**, posing the ***paradox***, for $^{3}_{h}\mathbf{N}_{\#}$'s notion of number 'Natural-ness', of 'non-increasive addition'. Solution[-set]: $x_1 = 0$, or $x_1 \in a \subset W$. This equation jumps us from $^{3}_{h}\mathbf{N}_{\#}$ to $^{3}_{h}\mathbf{W}_{\#}$;

2. $[w + x_2 = 0; x_2, w \neq 0]$, for $w > 0$ in **W**, posing the ***paradox***, for $^{3}_{h}\mathbf{W}$'s notion of number 'Whole-ness', of 'decreasive addition'. Solution-set: $x_2 \in m \subset Z$. This equation jumps us from $^{3}_{h}\mathbf{W}_{\#}$ to $^{3}_{h}\mathbf{Z}_{\#}$;

3. $[\,|x_3 \times z| < |z|; x_3, z \neq \pm 0\,]$, for **z** in **Z**, posing the ***paradox***, for $^{3}_{h}\mathbf{Z}_{\#}$'s notion of number '''integ[e]r-ity''', of 'decreasive multiplication'. Solution-set: $x_3 \in (\pm 0, +1)^* \subset f \subset Q$. This [in]equation in $^{3}_{h}\mathbf{Z}_{\#}$ jumps us into $^{3}_{h}\mathbf{Q}_{\#}$;

4. $[x_4^{\,2} - p = \pm 0]$, $\pm 0 < p \in Q$, **p** a **Q** prime number, posing the ***paradox***, for $^{3}_{h}\mathbf{Q}_{\#}$'s notion of number '*ratio*-nality', of "incommensurability". Solution-set: $x_4 \in \{ \pm\sqrt{p} \} \subset d \subset R$. This equation jumps us from $^{3}_{h}\mathbf{Q}_{\#}$ to $^{3}_{h}\mathbf{R}_{\#}$;

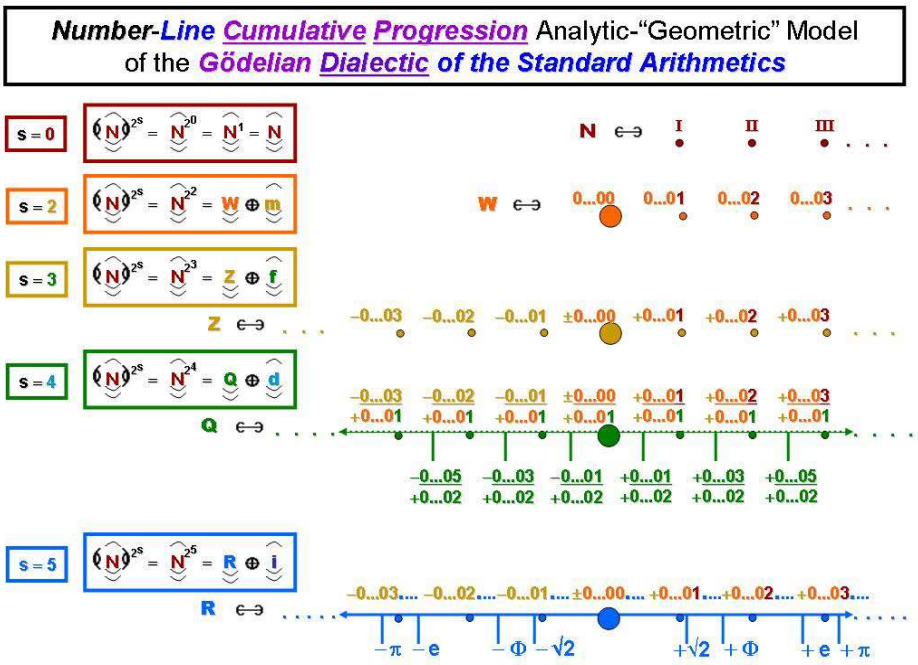
5. $[x_5^{\,2} + 1.0\ldots = \pm 0.0\ldots]$, or $[-x_5 = (+1.0\ldots)/x_5 = x_5^{\,-1}]$, posing the ***paradox***, for $^{3}_{h}\mathbf{R}_{\#}$'s notion of number 'Real-ity', of '*the* i *unit's* ***additive inverse/multiplicative inverse equality or identity***'. Solution-set: $x_5 \in \pm\sqrt{-1} \equiv \pm i \in i \subset C$. This equation jumps us from $^{3}_{h}\mathbf{R}_{\#}$ to $^{3}_{h}\mathbf{C}_{\#}$;

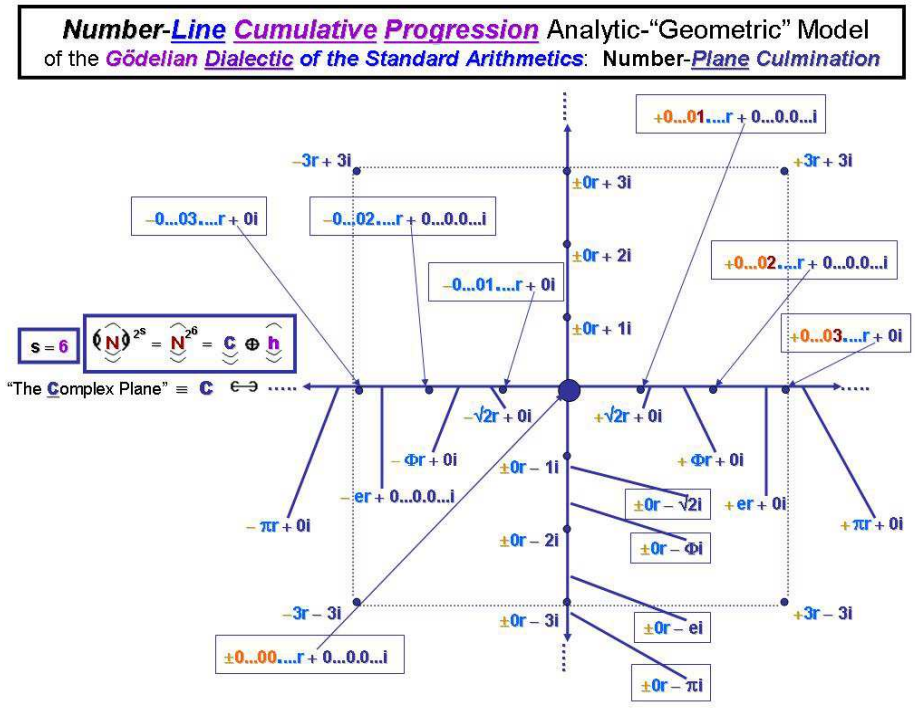
6. $[+x_6 y_6 = -y_6 x_6; x_6, y_6 \neq 0]$, posing the ***paradox***, for **C**'s notion of number '*Complex*-ity', of "***multiplicative anti-commutativity***", or of ***sign-reversal as a result of multiplicative factor-reversal***.
Solution-set: $\langle x_6, y_6 \rangle \in \{\langle i, i \rangle, \langle i, k \rangle, \langle j, k \rangle, \ldots\}$ [a set of "ordered pairs", $\langle a, b \rangle$, from the '''Cartesian product''' 'union space', $IJ \cup IK \cup JK$]. This equation jumps us from $^{3}_{h}\mathbf{C}_{\#}$ to $^{3}_{h}\mathbf{H}_{\#}$.

Equation **2** is ***in***expressible in $^{3}_{h}\mathbf{N}_{\#}$, because it involves $0 \notin N$. It would take us next to $^{3}_{h}\mathbf{Z}_{\#}$. If we started with equation **3**, which is not well-formed in $^{3}_{h}\mathbf{N}_{\#}$, because it uses the useless- or meaningless-in-$^{3}_{h}\mathbf{N}_{\#}$ "absolute value" operator, '|...|', it would take us next to $^{3}_{h}\mathbf{Q}_{\#}$. Equation **4** is ***in***expressible in $^{3}_{h}\mathbf{N}_{\#}$, because it involves $0 \notin N$. It would take us next to $^{3}_{h}\mathbf{R}_{\#}$. Equation **5** is also ***in***expressible in $^{3}_{h}\mathbf{N}_{\#}$, because it involves $0 \notin N$. It would takes us next us to $^{3}_{h}\mathbf{C}_{\#}$. If we started with equation **6**, which is ***not*** well-formed in $^{3}_{h}\mathbf{N}_{\#}$, because it uses the useless- or meaningless-in-$^{3}_{h}\mathbf{N}_{\#}$ "signs", '+' and '–', it would take us next to $^{3}_{h}\mathbf{H}_{\#}$! * [Note: '$(\pm 0, +1)$' means all **fractions between ±0 & +1**, i.e., excluding **±0 & +1**].

F.E.D. came to the conclusion that $^{3}_{h}\mathbf{N}_{\#}$, $^{3}_{h}\mathbf{W}_{\#}$, $^{3}_{h}\mathbf{Z}_{\#}$, $^{3}_{h}\mathbf{Q}_{\#}$, $^{3}_{h}\mathbf{R}_{\#}$, $^{3}_{h}\mathbf{C}_{\#}$, & $^{3}_{h}\mathbf{H}_{\#}$... were the right «*species*» for the «*genos*» of [counting] number, the best progressive partitioning of the generic "**Standard**" number concept, the best division [«***diairesis***»] of that «*genos*» of number into «*species*» of number, the best '''speciation''' of number-kinds for ready assimilation by those to whom we planned to present ***the*** "***Standard Arithmetics***", given the contemporary view of arithmetics, and of mathematics in general, and given the total psychohistorical / phenomic / ideological cognitive ***context*** of 'recent-modern' humanity. F.E.D. came to the further conclusion that the sequence given above was the right sequence of presentation of these number-«*species*», representing the right simplicity-to-complexity, abstractness-to-thought-concreteness gradient, with the right "consecutive **s**tep-sizes", in terms of the '''sizes''' of the qualitative increments in 'ideo-ontology', for optimal ease of assimilation. The inspiration for the ***order of presentation*** that F.E.D. has selected for this ***Standard Arithmetics***' systems-progression is partly pictorial. It is the perceived coherence of the "***order of filling-in***" of '***Standard-Numbers-Space***', as expressed by the 'spaces-progression' of the diagrams/depictions of the 'number-spaces', or of the '''analytical geometries''', of the "***Standard***" systems of arithmetic, as shown via the two graphics on the following page.

Note: We are herein rendering explicit throughout, in both our ideographic renderings of the individual numbers/-numerals of the various numbers-systems, and in our depictions thereof, below, certain key features [e.g., the "leading zeros" place-holders in **0...01** within **W**, for example], i.e., various syntactic attributes -- signifying semantic attributes of the concepts of those numbers -- which are usually, in common uses of these numbers, left implicit, or ignored, but which are of crucial conceptual significance in tracking the cumulative changes in 'number ideo-ontology' from each **S**tage and predecessor system of arithmetic to its successor **S**tage and system, throughout the entire progression of systems of ***the*** "***Standard Arithmetics***". We also, in the depictions below, use '⋹' as 'assignment sign', or 'interpretation sign', from one '''synchronic''', systematic representation to another.





We note, in passing, another dimension of the gradient -- the gradual gradations, escalating from the «***arché***»'s maximal relative simplicity, to the maximal relative complexity of the final 'counter-supplement' '***culminant***' that we consider, that runs through the entire inclusive 'qualitative interval', $\left(\,{}^{3}_{h}\underline{N}_{\#}\,,\,{}^{3}_{h}\underline{h}_{\#}\,\right)$, in the progress from ${}^{3}_{h}\underline{N}_{\#}$ to ${}^{3}_{h}\underline{h}_{\#}$. This further so-'gradated' dimension is that of the increasing relative thought-complexity of the ***arithmetical operations*** 'aporialized', or 'paradoxicalized', in the succession of "***un***solvable" diophantine algebraic equations that catapult our core section discourse from arithmetical system to next higher arithmetical system. Equations **1** & **2** involve ***paradoxes*** of the arithmetical ***addition*** operation. Equation **3** involves ***paradoxes*** of the arithmetical ***multiplication*** operation, definable, in the ***E.D.*** ***dialectic*** of such operations, as the 'meta-addition' operation. Equations **4** and **5** involve ***paradoxes*** of arithmetical ***exponentiation***, definable, in that same ***dialectic***, as the 'meta-multiplication' operation. Together with equation **6**, equations **4** and **5** involve ***quadratic nonlinearity*** -- **2**nd degree ***algebraic equation nonlinearity*** [albeit ***not integrodifferential equation nonlinearity***]. For more, see: http://feddialectics-miguel.blogspot.com/2013/08/part-iii-c-interlude-dialectic-of-basic.html.

B.θ. The 'Evolute-ness' and 'Cumulativity' of *Dialectical*, Ontological-Categorial Progressions.

The dialectical 'meta-models' rendered by 'dialectical meta-equations' formulated using the ${}_{N}\underline{O}_{\#}$ dialectical arithmetic, all take the form of '***categorial progressions***' -- of a stepwise-advancing, all-predecessors-containing, sequence of '"series"' [of inhomogeneous '"sums"'; of 'qualitative superpositions'], via 'category-symbolizing ideograms', or '***categorograms***' -- for whatever «***species***» of the «***genos***» of ***Dialectics***-in-general, in whatever «***species***» of '***the dialectic of the dialectic itself***' -- be it ***Systematic Dialectics***, ***Historical Dialectics***, '***Meta-Systematic Dialectics***', or '***Psychohistorical Dialectics***' -- that 'meta-model' resides.

¿But what is a '***categorial progression***'**?** It is an advancing, non-amalgamative '"sum"' of '''ontological categories''' symbols; i.e., of "*kind* of being" categories' symbols, wherein the symbol-represented "*kinds* of being" include, not only tangible, physical beings, but 'human-mental beings' -- 'ideo-beings'; 'idea-objects'; 'idea-material' -- as well.

¿Are there any common, everyday examples we can point to which involve, or instantiate, "progressions of ontological categories"**?** ¡Yes**!** Some of the best 'exhibitors' of such "advancing category sums" are books, especially books whose ***contents*** involve 'ideo-systematics', and 'ideo-taxonomics', such as Hegel's «***Logik***» and Marx's «***Kapital***». Consider the ***tables of contents*** of such books. Volume, Part, Section, Chapter, and sub-chapter ***titles*** in such books -- such as Quality [Determinateness], Being, Nothing, Becoming, Determinate Being, Being-for-self, Quantity, and Measure, or such as Commodities, Money, and "[The General Formula for]Capital", are also the ***names*** of the central, principal 'ideo[-physio]-ontological' ***categories*** that '''contain''', capture, classify, and comprehend the 'ideo-meta-anatomy', and the 'ideo-meta-physiology' of the universes of discourse that these books attempt to systematize and to comprehensively theorize. Such ***tables of contents*** thus spell out, as a whole, frozen at completion, the ***categorial progression*** that a reader experiences progressively as that reader reads through the contents of such books from start to finish. Such tables do so by listing those category-***names***, which also serve as division-***titles***, in the exact order of their presentation to such a reader.

A reader is not expected to entirely forget the contents of every chapter that the reader has read in such a book as soon as that reader reads it, as if the reader had ingested some [Transient Global Amnesia] TGA-inducing statin drug. Yes, the previous chapters read are expected to fade in the reader's mind, relative to the vividness of the chapter presently being read, and to merge into a background which frames the foreground of the chapter presently being read. But the full comprehension of present reading depends upon and presupposes the remembrance of material already read, even given the relative attenuation of that remembrance. The message of such a book is a ***cumulative*** one. The full communication of its content to the reader depends upon a partially unconscious '***semantic summation***', or '***qualitative superposition***', of the entire material already read, in the reader's mind, until the end of the reading, when the full '***cumulum***' of its meaning should have been conveyed to that reader.

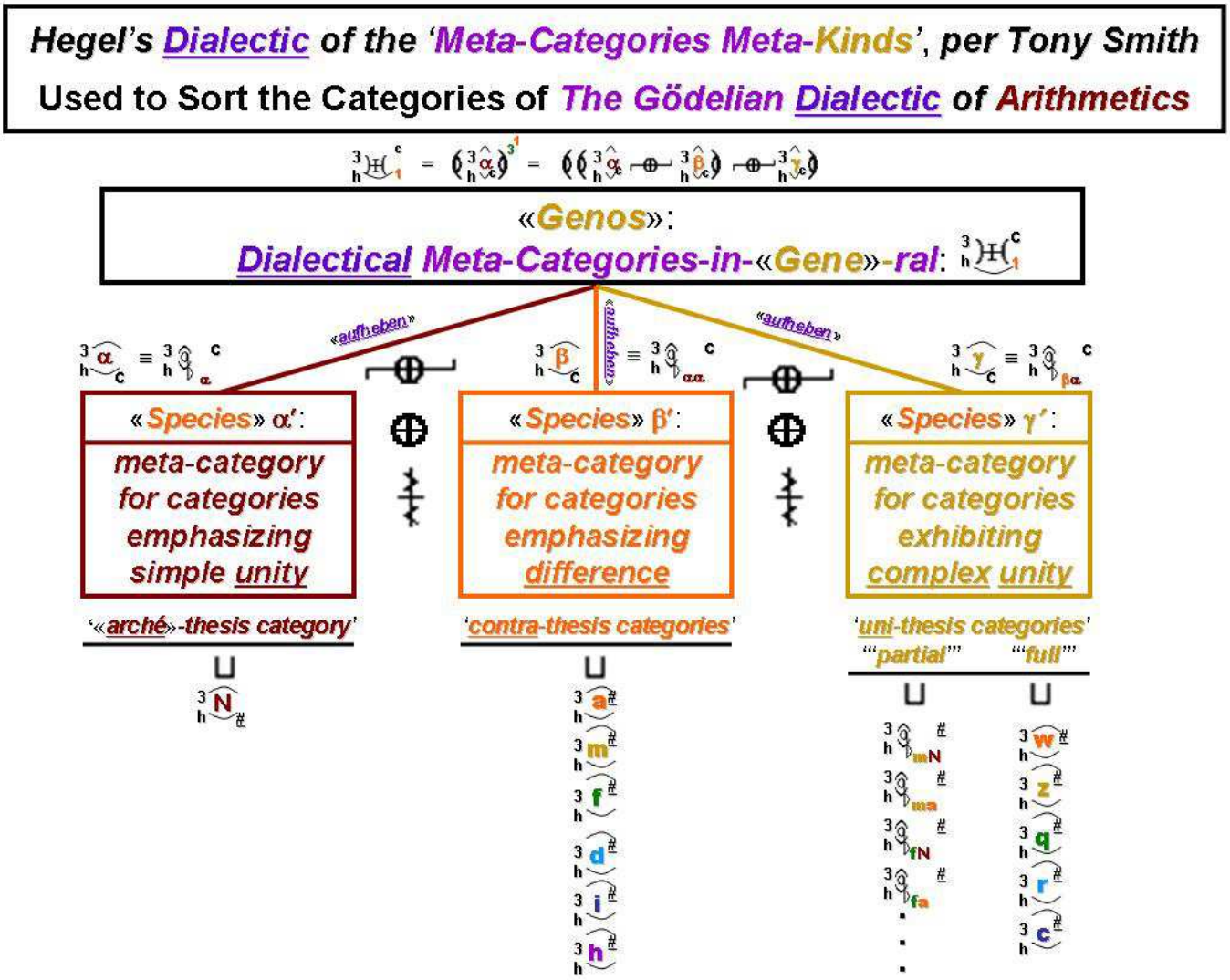
Each new chapter read is supposed to '''add''' itself to the cumulative effect of all of the previously-read chapters, in the mind of the reader; to 'cumulate' in the reader's mind, and to superimpose itself -- its new content, its incremental meaning -- on the retained content of each of the chapters read before it. Or, as with a lecture, the import of the whole previous discourse heard is supposed to 'cumulate' in the mind of the hearer, and each new enunciation is supposed to operate upon, in the hearer's memory, and to re-clarify, each element of that previous heard discourse.

Please consider, in the light of the examples given above, the following description of ***dialectical categorial progressions*** from the work of Tony Smith. Prof. Smith wrote as follows regarding "Systematic Dialectics", the categorial progression method of systematic presentation of theories advanced by Hegel: **"**Hegel attempted to provide an immanent ordering of the basic categories ... To see this we have first to consider what a category is. It is a principle (a universal) for unifying a manifold of some sort or other (different individuals, or particulars). A category thus articulates a structure with two poles, a pole of unity and a pole of differences. In Hegelian language this sort of structure, captured in some category, can be described as a unity of identity in difference, or as a reconciliation of universal and individuals. From this general notion of a category we can go on to derive three general types of categorial structures. In one the moment of unity is stressed, with the moment of differences implicit. In another the moment of difference is emphasized, with the moment of unity now being only implicit. In a third both unity and differences are made explicit together.**"** [Tony Smith, ***The Logic of Marx's Capital***: ***Replies to Hegelian Criticisms***, SUNY Press [NY: **1990**], pp. **5-8**].

Thusly, Dr. Smith notes what we term a 'trans-Platonian' «***arithmos eidetikos***», a ***dialectical*** 'ideo-systematic cumulum'. This «***arithmos***» is an 'ideo-taxonomy', in which the «***genos***» category-«***monad***» of '"categories-in-«***gene***»-ral"' is also the «***aufheben***» '***meta-***«***monad***»-***ization***' of its **3** «***species***» category-[sub-]«***monads***» -- «***species***» **α**., unity-biased categories; «***species***» **β**., difference(s)-biased categories, and; «***species***» **γ**., unity/difference(s) combined-emphasis categories. This «***arithmos eidetikos***» can be modeled by a ***Triadic Seldon Function*** --

$$ {}^{3}_{h}\mathcal{H}^{c}_{1} \;=\; \left({}^{3}_{h}\alpha_{c}\right)^{3^{1}} \;=\; \left(\left({}^{3}_{h}\alpha_{c} \oplus {}^{3}_{h}\beta_{c}\right) \oplus {}^{3}_{h}\gamma_{c}\right) $$

-- which forms a 'tri-oppositional synthesis-sum' for this dialectical theory of the tripartite 'ideo-ontology' of **c**ategories.

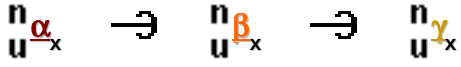


Prof. Smith notes next how these three category-kinds can be sequenced to constitute a dialectical logic of 'tri-categorial oppositions': **"**Hegel's next claim is that there is a systematic order immanently connecting these three categorial structures. A structure of unity in which differences are merely implicit is simpler than one in which differences are explicitly introduced; and one in which both unity and differences are explicit is yet more complex still. Similarly, the first sort of structure is the most abstract, while the other structures are successively more [**F**.*E*.*D*.: thought-]concrete. Yet another way of speaking about the immanent connections here is through the idea of a dialectical contradiction. Hegel's views on contradiction have been quite controversial. But at least in the context of constructing a systematic theory of categories he appears to have meant something fairly straightforward. If a category is in general a principle that unifies a manifold, then if a specific category only explicates the moment of unity, leaving the moment of difference implicit, then there is a "contradiction" between what it inherently is *qua* category (a unifier of a manifold) and what it is explicitly (the moment of unity alone). Overcoming this contradiction requires that the initial category be "negated" in the sense that a second category must be formulated that makes the moment of difference explicit. But when this is done the moment of difference will be emphasized at the cost of having the moment of unity made merely implicit. Once again there is a contradiction between what a category inherently is and what it is explicitly. Overcoming this contradiction demands that the second sort of category also be negated, and replaced with a category in which both poles, unity and difference, are each made explicit simultaneously. Hegel is well aware that "contradiction" and "negation" are not being used here in the sense given to them in formal logic. Following a tradition that goes back to Plato, he asserts that in the above usage "contradiction" and "negation" are logical operators for ordering categories systematically, as opposed to logical operators for making formal inferences. The logic with which we are concerned here is *dialectical* logic.**"** [Tony Smith, ***The Logic of Marx's Capital***, *ibid*., p. **6**].

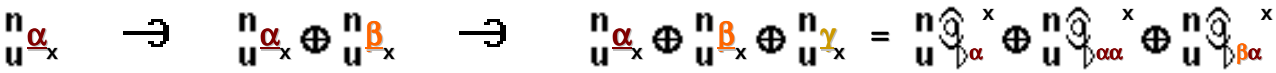
Thusly, Prof. Smith notes the 'qualitative gradient' of increasing complexity and of increasing 'thought-concreteness', or 'thought-determinateness' ['determinations-richness'] that runs through these three kinds of categories when arrayed in their native order. He also so narrates the dialectical logic of '''categorial dialectical contradiction''' -- one of categorial insufficiency and incompleteness -- and one of the categorial 'intra-duality' of unity versus differences, that moves our minds from **α**., to **β**., to **γ**., and beyond. Prof. Smith notes next how sequences of these three category-kinds can be combined in layers, ['qualo-fractal'] scales, or levels, to express a categorial-progression theorization, or comprehension, of a given [sub-]totality of human experience: **"**Since a category of unity-in-difference on one level can itself prove to be a category of simple unity from a higher-level perspective, thereby initiating another dialectical progression from unity through difference to unity-in-difference, we can construct a systematic theory of categories by employing the dialectical method. In this sort of theory we move in a step-by-step fashion from simple and abstract categories to those that are complex and [**F**.*E*.*D*.: thought-] concrete, with dialectical logic providing the warrant for each transition ... At the conclusion of the linear progression of categories we once again arrive at the initial starting point. But it has now been apprehended in thought. If dialectical logic is rigorously adhered to, the move from one category to the next is not *ad hoc*. The linear progression from a category of immediate unity to one of difference, and from there to a category of unity-in-difference, is not a mere formal schema imposed by Hegel externally. It is instead "the absolute method. . .[**F**.*E*.*D*.: which] does not behave like external reflection but takes the determinate element from its own subject matter, since it is itself that subject matter's immanent principle and soul." In this manner the object realm of [**F**.*E*.*D*.: "chaotic"[cf. Marx, ***Grundrisse***]] experience has been [**F**.*E*.*D*.: 'ideo-systematically' / 'ideo-taxonomically'/ 'well-orderedly'] reconstructed in thought [**F**.*E*.*D*.: i.e., comprehensively and 'comprendingly' ***theorized***]**."** [***The Logic of Marx's Capital***, *ibid*., pp. **7**-**8**].

But why do we bother at all, in Systematic Dialectics, i.e., in the dialectical theory-presentation method, with these known inadequacies of defective category-kinds **α**., to **β**.? Why don't we just jump from the **γ**. of one level or scale of the theory to the **γ**. of the next such level or scale? Or, why don't we just jump once, to the **γ**. of the highest scale of the theory, and be done with it? The reasons are pedagogical. Intelligibility for the learner is lost with such jumps. If you are not already a physicist, just try jumping to the penultimate or 'culminant' chapter of a modern physics textbook, without having read anything in the earlier chapters. If you're like most, you won't comprehend very much of modern physics from that chapter alone. New categories -- new concepts, and their definitions -- need to be introduced gradually, and with their introduction motivated, in part, by the need for them that is revealed by revealing the inadequacies of the old, preceding categories/concepts/definitions. Similar ***in***comprehension would arise if we started kindergartner's learning of arithmetic with **C** or **H**, instead of with **N**.

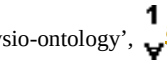
Now, to **F**.*E*.*D*.'s reading of Prof. Smith's account, extracted above, there is an ambiguity about what, from our perspective, is a key issue regarding the categorial progression method of theory formulation/presentation. The following question poses this issue most directly: ***¿***Is a ***categorial progression*** best described by a 'sequence of singletons'***?*** --



-- or, ***¿***Is a ***categorial progression*** best described by a 'sequence of series' / 'sequence of ***non***-amalgamative sums'***?*** --



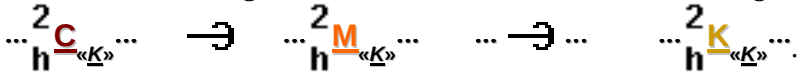
-- with all earlier-emerged categories «***aufheben***»-***conserved***, ‘‘‘externally’’’ **&** additively, *as well as* ‘‘‘internally’’’ [as notated ‘subscriptally’] -- implicitly also conserved “inside” / as-“contained”-in[-by], later-to-emerge categories.

Let’s consider an early phase of ‘‘‘***the dialectic of nature***’’’, for example, the ***self***-conversion of the early sub-atomic “particles” ‘physio-ontology’,  [using ‘∀’ to designate the **A**ll-inclusive level of the ***E.D.*** **Universal Taxonomy**], into that plus a new, ‘plasmic’, ‘‘‘ionized atoms/naked atomic nuclei”’ ‘physio-ontology’,  . We have therein a sub-phase of the ‘‘‘***formal*** *subsumption*’’’ of sub-atomic “particles” by atomic nuclei, called [primordial] “cosmological nucleosynthesis”, of ***proton***-«***monads***» and ***neutron***-«***monads***» «***aufheben***»-‘***self-meta-***«***monad***»-***izing***’ into, e.g., ionic, Helium-plasma atomic-nuclei. Next, we have a sub-phase of ‘‘‘***real*** *subsumption*’’’ of sub-atomic “particles” by atomic nuclei, called [primordial] “stellar nucleosynthesis”. In this sub-phase, those ***hybrid bodies/processes*** that we call ‘“first generation stars”’, made up out of ***both*** atomic nuclei, e.g., Helium ions, and, e.g., ***proton*** sub-atomic “particles” [i.e., ‘plasmic’, ***ionized Hydrogen*** “***atoms***”] convert, e.g., ***proton*** and ***neutron*** sub-atomic “particles’ into, e.g., even more Helium-ion atomic nuclei, and, later, convert Helium atomic nuclei into atomic nuclei of higher atomic «***species***», in these ‘‘‘first generation stars’’’. Note that, even in this ‘‘‘***real*** *subsumption*’’’ phase, sub-atomic matter does not disappear from the cosmos. Masses of matter organized only up to the sub-atomic level, but no higher, e.g., unbound ***protons***, continue to coexist with rising, much smaller, masses of matter organized up to the ionic-atomic level, but no higher. Thus, a ‘sequence of series’ description --

-- is more apt to the qualitative data of the successive phases of this growing ‘physio-ontological’ content of the cosmos in general, and of this phase of ‘universal meta-evolution’ in particular, than is a ‘sequence of singletons’ description --

Likewise, in the kind of example already considered at the outset of this sub-section, of an audience’s reception of a lecture-presentation, and of a reader’s reception of a book-presentation, the content currently being encountered should *eclipse*, but should ***not*** *erase*, the content encountered earlier. The whole advancing content should ‘cumulate’ in the reader’s mind. E.g., Chapter ***I***. of «***Kapital***», entitled “Commodities”, should ***not*** be extinguished from the reader’s mind in and by reading Chapter ***III***., entitled “Money...”, and, likewise, the ‘‘‘impressions’’’ left upon the reader’s mind after reading both the “Commodities” and “Money” Chapters should ***not*** be lost-to-mind, but, on the contrary, should inform, be ingredient in, the reading of Chapter ***IV***., entitled ‘‘‘...«Kapital»...’’’, like this --

-- ***not***, as if reading under the influence of an ingested substance such as some TGA-inducing statin drug, like this --



We of **F.*E*.*D***. term the former an ‘***evolute*** process’, after the name used by, e.g., paleontologists, to characterize the shell of a marine organism, in which successive whorls of that shell rise higher with each turn, thus rendering all earlier whorls visible concurrently to a sidewise view of that seashell. We term the latter a ‘***convolute*** process’, after the corresponding seashell-type, in which succeeding whorls do not rise, and hence cover-up all preceding whorls, hiding them from a sidewise view. The ‘***evolute***-tion’ description is codified in the ‘***double-conservation*** «***aufheben***» ***evolute*** ***product rule***’, axiom **§9** of the  axioms-system [see sub-section immediately after this for a rendering of the core axioms of].

The empirical arguments set forth above for modeling ***dialectic*** ‘***evolute***-ly’ would suffice. However, it is interesting to note that Hegel is repeatedly ***un***equivocal about the ‘***evolute***-ness’ of ***dialectic*** [although, of course, not using the term ‘**evolute**’]. For example, in a series of lectures on the «***Logik***», noted down by his son, and recently published in English translation, Hegel stated: **“**The ***first*** ***determination*** [**F.*E*.*D***.: ≈ ***first*** ‘«***speci***»-***fication***’] is ***im***mediate, while the ***second*** one constitutes the sphere posited in its ***differentiation*** *from* the ***first***.

Within every ***simple*** ***first*** ***determination***, [e.g., ***ground***,] what is ***determinately*** ***different*** from it [, e.g., the ***consequence*** of the ***ground***] is at once also present, but it is at first present without being ***explicitly posited***.

In the ***second*** ***determination***, finitude [and with it ***contradiction***] again ***enters***.

The ***third*** ***determination*** is the ***unity*** of the ***first*** and the ***second***, in which the ***contradiction*** is ***resolved***. ...

The ***progression*** is as follows. The ***beginning*** is ***simple***, ***im***mediate. ... Every newly emerging concept [F.E.D.: category] is ***more concretely determinate*** than its predecessor.

We are always carrying everything that went before along with ourselves into what is new, but everything prior is, within what is new, put in its determinate place. Whereas, in what preceded, each [momentarily ***im***mediate] ***determination*** … passed as ***ultimate***, it is now ***demoted*** into being *only a moment*.**”** [G. W. F. Hegel, ***Lectures on Logic***, Clark Butler, translator, *Introduction to the Lectures on Logic*, *More Exact Concept and Division of the Science of Logic*, [**I**. Being], Indiana U. Press [Indianapolis: **2008**], pp. **79**-**80**, **bold**, *italic*, underline, shadow, ***and color emphasis***, plus [F.E.D.-labeled square-bracketed commentary] added by F.E.D.].

At the very outset of his book «***Wissenshaft der Logik***» -- *“**The Science of Logic**”* -- Hegel also asserts what we call *‘the «**arché**»-onic’ and ‘‘‘**evolute**’’’ principle of **dialectical categorial progression**’*: **“**...the ***progress*** from that which forms the ***beginning*** is to be regarded as only a ***further determination*** of ***it***, hence ***that which forms the starting point of the development remains at the base of all that follows and does not vanish from it***. The ***progress*** does not consist merely in the derivation of an other, or in the effected transition into a genuine other; and in so far as this transition does occur it is equally sublated [F.E.D.: ‘«***aufheben***»-***ated***’] again. ***Thus the beginning ... is the foundation which is present and preserved throughout the entire subsequent development, remaining completely immanent in its further determinations.”*** [G. W. F. Hegel, ***Science of Logic***, Translated by A. V. Miller, Humanity Books [NY: **1969** [originally published in **1812**]], Volume One, ***With What Must The Science Begin*** [placed ***before*** *Chapter* ***1***], p. **71**, **bold**, *italic*, underline, shadow, ***and color emphasis***, plus [square-bracketed commentary] added by F.E.D.].

At the conclusion of the same book, «***Wissenshaft der Logik***», in its ***ultimate*** chapter, Hegel reiterates this ‘«***arché***»-*onic’* and ‘‘‘***evolute***’’’ principle of ***dialectical categorial progression*** once again: **“**... the ***determinateness*** [F.E.D.: ≈ ‘«***speci***»-***ficity***’] which was a ***result*** is itself, by virtue of the form of ***simplicity*** into which it has withdrawn, ***a fresh beginning***; as ***this beginning*** is ***distinguished*** from its ***predecessor*** precisely by that ***determinateness***, ***cognition rolls onward from content to content***. First of all, this ***advance*** is determined as beginning from ***simple determinatenesses***, the ***succeeding ones*** becoming ***ever*** *richer and more* [F.E.D.: *thought-*]***concrete***. For ***the result contains its beginning*** and its course has enriched it by a fresh ***determinateness***. The ***universal*** constitutes the ***foundation***; the ***advance*** is therefore ***not*** to be taken as a *flowing* from one *other* to the next *other*. In ***the absolute method*** the Notion *maintains* itself in its otherness, the ***universal*** in its ***particularization***, in judgment and reality; ***at each stage of its further determination*** [F.E.D.: ≈ ***further*** ‘«***speci***»-***fication***’] ***it raises the entire mass of its preceding content, and by its dialectical advance it not only does not lose anything or leave anything behind, but carries along with it all it has gained, and inwardly enriches and consolidates itself.”*** [G. W. F. Hegel, ***Science of Logic***, *ibid*., Volume Two, Section Three, *Chapter* ***3***, ***The Absolute Idea***, p. **840**, **bold**-***italic***, underline, shadow, ***and color emphasis***, plus [square-bracketed commentary] added by F.E.D.].

In our ‘ideo-perception’ of ***nature***’s ‘[*ideo-*]***physio-dialectic***’, as of its ‘***ideo***-[*physio-*]***dialectic***’, ***dialectical progression*** is ‘‘‘***evolute***’’’. ‘***Evolute****ness*’ is “***cumulative***”. ***Dialectical progression*** ‘***cumulates***’. It presents [*ideo-*][*physio-*]***ontological*** ‘***cumula***’, which ‘‘‘***contain***’’’ ***the past*** as the ‘***pre***-sent’, i.e., as the ‘sent to now’s now from ‘***pre***vious’, prior-nows’.

For more about the ***dialectic*** of the concepts ‘convolute’ and ‘evolute’, see --

http://www.dialectics.org/dialectics/Glossary_files/Glossary,The_Dialectic_of_the_%27Voluteness%27_of_Dialectical_Progressions_in_General,07FEB2014.jpg.

B.1. Rules of Computation of the *Dialectic* Rules-System: F.E.D.'s *First Dialectical Arithmetic*, $^{3}_{h,\mathbb{N}^{\#}_{O}}$.

The 'algorithmics' of **F.E.D.**'s '***First Dialectical Arithmetic***', $\mathbb{N}^{\#}_{O}$, undergird the Dyadic Seldon Function 'meta-model' of '***The Gödelian Dialectic of the Standard Arithmetics***', whose story is told, by phonetic prose, as well as by 'ideo/graph/ical' formulas and diagrams, in the core section of this essay. Some familiarization with those computational 'algorithmics' is therefore provided for you, the reader, in this near-final sub-section of the Background section. The **9** basic rules of the $\mathbb{N}^{\#}_{O}$ rules-system -- rules traditionally termed "postulates" or "axioms" -- are stated formally at the end of this sub-section. The subset of those **9** rules most essential for understanding the Dyadic Seldon Function 'meta-model' of the systematics of the Standard systems of Arithmetic can be informally summarized as follows. 'Dialectical Addition', $\oplus$, of ontological category-units in $\mathbb{N}^{\#}_{O}$ is "***non***-amalgamative" [cf. Musès], or is '***super***-amalgamative', but is never simply "amalgamative", as is **N** addition. 'Dialectical Addition' is, ***un***like the addition operation of Boolean algebra, "***non***-amalgamative" for '''***un***likes''' [for ***un***like category-units] -- $\mathbf{Y} \ddagger \mathbf{X} \Rightarrow \nexists \mathbf{Z} \in \mathbb{N}^{\#}_{O} \mid \mathbf{Z} = \mathbf{Y} \oplus \mathbf{X}$. As in [later] Boolean logic, it is '***super***-amalgamative', or "idempotent", for '''likes''' -- $\mathbf{X} \oplus \mathbf{X} = \mathbf{X}$; $\mathbf{Y} \oplus \mathbf{Y} = \mathbf{Y}$. 'Dialectical Multiplication' is '***contra***-Boolean'. It represents the core-***dialectical*** «***aufheben***» operation. It exhibits a 'double-ness' in the structure of its «***aufheben***» ***conservation*** moment, which consists of two 'sub-moments', one ***ex***ternal and the other ***in***ternal -- $\mathbf{X}^2 = \mathbf{X} \oplus \triangle\mathbf{X} = \mathbf{X} \oplus \mathbf{Y} \mid \mathbf{Y} \equiv \triangle\mathbf{X} = \mathfrak{g}_{XX} \ddagger \mathbf{X}$ & $\mathbb{q}_k^{\,2} = \mathbb{q}_k + \mathbb{q}_{k(+)k}$.

The ***ex***ternal ***conservation*** 'sub-moment' is called 'the Boolean term'. It grounds the '''***evolute***''' character of ***dialectic***, already explicated in the sub-section preceding this one. The ***in***ternal conservation 'sub-moment', with its supplementary term, $\triangle\mathbf{X}$, constitutes the ***dialectical corrective*** to the Boolean "fundamental law of thought", $\mathbf{X}^2 = \mathbf{X}$. It also grounds interpretation of the '"determinate ***negation***"' and '''***elevation***''' moments of the «***aufheben***» "times" operation. It does so via the concept of 'self-meta-***unit***-ization', or 'self-meta-«***monad***»-ization', in which heterogeneous multiplicities of the individuals constituent of the **X** ***onto***[***logical*** category] form the thus 'meta-'***individuals***/'meta-'***units*** of the $\triangle\mathbf{X}$ onto.

Axioms. The *core axioms sub-set* of the $\mathbb{N}^{\#}_{O}$ axioms-system for the generic **1**st ***dialectical arithmetic*** is the following --

(§1) $\mathbb{q}_1 \in \mathbb{N}_O$ [the axiom of «***arché***» inclusion].

(§2) $[\forall n \in \mathbf{N}][[\mathbb{q}_n \in \mathbb{N}_O] \Rightarrow [s\mathbb{q}_n = \mathbb{q}_{n+1} \in \mathbb{N}_O]]$ [the axiom of inclusion of ontological successors].

(§3) $[\forall j, k \in \mathbf{N}][[[\mathbb{q}_j, \mathbb{q}_k \in \mathbb{N}_O] \& [\mathbb{q}_j \ddagger \mathbb{q}_k]] \Rightarrow [s\mathbb{q}_j \ddagger s\mathbb{q}_k]]]$ [the axiom of categorial distinctness].

(§4) $[\forall x \in \mathbf{N}][\neg[\exists \mathbb{q}_x \in \mathbb{N}_O] \mid [s\mathbb{q}_x = \mathbb{q}_1]]$ [the axiom of the '***arché**onicity*' of the «***arché***»].

(§5) $[\forall n \in \mathbf{N}][\mathbb{q}_n \in \mathbb{N}_O]$ [the axiom of «***aufheben***» connexion / subsumption [of the subsumption of the **N** by the $\mathbb{N}_O$]].

(§6) $[\forall j, k \in \mathbf{N}][[j \gtrless k] \Rightarrow [\mathbb{q}_j \ddagger \mathbb{q}_k]]$ [the axiom of the ***qualitative inequality*** of distinct ontological-categorial ***qualifiers***].

(§7) $[\forall n \in \mathbf{N}][\mathbb{q}_n + \mathbb{q}_n = \mathbb{q}_n]$ [the axiom of *idempotent addition* / of ***ontological category*** [of ontological **q**ualifier] ***uniqueness***].

(§8) $[\forall i, j, k \in \mathbf{N}][[j \gtrless k] \Rightarrow [\mathbb{q}_j + \mathbb{q}_k \ddagger \mathbb{q}_i]]$ [the axiom of the ***irreducibility*** of ontological ***qualitative differences***].

(§9) $[\forall j, k \in \mathbf{N}][\mathbb{q}_j \times \mathbb{q}_k = \mathbb{q}_k + \mathbb{q}_{k+j}]$ [***double-conservation*** «***aufheben***» ***evolute product rule*** for ontological multiplication].

-- wherein **s** denotes the "Peano successor operator", $\mathbf{s(n) = n + 1}$, and wherein **S** denotes the $\mathbb{N}^{\#}_{O}$ version of that successor function, $S[\mathbb{q}_n] = \mathbb{q}_{s(n)} = \mathbb{q}_{n+1}$.

B.K. *Encyclopedia Dialectica*'s 'Organonic Algebraic Method' for the Solution of *Dialectical-Algebraic* 'Meta-Equations'. This sub-section is for the purpose of spelling-out the recipe -- the "algorithm" -- that we apply in the core section of this essay. We use that algorithm there to solve successively for the meanings of each term [other than for those of the **12** 'pre-given', stipulated-in-advance terms], that is generated up through its **6**th **s**tage/-**s**tep, by the Dyadic Seldon Function 'meta-model', already introduced above, of '***The Gödelian Dialectic of the Standard Arithmetics***'. That algorithm is called, by **F.*E*.*D*.**, the '**Organonic Algebraic Method**'.

The term 'organonic' in the phrase '**Organonic Algebraic Method**' hails from an old Greek term, «***organon***», meaning "instrument of thought or of knowledge", 'tool of cognition', 'organ of mind', or '"system of rules or principles of demonstration or investigation"'. The purpose of this 'synchronic/presentational' branch of the larger '**Organonic Algebraic Method**' that we will apply herein is to provide heuristic, "rule-of-thumb" support for the users of the $_{N}\underline{Q}_{\#}$ ***dialectical arithmetic*** when they use it to model systems synchronically. More precisely, this method provides checklists, hints, and clues, that can help to advance such a user's cognitive grasp of a Dyadic Seldon Function 'meta-model'. Such a user may initially have clarity on as little as the «***arché***» term of such a 'meta-model', plus on the epithet(s) of that «***arché***» only. The **Method**'s support elements may aid a user to move cognitively from the 'meta-model' for the simplest, most abstract, most generic pattern of ***dialectic***, as given by --

$$\underline{H}_{h_{T}} = [[\,T_{1}\,]]^{2^{h_{T}}}$$

-- to the more complex, more 'thought-concrete', more specific pattern implicit in a 'meta-model' of the form --

$$^{n}_{u}\underline{)H(}^{x}_{s_x T} = \left(^{n}_{u}\underline{\alpha}_{x}\right)^{2^{s_x T}}$$

-- if the user already knows at least the '''values'''/meanings of **n**, **u**, and **x**, and of $^{n}_{u}\underline{\alpha}_{x}$ [the «***arché***» *term*], and of the $s_x = 0$ *value* of that 'meta-model', which focuses on the 'meaning-value' of the «***arché***» in and by itself.

A schematic summary of this '**Organonic Algebraic Method**', in diagrammatic form, is provided at the end of this sub-section. The rest of the text of this sub-section is devoted to a narrative rendition of the algorithm that the diagram depicts. Both employ the "is equal to by definition" *relation* sign, '≡', as a definition *operator* sign, such that '≡(**a**)' means 'the definition of **a**', and such that '⊦. ∃≡(**a**)' asserts [⊦.] that 'a definition of **a** already exists for the user/is extant for the user, is known by the user, or has been discovered by the user'.

Algorithm Narrative.

A. Process **A**lpha: **START.**

α. GIVEN(s): The following two conditions must be given. Other givens may also apply, in particular cases.

α.1. $s_x = 0$: the **s**tep parameter is '''initialized''' to the «***arché***» **s**tep, whose value is the «***arché***» term alone --

$$T_{1} \Leftarrow\!\!\!\exists\; ^{n}_{u}\underline{)H(}^{x}_{0} = \,^{n}_{u}\underline{\alpha}_{x}.$$

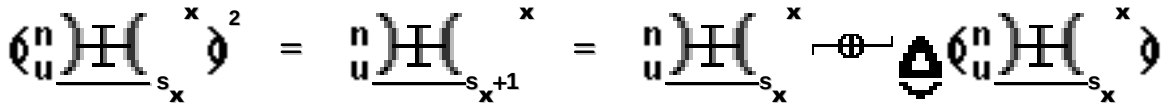
α.2. A definition of the «***arché***» *term*, $^{n}_{u}\underline{\alpha}_{x}$, is known to the user --

$$\vdash .\ \exists\equiv\left(^{n}_{u}\underline{)H(}^{x}_{0}\right).$$

β. COMPUTE:

β.1. $w = 2^{s_x}$: compute current value of loop control parameter, **w**, as indicated function of current value of s_x.

β.2. Square the Seldon Function value for the current value of s_x, to obtain its value for next value of s_x, $s_x + 1$ --



-- which stands for an *'antithesis-sum'*, that is, for an *'oppositional addition'* of the first, *un*-negated, *un*-elevated, '**Boolean**' conservation term of the right-most "side" of the equation above, 'opposed/added' to the second, '***delta***' (Δ) term, of that rightmost "side".

β.3. Reset the current value of s_x, to its next/Peano successor value, $s_x + 1$.

γ. COMPUTE:

γ.1. Reset the current value of **w**, to its next/ Peano successor value, $w + 1$.

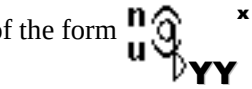
δ. DECIDE: ¿Is **w** greater than 2^{s_x}?

δ.1. If answer to question δ. is **YES**, go to Process ι.

δ.2. If answer to question δ. is **NO**, go to Process ε.

ε. DECIDE: ¿Is the **w**th term in  a '***self***-hybrid' term?

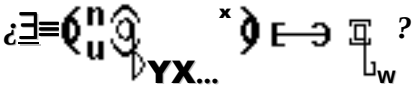
Commentary: A 'self-hybrid' term denotes a 'contra-thesis', 'contra-category', 'contra-system', or counter-example.

It is of the form  in the generic, minimally-interpreted arithmetic, and is of the form  in the interpreted/assigned arithmetic/algebra, where  is the already known, just-prior-step's 'self-hybrid' term.

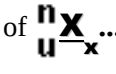
ε.1. If answer to question ε. is **YES**, go to Process η.

ε.2. If answer to question ε. is **NO**, go to Process ζ.

ζ. DECIDE: ¿Does user know of an apt definition for the '''hybrid''' algebraic term mapped to the arithmetical  ?

I.e.: 

I.e: ¿Can the user define this **w**th term, either because its meaning is given, or via the generic clues listed below? --

- This term aptly represents the '***complex unification***', ***dialectical synthesis***, or ***reconciliation*** of  and/with ;
- This term aptly represents the '''***real subsumption***''', ***assimilation***, ***adjustment***, or ***adaptation*** of  by/to ;
- This term aptly represents the ***conversion***, by , of [some of] the units constituting into units of ;
- This term aptly represents the ***appropriation***, or ***subordination***, by , of [[some of] the units constituting] ;
- This term aptly represents ***hybridization***/formation of '***hybrid units***', hybridizing units of with units of .

ζ.1. If answer to question ζ. is **YES**, go to Process θ.

ζ.2. If answer to question ζ. is **NO**, go to Process γ.

η. DECIDE: ¿Does user know of an apt definition for the '***self***-hybrid' algebraic term mapped to the arithmetical ?

I.e.: ¿

I.e: ¿Can the user define this **w**th term, either because its meaning is given, or via the generic clues listed below**?** --

- This term aptly represents the ***immanent critique***, ***self-critique***, ***self-reflexion***, ***self-inspection***, or «***aufheben***» ***self-negation*** of ;
- This term aptly represents '***self-subsumption***', '***self-incorporation***', or '***self-internalization***' of ;
- This term signs the '''***self-re-entry***''' of logical-individuals/units/elements of , generating ;
- This term aptly represents the '***self-conversion***', by , of [some of the] units constituting ;
- This term aptly represents the '***self-appropriation***', by , of [some of the] units constituting] ;
- This term aptly represents formation of '***meta***-units' of units, each made up out of a multiplicity of units.
- The term is a '***supplementary opposite***' of the term ;
- The term '''***outers***'''/externalizes/'explicitizes' an ***inner/internal/implicit duality*** of the term ;

η.1. If answer to question **η**. is **YES**, go to Process **θ**.
η.2. If answer to question **η**. is **NO**, go to Process **γ**.

θ. NAMING: The user '''names''' this **w**th term, by means of a single-letter '*mnemonic epithet*', taken from the phrase or word which most aptly describes the meaning of this term to the best present knowledge of the user.

Then shift control to Process **γ**.

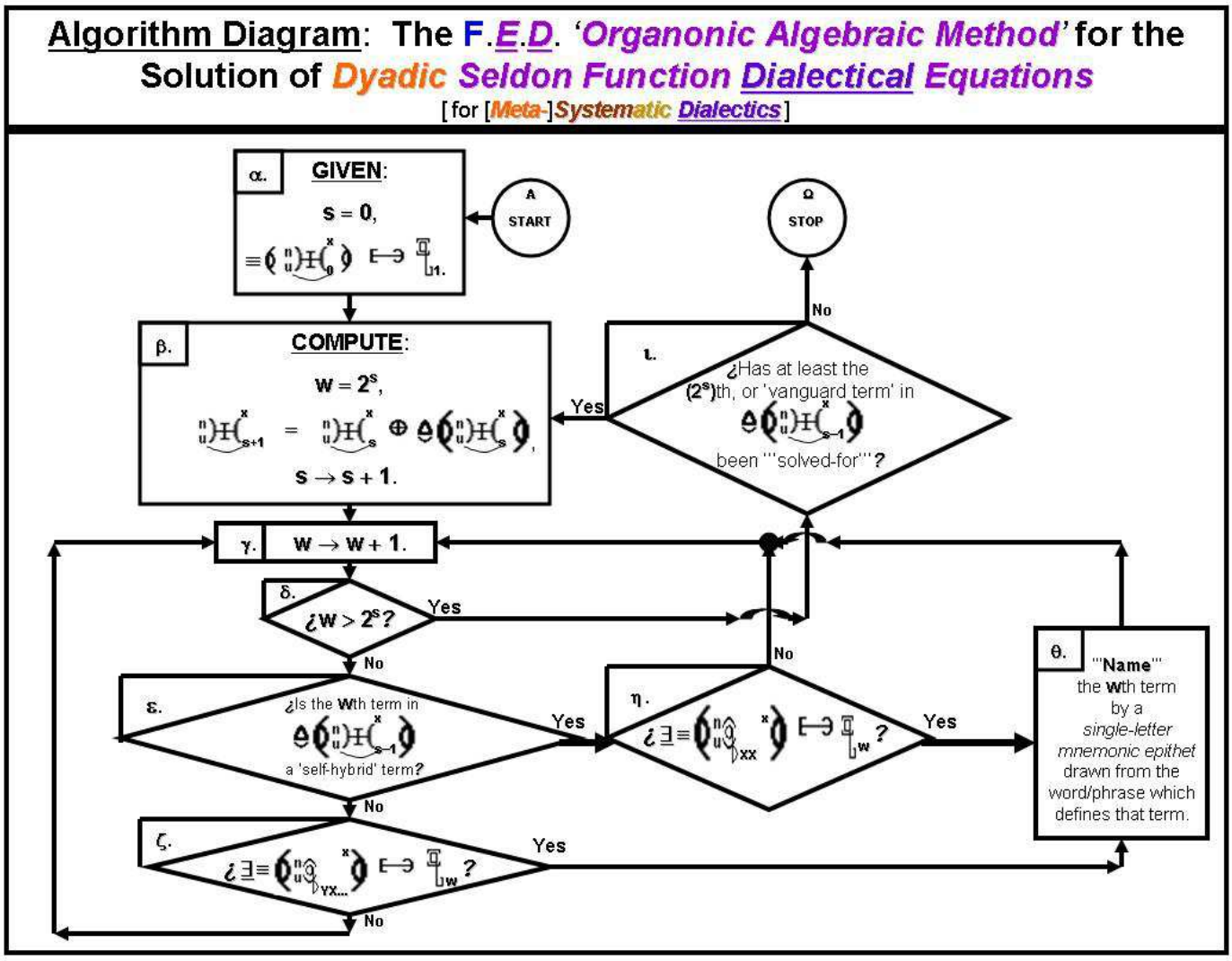
ι. DECIDE: ¿Has ***at least*** the th 'vanguard term', or 'meta-meristemal'/highest term, in

been '''solved for'''/'semantified'/ceased to be [an] "***un***known"/determined as to its meaning to user's satisfaction**?**

ι.1. If answer to question **ι**. is **YES**, go to Process **β**.
ι.2. If answer to question **ι**. is **NO**, go to Process **Ω**.

Ω. Process Omega: **STOP**.

Caveat: We will not always, in the core section of this essay, implement Process **θ**. Often, instead, we will leave a '''hybrid''' term expressed in its full subscript-epithetic regalia, even after having decoded it in Decision **ζ**., because of the mnemonic value, for the reader, of retaining the subscripted epithets.



The ***Encyclopedia Dialectica*** '**Organonic Algebraic Method**' for the solution of 'Dialectical Meta-Equations' is ***not*** entirely the same for solving [*Meta-*]***Systematic-Dialectical*** 'Meta-Equations' as it is for solving -- for the 'semantification' of the terms of -- [*Psycho*]***Historical-Dialectical*** 'Meta-Equations'.

'**Organonic Algebraic Method**', as it is applied to the 'Qualitative [Meta-]Equations' of [*Psycho*]***Historical-Dialectics***, to the solution/'semantification' of each term of the progression-series/"non-amalgamative sum" [cf. Musès] generated for each epoch, **τ**, by a 'Dyadic Seldon Function meta-model' of that [*Psycho*]***Historical-Dialectic***, is a matter of mapping/correlating each term to its *physically evident counterpart*(*s*) in the empirical, external-phenomenal world, ***if any such counterpart is known***.

This same 'Organonic Method', but as applied to [*Meta-*]***Systematic-Dialectical***, '''Method-of-Presentation''' '[Meta-]Equations', for 'Dyadic Seldon Function meta-equation meta-models', typically involves *pedagogical choices*, and stipulated pre-assignments of the meanings of at least the '«***arché***»-thesis' term, and perhaps also of *some* of the 'contra-thesis' terms, and perhaps even of *some* -- e.g., of the '''culminating''' -- full 'uni-thesis' terms, so that solution is a matter of filling-in definitions, identifying, or solving-for, the implied but initially, explicitly 'un[re-]cognized', '''algebraically ***un***known''', meanings of the terms that are intermediate between the pre-stipulated meanings of the pre-assigned terms.

Relative to cases of the '**Organonic Algebraic Method**' as applied to the solution of the 'purely-qualitative Meta-Equations' of [*Psycho*]***Historical*** ***Dialectics***, the question here is an easier one. That question is: ***¿If*** **7** of our **64** terms -- $^3_h\mathbf{N}_\# \vdash\!\ni q_{1}$, $^3_h\mathbf{a}_\# \vdash\!\ni q_{2}$, $^3_h\mathbf{m}_\# \vdash\!\ni q_{4}$, $^3_h\mathbf{f}_\# \vdash\!\ni q_{8}$, $^3_h\mathbf{d}_\# \vdash\!\ni q_{16}$, $^3_h\mathbf{i}_\# \vdash\!\ni q_{32}$, & $^3_h\mathbf{h}_\# \vdash\!\ni q_{64}$ -- mean what we have chosen, in advance, that they should mean, ***then*** what is ***implied*** as to the best meaning to be assigned to/"solved" for each of the terms remaining to be solved-for/assigned, i.e., to the **57** *generic* 'ideo-ontological qualifiers' in the inclusive interval $[\![q_{1}, q_{64}]\!]$, as mapped to the **57** *specific*, 'connotogramic' "***un***knowns" in the ***Standard-Arithmetics***-interpreted interval $(\!(^3_h\mathbf{N}_\#, {}^3_h\mathbf{h}_\#)\!)$? But even for a ***Meta-Systematic-Dialectical*** Dyadic Seldon Function 'Meta-Equation' such as the one we are setting-up to solve herein, a kind of 'Meta-Lagrangian Principle' applies, which also applies -- perhaps more strongly -- for [*Psycho*]***Historical-Dialectical*** Equations. This principle arises from the '***Possibility-Space***' interpretation of the qualitatively different 'ontological categories **q**ualifiers-sums' that the 'Dyadic Seldon Function' generates for each $\geq$ **1** value of its 'self-iteration parameter' -- its 'epoch parameter', τ, or '**s**tep/**s**tage parameter', **s**. It means that ***not all*** of the ***possible***, consecutive subscript '«***gene***»-ric' terms in, e.g., the interval $[\![q_{1}, q_{64}]\!]$, will ***actualize*** -- will necessarily be instantiated -- in any given '«***speci***»-fic dialectical meta-model'. The Lagrange equations of motion of classical mechanics describe a generic dynamical system, a ***generic*** "connected mechanical system", by means of "generalized coordinates" and "generalized velocities". Those equations' spectrum of such "generalized" variables are intended to encompass ***all possible*** "moving material systems". But ***not*** all ***specific*** kinds of such systems will ***actually*** embody all of those ***possible*** variables. Thomas K. Simpson describes, as follows, the process by which James Clerk Maxwell derived the dynamical equations of the electromagnetic field, using the Lagrange equations. Maxwell did so by honing down the full ***possible*** ensemble of terms of the latter to those that were ***actual*** for electromagnetic field dynamics:

"...Maxwell approaches the construction of his own electromagnetic theory with a clear initial vision of the shape it must take. He does not begin with a collection of basic empirical results and seek a merely complete and convenient set of equations which will save the appearances. Maxwell knows at the outset that his theory must take the form of the equations of motion of a moving material system; these, as we have seen, are Lagrange's equations of motion, which in Maxwell's view simply explicate mathematically our *a priori* concept of matter in motion. *A priori*, Maxwell's equations are merely a ***speci***al case of Lagrange's equations. Therefore, Maxwell's program for a "dynamical" approach to electromagnetism must be this: beginning with Lagrange's equations of motion, identify the ***generalized*** coordinates and velocities which characterize an electromagnetic system, and then determine by experiment which of the ***possible*** coefficients are ***actually operative*** in this ***particular*** science, and what relationships exist among the coefficients and the coordinates. Lagrange's equations, thus related to electromagnetism and ***sifted of inoperative terms***, will be the basic equations of electromagnetism. At the same time, they will characterize in broad strokes a ***particular*** form of connected system.**"** [Thomas K. Simpson, ***Maxwell's Mathematical Rhetoric***: ***Rethinking the Treatise on Electricity and Magnetism***, Green Lion Press [Santa Fe: **2010**], pages **272-273**, ***emphases added***].

By analogy with this Maxwellian procedure with respect to the Lagrange equations, our "sifting", in the core section if this essay, of the **64** ***possibly-Standard-Arithmetics-meaningful*** terms generated by --

$$ ^3_h)\!H\!(^{\#}_{6} \;=\; (\!(^3_h\mathbf{N}_\#)\!)^{2^6} $$

-- may find that ***some*** of the '«***gene***»-ric' terms $q_{k} \mid \mathbf{k} \in [\mathbf{1}, \mathbf{2^6} = \mathbf{64}]$*, for '***dialectic***-in-«***gene***»-ral', are "***in***operative" in the '«***speci***»-fic' case of ***the dialectic of the Standard Arithmetics***. ¡Or maybe ***not***!

*[in the expression above, '|' is used to replace the phrase "such that". Also, in general, '**[a, b]**' denotes the **in**clusive interval between "**N**atural" numbers **a** and **b**].

Summary. Each new **Q**-algebraic "***unknown***" arises, in its "debut" iteration of our Seldon Function 'meta-model', as a ***new*** combination of "***old***", already "***known***" epithets/predicate-letters/intensions/connotations. The **Q**-*algebraic* task -- the solution-task of the ***dialectical-algebraic*** *equation* of that iteration/**s**tage -- is to discern the best meaning for each such term [if any; if not the null/"***in***operative" meaning, corresponding to '''existential ***im***possibility''', or to ***non***-instantiation], the one that best fits the epithets of that new term, in the context of one's experience and knowledge of the totality/"object realm" being '''theorized''' by use of that 'meta-model'.

B.λ. From '''Formal Subsumption''' to '''Real Subsumption''', and the 'Culminant'/Epitome of the Latter.

The Dyadic Seldon Function 'meta-model' of *the Gödelian Dialectic of the Standard Arithmetics* is ***very*** '"Gödelian"'. It unfolds as an $\mathbf{S}_{\#}$-escalation-driven sequence of series -- of 'qualitative, "***un***addable" sums' [i.e., of 'ideo-ontological, ***non***-amalgamative **q**ualifier-sums'], or 'cumula' -- each of which, each result of the **S**tep-calculation for a given value of $\mathbf{S}_{\#}$, contains, as its final term, a qualitatively new -- 'ideo-ontologically new' -- 'contra-thesis', 'oppositionally added to', and qualitatively opposing, as a ***counter-example*** to [cf. Imre Lakatos], and as a partial refutation/denial of, the joint meaning of all of the terms that came prior to its coming; of the entire 'cumulum'/'qualitative super[im]position' of number-meanings that arose before it, in its own **S**tep, and[/or] in all previous **S**teps ["/or" for $\mathbf{S}_{\#} < 2$ only].

Every **S**tep contains the solution to the previous «***aporia***», but a solution that is also then 'oppositionally added' to that solution's own, ***new*** 'counter-term', thus forming yet a ***new***, as yet ***un***solved, «***aporia***».

Every *new* **S**tep contains the solution to that «***aporia***» which was definitive of its immediately-prior **S**tep, in the form of the '''union''' of the meanings of all of the terms first arisen in its own, *new* **S**tep, and in that prior **S**tep, and[/or] in all previous **S**teps, ***except for/excluding the final term*** of the *new* **S**tep. I.e., each *new* **S**tep's solution to the thereby *just-past* problem arises in conjunction with an assertion of the *next* problem, which is the Gödelian ***in***completeness of that *new* **S**tep's *new* solution itself. This *next* problem is asserted via the *final* term of the *new* **S**tep, a *final* term which is 'oppositionally added' to the univocal symbol standing for the 'solutionary' sum[mary] of all of that *new final* term's preceding terms. Together, that summary term for the meaning of all previous terms, plus the new final term, form the *new* «***aporia***», the «***aporia***» that is definitive of the *new* **S**tep itself. Every *new* **S**tep contains the completion of the prior **S**tep's defining self-***in***completeness, '''plus''' the assertion of the ***self-in***completeness of that completion of the just-prior ***self-in***completeness. Each *new* **S**tep-calculation for $\mathbf{S}_{\#} > 1$ thus issues forth a *new* 'antithesis-sum' -- the 'oppositional addition' of its *new final* term to all of its preceding terms in that *new* **S**tep, with that sum of all such preceding terms constituting the 'synthesis-sum' that resolves the prior **S**tep's 'antithesis-sum'.

In ***Encyclopedia Dialectica*** parlance, the emergence, in each *new* **S**tep, of a *new* 'contra-thesis' *final* term, already constitutes the '''***formal subsumption***''' of all preceding terms in that *new* **S**tep by that *new* 'vanguard term', that *new* '*meta-**meristemal** term*'*. In each **S**tep's *next* **S**tep, all of the 'ideo-ontological increment' of terms *new* with that *next* **S**tep, ***except the final incremental term***, represent the «***aufheben***»***-negation***/-***elevation***/-***conservation***, ***of*** every '''old''' term, ***by*** the prior **S**tep's newest, *final* term. That is, those next **S**tep *new* terms represent the '''subsumption''', '''conversion''', '''adaptation''', '''adjustment''', '''assimilation''', '''appropriation''', and/or '''subordination''', of each older term to the prior **S**tep's subsuming 'contra-thesis' *final* term, or the '[***partial***] ***synthesis***'/'ideo-ontological ***hybridization***' of each older term with that prior **S**tep's *final* term, a *final* term which is, itself, the '***self-subsumption***' of ***its*** prior **S**tep's 'contra-thesis' *final* term.

We say that, together, the ***non***-amalgamative sum, or 'cumulum', of all of these 'other-subsumptions' -- the subsumption, by a given **S**tep's 'contra-thesis' term, in the given **S**tep's *next* **S**tep, of all of the given **S**tep's '''other terms''' -- constitutes the '''***real subsumption***''' ***of*** all of the preceding 'number ideo-ontology' terms, ***by*** that *new*-with-the-given-**S**tep 'number ideo-ontology' term, which is the given **S**tep's *final* term.

For example, as soon as $\mathbf{m}_{\#}^{3}\mathbf{h}$ irrupts in **S**tep 2 --

$$\mathbf{H}_{2}^{\#}{}^{3}\mathbf{h} = \left(\mathbf{N}_{\#}^{3}\mathbf{h}\right)^{2} = \left(\mathbf{N}_{\#}^{3}\mathbf{h} \oplus \mathbf{a}_{\#}^{3}\mathbf{h} \oplus \mathbf{g}_{\mathbf{aN}}^{\#}{}^{3}\mathbf{h}\right) \multimap \mathbf{m}_{\#}^{3}\mathbf{h} \equiv \mathbf{W}_{\#}^{3}\mathbf{h} \multimap \mathbf{m}_{\#}^{3}\mathbf{h}$$

*[The botanical term '***meristem***' refers to the 'advancing edge' of a living plant; to the actively-dividing plant tissues of the growing tips of roots and stems].

-- as the new '*meta-**meristemal** term*', or 'vanguard term', every term that has emerged before its emergence is thereby "demoted" [cf. Hegel, as quoted in sub-section **B**.θ hereof] *relative* to it, inaugurating a presentational **s**tage of the '''***formal subsumption***''' ***of*** $^{3}_{h}\mathbf{W}_{\#}$ -- i.e., of $^{3}_{h}\mathbf{N}_{\#}$, $^{3}_{h}\mathbf{a}_{\#}$, & $^{3}_{h}\mathfrak{g}_{\mathbf{aN}}^{\#}$ -- ***by*** $^{3}_{h}\mathbf{m}_{\#}$.

In this **s**tage **2**, $^{3}_{h}\mathbf{W}_{\#}$, connoting the axiomatic system for the arithmetic of the space of the "**W**hole numbers", $\mathbf{W} \equiv \{\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{...}\}$, i.e., the system constituted by the sum of the meanings of the **3** '''determinations''' in $(^{3}_{h}\mathbf{N}_{\#} \oplus ^{3}_{h}\mathbf{a}_{\#} \oplus ^{3}_{h}\mathfrak{g}_{\mathbf{aN}}^{\#})$, is the solution to the «***aporia***» of the previous **s**tage, **s**tage **1** --

$$^{3}_{h}\mathcal{H}_{1}^{\#} = (^{3}_{h}\mathbf{N}_{\#})^{2^{1}} = {}^{3}_{h}\mathbf{N}_{\#} \multimap {}^{3}_{h}\mathbf{a}_{\#}$$

-- which also signifies the '''***formal subsumption***''' of the axiomatic system for the arithmetic of the space of "**N**atural numbers", $\mathbf{N} \equiv \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{...}\}$, by the concept of **0** as a "full-regalia" number, via its formulation as an axioms-system-component, connoted by $^{3}_{h}\mathbf{a}_{\#}$, for the "**a**ughts", $\mathbf{a} \equiv \{\mathbf{(I–I)}, \mathbf{(II–II)}, \mathbf{(III–III)}, \mathbf{...}\}$.

The '''***real subsumption***''' of $^{3}_{h}\mathbf{N}_{\#}$ by $^{3}_{h}\mathbf{a}_{\#}$, arrives only in **s**tep **2**, with, and by means of, the adding-in of $^{3}_{h}\mathfrak{g}_{\mathbf{aN}}^{\#}$, the ***synthesis*** of $^{3}_{h}\mathbf{a}_{\#}$ and $^{3}_{h}\mathbf{N}_{\#}$. Analogously, the '''***real subsumption***''' of $^{3}_{h}\mathbf{W}_{\#}$ by $^{3}_{h}\mathbf{m}_{\#}$ arrives only in the next **s**tep, **s**tep **3**, with the irruption of $(^{3}_{h}\mathfrak{g}_{\mathbf{mN}}^{\#} \oplus {}^{3}_{h}\mathfrak{g}_{\mathbf{ma}}^{\#} \oplus {}^{3}_{h}\mathfrak{g}_{\mathbf{maN}}^{\#})$, namely, of **(1)** $^{3}_{h}\mathfrak{g}_{\mathbf{mN}}^{\#}$, as '*partial* synthesis'/*partial* 'uni-thesis' of $^{3}_{h}\mathbf{m}_{\#}$ with $^{3}_{h}\mathbf{N}_{\#}$, plus of **(2)** $^{3}_{h}\mathfrak{g}_{\mathbf{ma}}^{\#}$, as the '*partial* synthesis'/*partial* 'uni-thesis' of $^{3}_{h}\mathbf{m}_{\#}$ with $^{3}_{h}\mathbf{a}_{\#}$, plus of **(3)** the '***full*** uni-thesis', $^{3}_{h}\mathfrak{g}_{\mathbf{maN}}^{\#}$, unifying $^{3}_{h}\mathbf{m}_{\#}$ with the prior '***full*** uni-thesis', that of $^{3}_{h}\mathbf{a}_{\#}$ ***and***/with $^{3}_{h}\mathbf{N}_{\#}$, thus combining all 'priorly-extant' 'contra-thesis' & «***arché***» 'subscripted epithets', namely, subscripted-**m**, subscripted-**a**, & subscripted-**N** -- into a new, single unit-term, as well as 'oppositionally adding-in' yet a new 'contra-thesis' term, $^{3}_{h}\mathbf{f}_{\#}$, so as to constitute the next, *new* «***aporia***»:

$$^{3}_{h}\mathcal{H}_{3}^{\#} = (^{3}_{h}\mathbf{N}_{\#})^{2^{3}} =$$

$$(^{3}_{h}\mathbf{N}_{\#} \oplus {}^{3}_{h}\mathbf{a}_{\#} \oplus {}^{3}_{h}\mathfrak{g}_{\mathbf{aN}}^{\#} \oplus {}^{3}_{h}\mathbf{m}_{\#} \oplus {}^{3}_{h}\mathfrak{g}_{\mathbf{mN}}^{\#} \oplus {}^{3}_{h}\mathfrak{g}_{\mathbf{ma}}^{\#} \oplus {}^{3}_{h}\mathfrak{g}_{\mathbf{maN}}^{\#}) \multimap {}^{3}_{h}\mathbf{f}_{\#} \equiv$$

$$^{3}_{h}\mathbf{Z}_{\#} \multimap {}^{3}_{h}\mathbf{f}_{\#}.$$

The terminology of ‘‘‘***formal subsumption***’’’ vs. ‘‘‘***real subsumption***’’’ is adapted from, and generalized from, a text by Karl Marx. Marx theorizes, in a text sometimes termed “the unpublished sixth chapter” of ***Capital*** [in relation to the table of contents of an earlier draft of ***Capital***, not to that of ***Capital*** as published], and entitled *“Results of the Immediate Process of Production”*, a text which presents a two-stage model of the historical development of the historically-specific global human socio-politico-economic system based, initially, upon competing individual [industrial] capitals.

In that manuscript, Marx defines the phase of the *“formal subsumption of labor under capital”* in the following terms -- **“**...an ***essential change*** in the ***real manner and mode*** of the process of labor, of the ***actual*** process of production, has ***not*** at all [yet] made its appearance... On the contrary... since the subsumption of the process of labour under capital makes its appearance -- on the basis of a *process of labour which is at hand*, ***which obtained prior to this subsumption under capital***, which has shaped itself on the basis of processes of production which were prior and various and of other preconditions of production... it is a *process of labour which is given and at hand* that capital subsumes to itself; thus, for example, artisan-work...[or] the mode of agriculture which corresponds to the small, self-sufficient peasant-economy. If changes in these ***traditional*** processes of labour which have been brought by capital under its command make their appearance, then these modifications can only be gradual consequences of ***the subsumption of given, traditional processes of labour*** under capital....**”** [Albert Dragstedt, translator and editor, ***Value***: ***Studies by Karl Marx***, *Results of the Immediate Process of Production*, New Park Publications, [London: **1976**], p. **117**; ***bold-italic*** and ***underline*** emphasis and square-bracketed inserts added by **F.E.D.**].

There also, Marx defines the *“real subsumption of labour under capital or the specifically capitalist mode of production”* in the following way -- **“**...the entire ***real*** shape of the mode of production changes and a ***speci***fically capitalist mode of production arises (technologically, too), upon whose basis and with which also those ***relationships of production*** between the different ***agents of production***, and especially between capitalist and wage-labourer, which correspond to the capitalist process of production simultaneously develop. ... With ***the real subsumption of labour under capital***, a complete (and constantly advancing and self-repeating) ***revolution*** in the mode of production itself takes place, in the ***productivity of labour*** and in the ***relationship*** of capitalist and worker. Upon ***the real subsumption of labour under capital***, all those *‘changes’* in the labour-process itself which we developed earlier enter the scene. The *social productive-powers of labour* are developed and also ***the application of science and machinery to the immediate production***, as work begins to take place on a large scale. On the other hand, the capitalist mode of production (which now structures itself as a mode of production *sui generis*) creates an altered structure of material production. On the other hand, this alteration in the material structure forms the basis for the development of ***the capital-relationship***, whose adequate structure consequently corresponds to a particular degree of development of the productive-powers of labour. ... What is revealed at this point is how even ***economic categories*** [such as that of *“**the commodity-relationship**”* and that of *“**the money-relationship**”*] ***which belong to*** [i.e., which first arise in, and inhere in] ***earlier epochs*** of production ***acquire a specifically different, historical character*** on the basis of the capitalist mode of production.**”** [*ibid*., pp. **120**, **130**, **160**; ***bold-italic***, ***color***-ordinal, and ***underline*** emphasis and square-bracketed inserts added by **F.E.D.**].

Regarding the point of the final sentence of the quoted passage above, more amplitude is to be found in the ***Grundrisse*** manuscripts [“***Foundations*** of the Critique of Political Economy [Rough Draft]”] by Karl Marx[, which are the original source manuscripts, ***un***published by Marx, for the works that he ***did*** publish on his immanent critique of the ideology-adulterated science of classical political economy], viz.: **“**Bourgeois society is the most developed and the most complex historic organization of production [**F.E.D.**: so far]. The ***categories*** which express its ***relations***, the ***comprehension of its structure***, thereby also allow insights into the ***structure*** and ***relations of production*** of all the ***vanished*** social formations ***out of whose ruins and elements it built itself up, whose partly still unconquered remnants are carried along within it, whose mere nuances have developed explicit significance within it***, etc. ... While in the completed bourgeois system every economic relation presupposes every other in its bourgeois economic form, and everything posited is thus also a presupposition, this is the case with every ***organic system***. This ***organic system*** itself, as a ***totality***, has its presuppositions, and ***its development to its totality consists precisely in subordinating all elements of society to itself, or in creating out of it the organs which it still lacks. This is historically how it becomes a totality.”*** [Karl Marx, ***Grundrisse***, Martin Nicolaus, translator, New Left Review, [London: **1973**], pp. **105**, **278**; ***bold-italic***, shadow, and ***underline*** emphasis and square-bracketed inserts added by **F.E.D.**].

At this point, I think that we are now in a position to helpfully illustrate this ***gene***ric principle of '''***formal subsumption***''' vs. '''***real subsumption***''', and its role in the solution/'semantification' of the terms whose semantics are initially ***un***known, that arise in our model presentation of the '***Gödelian Ideo-Meta-Dynamic of the Standard Arithmetics***'. We can do so by illustrating this principle in a different context, by means of an ${}_{N}Q_{\#}$ ***dialectical*** '***meta-model***' of the ***systematic-dialectical method of presentation*** applied by Marx in his masterpiece, «***Das Kapital***». True, a *more direct* illustration of these principles would arise in the context of an ${}_{N}Q_{\#}$ ***dialectical*** '***meta-model***' of the ***history*** of the ***systems*** of human-***social Relations of*** [human societies' expanding self-re-]***production***, such as in the '***meta-model***' --

$$ {}^{2}_{h}\rangle\!-\!|\!-\!\langle^{R}_{\tau_R}\uparrow \;=\; \left\langle {}^{2}_{h}A_{R} \right\rangle^{2^{\tau_R}\uparrow} $$

-- in which the «***arché***» category, ${}^{2}_{h}A_{R}$, connotes the **A**ppropriation, in their "raw" form -- a form ***un***improved, by human labor, for human consumption -- of the products of **Nature**, that is, the mode of production of ***predation***, of ***foraging***, and of ***scavenging*** -- of "***hunting and gathering***". In this '***meta-model***', the '''***formal subsumption***''' of all previously-arisen ***social Relations of production*** by ***the capital social Relation of production*** arises in and as its model historical epoch $\tau_R = 4$, and the '''***real subsumption***''' of all previously-arisen such ***social Relations*** by ***the capital-Relation*** arises in its model historical epoch $\tau_R = 5$.

However, keeping within the '[***meta-***]***systematic-dialectical***' domain of ***dialectical*** [***meta-***]***modeling*** -- that of our core '***meta-model***' of the "***Standard Arithmetics***", we can nevertheless find useful ***analogues*** of both this '''***formal subsumption***''', and this '''***real subsumption***''', in an ${}_{N}Q_{\#}$ ***dialectical*** '***meta-model***' of the ***systematic-dialectical method of presentation*** of Marx's «***magnum opus***», «***Das Kapital***», and, specifically, even in a ***dialectical*** '***meta-model***' of ***only*** the "value-form" content of that potentially world-liberating treatise:

$$ {}^{2}_{h})\text{H}(^{«K»}_{s_{«K»}}\uparrow \;=\; \left({}^{2}_{h}A_{«K»} \right)^{2^{s_{«K»}}\uparrow} $$

-- wherein the «***arché***» category, ${}^{2}_{h}A_{«K»}$, connotes the content of sub-section ***A***. of Section **3**. ["The Form of Value or Exchange-Value"] of Chapter ***I***. ["Commodities"] of Part **I** ["**Commodities and Money**"] of Volume **I** ['''***The Productions-Process of Capitals***'''] of «***Das Kapital***». Sub-section ***A***. is entitled "Elementary or Accidental Form of Value". Of the centrality of this ***social relations of production category*** to his treatise, as, in effect, the «***arché***» of its entire exposition, Marx writes as follows, in his Preface to the First German Edition: **"**...in bourgeois society the commodity-form of the product of labour -- or the value-form of the commodity -- is the economic cell-form.**"** [Karl Marx, ***Capital***: ***A Critique of Political Economy***, International Publishers [NY: **1967**], p. **8**]. Those who live in capital-based society, acquire, *per force*, from experience, a knowledge of its phenomena -- a veritable '''phenomenology''' of capital -- however "chaotic" [Marx] may be the conception of that modern world as a whole that their experience, if '***un***theorized' and '***un***systematized' by them, may afford them. It can be demonstrated, for them, that the ***immanent*** critique [or ***self***-critique] of the category "Elementary Form of [Commodity-]Value", as candidate for a ***complete*** categorial comprehension of capital-based society, elicits "The Total or Expanded Form of [Commodity-]Value" category, covered in Marx's sub-section **3**.***B***., as supplementary *counter*-category, and that the synthesis/critique/'''***real subsumption***''' of the "Elementary Form" category, with/by/by the "Expanded Form" category, elicits the "General Form of [Commodity-]Value" category, covered in Marx's sub-section **3**.***C***., which leads to "The Money-Form" of value category, covered in Marx's sub-section **3**.***D***.

The ‘partial syntheses’/critiques/‘‘‘*real subsumptions*’’’, by “The Money Form” category, with/of/of each of the three prior Commodity Forms of Value categories, leads to the three categories of the functions of Money that constitute Chapter ***III***., entitled “Money, or the Circulation of Commodities”, in its Sections **1**., **2**., **& 3.b**.

The immanent critique [or self-critique] of the category of “The Money-Form” of value itself, as candidate for a ***complete*** categorial comprehension of the ‘‘‘phenomenology’’’ of capital-based society, elicits “Capital”, first covered in Chapter ***IV***., entitled “The General Formula for Capital”, as its supplementary counter-category.

The ‘partial syntheses’/critiques/‘‘‘*real subsumptions*’’’, by the Capitals category with/of/of each of the three Commodities sub-categories yields the sub-categories of “Commodity-Capital”, covered in Chapter ***III***., entitled “The Circuit of Commodity-Capital”, of Part **I**, of Volume **II** of ***Capital***.

The ‘partial synthesis’/critique/‘‘‘*real subsumption*’’’ of the Money Value-Form category with/by the Capital Value-Form category yields the category of “Money-Capital”, covered in Chapter ***I***., entitled “The Circuit of Money-Capital”, also of Part **I**, entitled “**The Metamorphoses of Capital and Their Circuits**”, of Volume **II** of ***Capital***, entitled ‘‘‘***The Circulations-Process of Capitals***’’’.

The full-synthesis/critique/‘‘‘*real subsumption*’’’ of the Money and Commodity Value-Forms categories with/by the Capital Value-Form category -- the synthesis/critique/subsumption of the previous full-synthesis/-/critique/subsumption category, that of ***the money-mediated circulation of commodities***, with/by the Capital Value-Form category -- yields the category of ***the money-capital-mediated and commodity-capital-mediated self-circulation of the total social capital***, covered in Part **III**, entitled “**The Reproduction and Circulation of the Aggregate Social Capital**”, the final Part of Volume **II** of ***Capital***.

The ***immanent*** critique [or ***self***-critique] of the category of “Capital” itself, or of “The Capital Relation”, as predominant social relation of production, leads cognition to the predicted historical future-boundary of the ***system*** of capitals, and to a glimpse beyond that boundary -- in Chapter ***XXXII***. of Volume **I**, entitled “Historical Tendency of Capitalist Accumulation”, and, at a ‘sub-tabular level’ [i.e, at a level lower than that of any explicit division of the text; at a level below any [entitled] sub-section or sub-sub-section that is explicitly listed in the Table of Contents], in Volume **III**, Chapter ***XXVII***., entitled “The Role of Credit in Capitalist Production”, and, also ‘sub-tabularly’, elsewhere in Marx’s text -- all intimating as to how the self-reproduction of the capitalist system impends an eventual critical point of historical ‘meta-system transition’ [cf. Turchin], by that system, to out of and beyond that ***system***, to inside a new and higher ***system*** of human-social relations of production, self-constituted from out of the self-dissolution of the capitals-system, and founded upon a new, higher level of growth of the human-social forces of production than any that a capitals-system can contain.

In ideographical symbols, the above narrative can be re-rendered, per the $_{\underline{N}}\underline{Q}_{\#}$ ***dialectical*** ‘***meta-model***’ of the ***systematic-dialectical method of presentation*** of «***Das Kapital***» presented above, as --

$${}^{2}_{h}\text{)H(}^{«K»}_{0} = \left({}^{2}_{h}\underline{A}_{«K»} \right)^{2^{0}} = \left({}^{2}_{h}\underline{A}_{«K»} \right)^{1} = {}^{2}_{h}\underline{A}_{«K»};$$

$${}^{2}_{h}\text{)H(}^{«K»}_{1} = \left({}^{2}_{h}\underline{A}_{«K»} \right)^{2^{1}} = \left({}^{2}_{h}\underline{A}_{«K»} \right)^{2} = \left({}^{2}_{h}\underline{A}_{«K»} \oplus {}^{2}_{h}\mathcal{Q}^{«K»}_{AA} \right) \equiv {}^{2}_{h}\underline{A}_{«K»} \oplus {}^{2}_{h}\underline{B}_{«K»};$$

$${}^{2}_{h}\text{)H(}^{«K»}_{2} = \left({}^{2}_{h}\underline{A}_{«K»} \right)^{2^{2}} = \left({}^{2}_{h}\underline{A}_{«K»} \right)^{4} = \left({}^{2}_{h}\underline{A}_{«K»} \oplus {}^{2}_{h}\underline{B}_{«K»} \right)^{2} =$$

$$\left(\left({}^{2}_{h}\underline{A}_{«K»} \oplus {}^{2}_{h}\underline{B}_{«K»} \oplus {}^{2}_{h}\mathcal{Q}^{«K»}_{BA} \right) \oplus {}^{2}_{h}\mathcal{Q}^{«K»}_{BB} \right) \equiv \left({}^{2}_{h}\underline{A}_{«K»} \oplus {}^{2}_{h}\underline{B}_{«K»} \oplus {}^{2}_{h}\underline{C}_{«K»} \oplus {}^{2}_{h}\underline{M}_{«K»} \right);$$

$${}^{2}_{h})\!\mathrm{H}\!(_{3}^{\text{«K»}} = \left({}^{2}_{h}\underline{A}_{\text{«K»}}\right)^{2^{3}} = \left({}^{2}_{h}\underline{A}_{\text{«K»}}\right)^{8} = \left({}^{2}_{h}\underline{A}_{\text{«K»}} \oplus {}^{2}_{h}\underline{B}_{\text{«K»}} \oplus {}^{2}_{h}\underline{C}_{\text{«K»}} \oplus {}^{2}_{h}\underline{M}_{\text{«K»}}\right)^{2} =$$

$${}^{2}_{h}\underline{A}_{\text{«K»}} \oplus {}^{2}_{h}\underline{B}_{\text{«K»}} \oplus {}^{2}_{h}\underline{C}_{\text{«K»}} \oplus {}^{2}_{h}\underline{M}_{\text{«K»}} \oplus {}^{2}_{h}\hat{q}_{MA}^{\text{«K»}} \oplus {}^{2}_{h}\hat{q}_{MB}^{\text{«K»}} \oplus {}^{2}_{h}\hat{q}_{MC}^{\text{«K»}} -\!\oplus\!- {}^{2}_{h}\hat{q}_{MM}^{\text{«K»}}$$

$$\equiv$$

$${}^{2}_{h}\underline{A}_{\text{«K»}} \oplus {}^{2}_{h}\underline{B}_{\text{«K»}} \oplus {}^{2}_{h}\underline{C}_{\text{«K»}} \oplus {}^{2}_{h}\underline{M}_{\text{«K»}} \oplus {}^{2}_{h}\hat{q}_{MA}^{\text{«K»}} \oplus {}^{2}_{h}\hat{q}_{MB}^{\text{«K»}} \oplus {}^{2}_{h}\hat{q}_{MC}^{\text{«K»}} -\!\oplus\!- {}^{2}_{h}\underline{K}_{\text{«K»}};$$

$${}^{2}_{h})\!\mathrm{H}\!(_{4}^{\text{«K»}} = \left({}^{2}_{h}\underline{A}_{\text{«K»}}\right)^{2^{4}} = \left({}^{2}_{h}\underline{A}_{\text{«K»}}\right)^{16} =$$

$$\left({}^{2}_{h}\underline{A}_{\text{«K»}} \oplus {}^{2}_{h}\underline{B}_{\text{«K»}} \oplus {}^{2}_{h}\underline{C}_{\text{«K»}} \oplus {}^{2}_{h}\underline{M}_{\text{«K»}} \oplus {}^{2}_{h}\hat{q}_{MA}^{\text{«K»}} \oplus {}^{2}_{h}\hat{q}_{MB}^{\text{«K»}} \oplus {}^{2}_{h}\hat{q}_{MC}^{\text{«K»}} -\!\oplus\!- {}^{2}_{h}\underline{K}_{\text{«K»}}\right)^{2}$$

$$\equiv$$

$${}^{2}_{h}\underline{A}_{\text{«K»}} \oplus {}^{2}_{h}\underline{B}_{\text{«K»}} \oplus {}^{2}_{h}\underline{C}_{\text{«K»}} \oplus {}^{2}_{h}\underline{M}_{\text{«K»}} \oplus {}^{2}_{h}\hat{q}_{MA}^{\text{«K»}} \oplus {}^{2}_{h}\hat{q}_{MB}^{\text{«K»}} \oplus {}^{2}_{h}\hat{q}_{MC}^{\text{«K»}} \oplus {}^{2}_{h}\underline{K}_{\text{«K»}} \oplus$$
$${}^{2}_{h}\hat{q}_{KA}^{\text{«K»}} \oplus {}^{2}_{h}\hat{q}_{KB}^{\text{«K»}} \oplus {}^{2}_{h}\hat{q}_{KC}^{\text{«K»}} \oplus {}^{2}_{h}\hat{q}_{KM}^{\text{«K»}} \oplus {}^{2}_{h}\hat{q}_{KMA}^{\text{«K»}} \oplus {}^{2}_{h}\hat{q}_{KMB}^{\text{«K»}} \oplus$$
$${}^{2}_{h}\hat{q}_{KMC}^{\text{«K»}} -\!\oplus\!- {}^{2}_{h}\hat{q}_{KK}^{\text{«K»}}$$

-- with the following solution to the last of the five Seldon Equations listed above, for the fifteen terms of initially ***un***known meaning, that come after the initial term, ${}^{2}_{h}\underline{A}_{\text{«K»}} \longmapsto \mathrm{q}_{1}$. The latter, «***arché***» term is already meaning-*stipulated* and meaning-*given* as connoting the content of the sub-section **3**.***A***., Chapter ***I***., Volume **I** of ***Capital***, entitled "Elementary or Accidental Form of Value" --

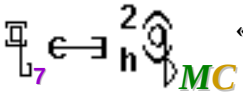
$\mathrm{q}_{2} \longleftarrow {}^{2}_{h}\underline{B}_{\text{«K»}}$: connotes the category for the content of sub-section **3**.***B***. of Chapter ***I***., entitled "Total or Expanded Form of Value";

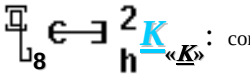
$\mathrm{q}_{3} \longleftarrow {}^{2}_{h}\underline{C}_{\text{«K»}}$: connotes the category for the content of sub-section **3**.***C***. of Chapter ***I***., entitled "The General Form of Value";

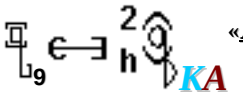
$\mathrm{q}_{4} \longleftarrow {}^{2}_{h}\underline{M}_{\text{«K»}}$: connotes the category for the content of sub-section **3**.***D***. of Chapter ***I***., entitled "The Money Form";

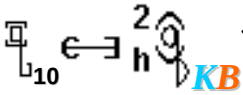
$\mathrm{q}_{5} \longleftarrow {}^{2}_{h}\hat{q}_{MA}^{\text{«K»}}$: connotes the category for the content of Section **1**. of Chapter ***III***., entitled "The Measure of Values"; the category of Money's function as Value-measurer, the categorial '''***real subsumption***''' of the Elementary Value-Form category by the Money Value-Form category, given that the LHS Commodity of the Elementary Form equation measures the value of the LHS Commodity.

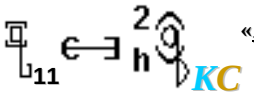
$\mathrm{q}_{6} \longleftarrow {}^{2}_{h}\hat{q}_{MB}^{\text{«K»}}$: connotes the category for the content of Section **2**. of Chapter ***III***., entitled "The Medium of Circulation"; the category of Money's function as circulation-Medium for Commodities, the categorial '''***real subsumption***''' of the Expanded Value-Form category by the Money Form category, as the Expanded Form is circulation with the mediating role of the Money-Commodity abstracted away;

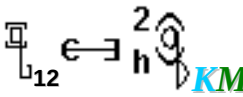

: connotes the category for content of sub-section **3.b.** of Chapter ***III***., the category for the "The Means of Payment" function of the Money-Form, the '''***real subsumption***''' of the General Value-Form category by the Money Value-Form category, as the General Form describes a "universal equivalent" Form -- not yet a single, universal Money-Commodity -- which is thus already a potential 'Payment-Form';

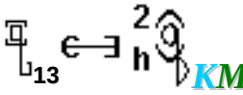

: connotes the category for content of Chapter ***IV***., entitled "The General Formula for Capital";

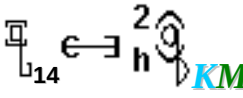

: connotes the category for 'sub-tabular'/"***in***operative" content of Vol. **II**, Chapter ***III***., "The Circuit of Commodity-Capital", for the categorial '''***real subsumption***''' of the Elementary Form of Commodity-Value category by the «***K***apital»-Value-Form category, i.e., recognition of [most of] the Commodities of the Elementary Value-Form category as Commodity-***Capital***;

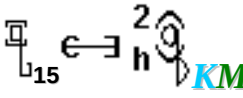

: connotes the category for 'sub-tabular'/"***in***operative" content of Vol. **II**, Chapter ***III***., "The Circuit of Commodity-Capital", for the categorial '''***real subsumption***''' of the Expanded Form of Commodity-Value category by the «***K***apital»-Value-Form category, i.e., recognition of [most of] the Commodities of the Expanded Form category as Commodity-***Capital***;

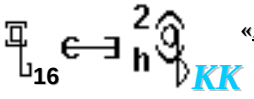

: connotes the category for 'sub-tabular'/"***in***operative" content of Vol. **II**, Chapter ***III***., "The Circuit of Commodity-Capital"; for the categorial '''***real subsumption***''' of the General Form of Commodity-Value category by the «***K***apital»-Value-Form category, i.e., recognition of [most of] the Commodities of the General Value-Form category as Commodity-***Capital***;


: connotes the category for content of Vol. **II**, Chapter ***I***., "The Circuit of Money-Capital", for the categorial '''***real subsumption***''' of the Money-Value-Form category by the «***K***apital»-Value-Form category, i.e., recognition of the Money-Value-Form category as [largely] one of Money-***Capital***;


: connotes the category for 'sub-tabular'/"***in***operative" content of Vol. **II**, Chapter ***I***., "The Circuit of Money-Capital", for the categorial '''***real subsumption***''' of the Money-Value-Form category, in the sub-category of its function as Measure of Values, by the «***K***apital»-Value-Form category, i.e., recognition of the function represented by the Measure of Values 'Money-sub-Form' sub-category as being [most often] that of "Money" which is, in fact, not ***just*** "Money", but Money-***Capital***;


: connotes the category for 'sub-tabular'/"***in***operative" content of Vol. **II**, Chapter ***I***., "The Circuit of Money-Capital", for the categorial '''***real subsumption***''' of the Money-Value-Form category, in its Commodity Circulation-Medium function sub-category, by the «***K***apital»-Value-Form category, i.e., recognition of the function represented by the Medium of Circulation 'Value-sub-Form' sub-category of the Money category as being [as most often experienced in practice] that of Money which, in fact, belongs to the Money-***Capital*** category;


: connotes the category for content of Vol. **II**, Part **III**., "**The Reproduction and Circulation of the Aggregate Social Capital**", the final Part of Vol. **II**, regarding ***money-capital-mediated and commodity-capital-mediated self-circulation of the total social capital***; the «***K***apital»-Value-Form category's '''***real subsumption***''' of the previous 'full synthesis'/'full uni-thesis' process category, namely  , connoting the category for the process of the ***M***oney-mediated circulation of ***C***ommodities, i.e., recognizing that the largest part of modern social circulation is ***M***onies-and-***C***ommodities-mediated circulation of ***Capitals***;


: connotes the category for content of Vol. **I**, Chapter ***XXXII***., "Historical Tendency of Capitalist Accumulation", as well as for 'sub-tabular' content of Vol. **III**, Chapter ***XXVII***., "The Role of Credit in Capitalist Production", etc., regarding the category of the potential '''meta-system transition''' [cf. Turchin] to a higher prevailing social relation of production, predicted on the basis of the self-subsumption/immanent critique [or self-critique] of the category of the «***K***apital»-Value-Form, the latter as a failed candidate for a ***complete*** categorial comprehension of capital-centered society, given that the category of the gradually and ultimately crisis-causing, self-destroying self-reproduction-process of that societal ***system*** already points to the category of the capital-centered-system's own self-***non***-reproduction -- to its own, immanent-crisis-induced, growth-of-the-productive-forces-induced, self-induced breakdown -- and to the ***possible*** category of the formation of a higher societal system, beyond the capital-centered-system itself.

We note, in passing, an aspect of the '***homeomorphic defect***' of this '***meta-model***' -- that the 'Money-Value-Form' is modeled as issuing from the self-subsumption/immanent critique [or self-critique] of the "Total or Expanded Form of Value", ${}^{2}_{h}\underline{B}_{«\underline{K}»}$, as ${}^{2}_{h}\mathcal{G}_{BB}^{«\underline{K}»} \equiv {}^{2}_{h}\underline{M}_{«\underline{K}»}$, rather than from a self-subsumption/immanent critique [or self-critique] of the "General Form of Value", ${}^{2}_{h}\mathcal{G}_{BA}^{«\underline{K}»} \equiv {}^{2}_{h}\underline{C}_{«\underline{K}»}$, as ${}^{2}_{h}\mathcal{G}_{CC}^{«\underline{K}»} \equiv {}^{2}_{h}\underline{M}_{«\underline{K}»}$ [which ${}^{2}_{h}\underline{M}_{«\underline{K}»}$ does ***not*** equal, in this '***meta-model***']. Exploration of possible routes to the redress of this defect, e.g., by modeling using the 'Triadic Seldon Function', instead of the 'Dyadic Seldon Function', or by other means, are outside the scope of this essay. Exploration of this issue is planned in the context of future, separate publications. We note also that the '***Dialectical Meta-Equation***' --

$$ {}^{2}_{h}\underline{)\!\!H\!\!(}_{4}^{«\underline{K}»} = \left({}^{2}_{h}\underline{A}_{«\underline{K}»} \right)^{2^{4}} $$

-- and its **4** symbol-elements core expression --

$$ \underline{A}^{2^{4}} $$

constitute a summary of the content of the three(+) volumes of Marx's treatise, ***Capital***, in an ***ex***plicit sense, and with regard to its Value-Form content [only], ***only*** **(1)** in that the content of its first chapter's first sub-section, on the "Elementary or Accidental Form of Value", connoted in that '***Dialectical Meta-Equation meta-model***' by $\underline{A}$, is the '''seed''', or "cell-form" [Marx], of all the ***Value-Form*** *content* that follows in that treatise, and; **(2)** in that the 'exponented exponent' form of that core expression affords it a '''***self-reflexive function***''' format. That format signifies that the '***ex***plicitization' of all of the ***im***plicit content of this core expression arises by way of a *self*-iterated *self*-critique of all that has been '***ex***plicitized' from that '''seed''' in previous iterations of that immanent, or self-, critique, tracking all the way back to $\underline{A}$ as its 'content-seed', i.e., as its 'maximal-***im***plicitude' '***dialectical premise***', or «***arché***», from which the progressing **s**tages of self-critique were originally launched. I have already, elsewhere, demonstrated in detail the '***meta-monadicity***' of the **2**nd term of the **1**st ***dyad*** of the **1**st ***triad*** of Marx's ***Capital***, and the '*ontological* ***hybridicity***' of the **3**rd term of that ***triad*** --

'Meta-Monadicity' in the First Triad of Marx's Immanent Critique of Political Economy --
http://www.dialectics.org/dialectics/Aoristoss_Blog/Entries/2011/12/12_Part_II._B._Intuitive_Account_of_the_Universality_of_F.E.D.s_Generic_Heuristic_Algorithm_for_Dialectic_-_The_Opening_Triad_of_Marxs___Capital___-_86th_Blog_Entry.html
'meta-monadization', definition 1 -- http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/Meta/Meta.htm
'meta-monadization', definition 2 -- http://point-of-departure.org/Point-Of-Departure/ClarificationsArchive/MetaMonadization/MetaMonadization.htm
See also the '*Meta-Monadology*' guest-authors' versions of these two '*meta-models*' --
http://adventures-in-dialectics.org/Adventures-In-Dialectics/MetaMonadology/MetaMonadology-entry.htm

There is a much simpler, **8**-term '***meta-model***' of this '''Value-Form ***Dialectic***''' content of Marx's ***Capital***.

This simpler '***meta-model***' averts the element of '***homeomorphic defect***' noted above. It is also far less detailed in its coverage of that content, due to its issuing from a '''shallower''' «***arché***». The «***arché***» of this simpler '***meta-model***' is the alternative «***arché***»/"cell-form" that Marx mentions in the passage from his Preface to ***Capital***, quoted above: **"**...in bourgeois society ***the commodity-form of the product of labour*** -- or the value-form of the commodity -- ***is the economic cell-form*.**" [***emphasis*** added]. This simpler, **8**-term '***meta-model***' also averts the issues of 'sub-tabular correspondences of terms' raised by the more complex, **16**-term '***meta-model***', set forth above, which issues from Marx's '''deeper''', Elementary Value-Form «***arché***»/"cell-form" alternative.

In the '***meta-model***' issuing from the '''shallower''' «***arché***», of the '''**C**ommodity''' category as a whole, ${}^{2}_{h}\underline{C}_{«\underline{K}»}$, each term maps to no '''deeper''' than the Chapter level [albeit with some '''straddling''' of pairs of Chapters] -- or even to the level of an entire "Part", in one case, the case of its final 'full synthesis' term. We "solve" this '***meta-model***'s' '***Dialectical Meta-Equation***' -- map its **8** terms to Marx's Table(s) of Contents -- as set forth below. First, let's view the full '***Dialectical Meta-Equation***', deployed via its '***Equation-valued***' **s**tages for the various values of its **W**hole-number-valued $\mathbf{s}_{«\underline{K}»}$:

$${}^{2}_{h}\underline{)\!\!H\!\!(}_{1}^{\text{«}\underline{K}\text{»}} = \left({}^{2}_{h}\underline{C}_{\text{«}\underline{K}\text{»}}\right)^{2^{1}} = \left({}^{2}_{h}\underline{C}_{\text{«}\underline{K}\text{»}}\right)^{2} = \left({}^{2}_{h}\underline{C}_{\text{«}\underline{K}\text{»}} \vdash\!\oplus\!\dashv {}^{2}_{h}\mathfrak{g}_{CC}^{\text{«}\underline{K}\text{»}}\right) \equiv {}^{2}_{h}\underline{C}_{\text{«}\underline{K}\text{»}} \vdash\!\oplus\!\dashv {}^{2}_{h}\underline{M}_{\text{«}\underline{K}\text{»}};$$

$${}^{2}_{h}\underline{)\!\!H\!\!(}_{2}^{\text{«}\underline{K}\text{»}} = \left({}^{2}_{h}\underline{C}_{\text{«}\underline{K}\text{»}}\right)^{2^{2}} = \left({}^{2}_{h}\underline{C}_{\text{«}\underline{K}\text{»}}\right)^{4} = \left({}^{2}_{h}\underline{C}_{\text{«}\underline{K}\text{»}} \vdash\!\oplus\!\dashv {}^{2}_{h}\underline{M}_{\text{«}\underline{K}\text{»}}\right)^{2} =$$

$$\left(\left({}^{2}_{h}\underline{C}_{\text{«}\underline{K}\text{»}} \oplus {}^{2}_{h}\underline{M}_{\text{«}\underline{K}\text{»}} \oplus {}^{2}_{h}\mathfrak{g}_{MC}^{\text{«}\underline{K}\text{»}}\right) \vdash\!\oplus\!\dashv {}^{2}_{h}\mathfrak{g}_{MM}^{\text{«}\underline{K}\text{»}}\right) \equiv$$

$$\left(\left({}^{2}_{h}\underline{C}_{\text{«}\underline{K}\text{»}} \oplus {}^{2}_{h}\underline{M}_{\text{«}\underline{K}\text{»}} \oplus {}^{2}_{h}\mathfrak{g}_{MC}^{\text{«}\underline{K}\text{»}}\right) \vdash\!\oplus\!\dashv {}^{2}_{h}\underline{K}_{\text{«}\underline{K}\text{»}}\right);$$

$${}^{2}_{h}\underline{)\!\!H\!\!(}_{3}^{\text{«}\underline{K}\text{»}} = \left({}^{2}_{h}\underline{C}_{\text{«}\underline{K}\text{»}}\right)^{2^{3}} = \left({}^{2}_{h}\underline{C}_{\text{«}\underline{K}\text{»}}\right)^{8} = \left({}^{2}_{h}\underline{C}_{\text{«}\underline{K}\text{»}} \oplus {}^{2}_{h}\underline{M}_{\text{«}\underline{K}\text{»}} \oplus {}^{2}_{h}\mathfrak{g}_{MC}^{\text{«}\underline{K}\text{»}} \oplus {}^{2}_{h}\underline{K}_{\text{«}\underline{K}\text{»}}\right)^{2} =$$

$$\left(\left({}^{2}_{h}\underline{C}_{\text{«}\underline{K}\text{»}} \oplus {}^{2}_{h}\underline{M}_{\text{«}\underline{K}\text{»}} \oplus {}^{2}_{h}\mathfrak{g}_{MC}^{\text{«}\underline{K}\text{»}} \oplus {}^{2}_{h}\underline{K}_{\text{«}\underline{K}\text{»}} \oplus {}^{2}_{h}\mathfrak{g}_{KC}^{\text{«}\underline{K}\text{»}} \oplus {}^{2}_{h}\mathfrak{g}_{KM}^{\text{«}\underline{K}\text{»}} \oplus {}^{2}_{h}\mathfrak{g}_{KMC}^{\text{«}\underline{K}\text{»}}\right)\right.$$

$$\left.\vdash\!\oplus\!\dashv {}^{2}_{h}\mathfrak{g}_{KK}^{\text{«}\underline{K}\text{»}}\right).$$

The solution-definition of each of the **7** terms, derived from the «***arché***», already defined, are the following:

[$\mathfrak{T}_{1} \Leftarrow\!\!\Rightarrow {}^{2}_{h}\underline{C}_{\text{«}\underline{K}\text{»}} \Leftrightarrow$ Volume **I**, Chapter ***I***., entitled "**C**ommodities" -- *given* / *pre-stipulated*;]

$\mathfrak{T}_{2} \Leftarrow\!\!\Rightarrow {}^{2}_{h}\underline{M}_{\text{«}\underline{K}\text{»}} \Leftrightarrow$ [Part of] Volume **I**, Chapter ***III***., entitled "**M**oney, or the Circulation of Commodities", and Chapter ***I***., Section **3**., sub-section ***D***., entitled "The Money-Form";

$\mathfrak{T}_{3} \Leftarrow\!\!\Rightarrow {}^{2}_{h}\mathfrak{g}_{MC}^{\text{«}\underline{K}\text{»}} \Leftrightarrow$ Volume **I**, Chapter ***III***., "Money, or the Circulation of Commodities", Section **2**., "The Medium of Circulation", sub-section ***a***.: "The Metamorphosis of Commodities": the '''complex unity'''/'''synthesis''' of Money and Commodities as the Process of their '''Circulatory''' Alternation;

$\mathfrak{T}_{4} \Leftarrow\!\!\Rightarrow {}^{2}_{h}\underline{K}_{\text{«}\underline{K}\text{»}} \Leftrightarrow$ Volume **I**, Chapter ***IV***., entitled "The General Formula for Capital" ['''Capital(s)-in-General'''];

$\mathfrak{T}_{5} \Leftarrow\!\!\Rightarrow {}^{2}_{h}\mathfrak{g}_{KC}^{\text{«}\underline{K}\text{»}} \Leftrightarrow$ Volume **II**, Chapter ***III***., "The Circuit of **C**ommodity-Capital";

$\mathfrak{T}_{6} \Leftarrow\!\!\Rightarrow {}^{2}_{h}\mathfrak{g}_{KM}^{\text{«}\underline{K}\text{»}} \Leftrightarrow$ Volume **II**, Chapter ***I***., "The Circuit of **M**oney-Capital";

$\mathfrak{T}_{7} \Leftarrow\!\!\Rightarrow {}^{2}_{h}\mathfrak{g}_{KMC}^{\text{«}\underline{K}\text{»}} \Leftrightarrow$ Volume **II**, Part **III**., "**The Reproduction and Circulation of the Aggregate Social Capital**", the final Part of Vol. **II**, regarding the category of the ***Money-capital-mediated and Commodity-capital-mediated self-circulation of the total social*** «***Kapital***»; the «**K**apital»-Value-Form category's categorial '''***real subsumption***''' of the previous 'full synthesis' process category, the category of the process ***of the money-mediated circulation of commodities***, to form the final 'full synthesis'/'"complex unity"' value-form category ***within*** the value-form categories-system of capital;

$\mathfrak{T}_{8} \Leftarrow\!\!\Rightarrow {}^{2}_{h}\mathfrak{g}_{KK}^{\text{«}\underline{K}\text{»}} \Leftrightarrow$ Volume **I**, Chapter ***XXXII***., "Historical Tendency of Capitalist Accumulation"; the ***self***-critique of the «***Kapital***»-relation; the expropriation operator category that is the «**K**apital»-Value-Form/«**K**apital»-relation-of-production category, self-applied, i.e., applied to that very operator category -- to the «**K**apital»-Value-Form category itself -- connoting, e.g., the up-rising, *within* ***total capital***, of ***variable capital***, «***aufheben***»-***negating*** capital ***as capital*** by «***aufheben***»-negating/**ex**propriating/-**a**ppropriating ***constant capital*** and ***fixed capital***, «***aufheben***»-negating *them* ***as capital***, with or without compensation to their former owners, thus also «***aufheben***»-***negating itself as*** [***variable***] ***capital***, and also as ***wage***-labor, constituting itself as "***the associated producers***" [Marx].

‘Dialogue-ic’ Translation of the **8**-Category ‘***Dialectical-Algebraic Meta-Model***’ of the ‘***Value-Form Dialectic***’ of Marx’s ***Capital***.

The eight-term ‘***dialectical-ideographical meta-model***’ of the ‘‘‘***Value-Form Dialectic***’’’ content of Marx’s ***Capital***, rendered above, can be translated into a narrative, two-party, “**Q&A**”-format, ‘dialogue-model’, one with the same essential content as the ‘***dialectical-ideographical meta-model***’. A transcript-excerpt of such a ‘dialogical narrative-model’ is exhibited below: the above ‘***dialectical-ideographical meta-model***’ of Capital ‘re-presented’ as a prose dialogue. This dialogue can also usefully be read as a ***self***-questioning and ***self***-answering of a single, fictive/-representative observer/participant in the presently-existing system of social economy. A phrase used repeatedly in the dialogue below is “[the ***dialectical***] ***analysis of a*** [category]”, whose Marxian usage has special features *vis-à-vis* its usual connotations. The Marxian, ‘‘‘***dialectical analysis***’’’ of a category is the detailed ‘***ex***plicitization’ of its initially only ***im***plicit content -- of all that the category being analyzed ‘‘‘contains’’’; of all that is immanent in that category as conceived -- prior to such analysis -- as a whole, but without any ***ex***plicit articulation of its parts. The ‘‘‘***dialectical analysis***’’’ of a category is thus the delineation of its detailed internal constitution; the ‘***ideo-taxonomic***’ delineation of its ***im***plied composition, in terms of the ***sub***-categories of that category; of the «***species***» of that category, taken as «***genos***»; the formulation of its ‘sub-categories sum’, of which it, as a whole, is, ***im***plicitly, the ‘***cumulum***’.

Q1. ¿What is the simplest category that grasps the totality of our ‘***un***theorized’, ‘***un***systematized’, “chaotic” [Marx], “raw” experience of our present/capitalist system of societal self-reproduction in which -- and as part of which -- we live**?**

A1. The “social relation of production” category of “**Commodities**”. Indeed, “The wealth of those societies in which the capitalist mode of production prevails, presents itself as “an immense accumulation of commodities,” its unit being a single commodity. Our investigation must therefore begin with the analysis of a commodity.” [K. Marx, opening words of ***Capital***, volume **I**.]. That analysis is as follows: **....** The foregoing analysis of a representative unit of this category of “**Commodities**”, an analysis that ***un***packs its ***im***plicit content, thus already leads to a detailed grasp/explanation of much of our experience of our modern social system in which, and as part of which, we live.

Q2. ¿Did that analysis of the “**Commodities**” category completely exhaust our experience and knowledge of our existing system of societal self-reproduction, or are there other categories whose content is ***not*** explicitly covered by that of the category of “**Commodities**”, whose analysis could expand our comprehension of our system of social life, and of the range of its human, social phenomena that we have experienced**?**

A2. The “**Commodities**” category, taken alone, leaves out any explicit coverage of a key part of our experience, that has to do with **money**. The category of ‘‘‘**Monies**’’’ comprehends explicitly a vast portion of our experience of our present system that is ***not*** captured explicitly by the “**Commodities**” category. The analysis of the category of ‘‘‘**Monies**’’’ is as follows: **....** The foregoing analysis of the content implicit in the “social relation of production” category of ‘‘‘**Monies**’’’ thus accomplishes a further advance in our detailed, scientific grasp/explanation of our experience of life within our present system of self-reproduction of human-social life.

Q3. ¿Are “**Commodities**” and ‘‘‘**Monies**’’’ just two ***absolutely separate***, and, in some ways, diametrically, qualitatively ***opposite*** categories of ‘human socio-economic ontology’**?** ¿Or do these two categories interrelate, and even ***combine***, in descriptions of our human-society self-reproductive process that are adequate to our experience and knowledge thereof**?**

A3. “**Yes**” they are different, even qualitatively ***opposite***, categories. But also “**No**”, they are ***not*** absolutely separate or separable means of grasping our experience of modern society. Indeed, the two categories are inseparable in accurate-to-experience descriptions of the buying and selling [value-exchange, ‘⇋’] activities which form a core component of our “raw” experience and fragmentary knowledge as to how our present system of human-social life reproduces itself, continues itself, extends itself to fill the “space” of advancing historical time. “**Commodities**” are sold for ‘‘‘**Monies**’’’. Those ‘‘‘**Monies**’’’ are then often used to buy other “**Commodities**”. These “**Commodities**” are then often used to produce other “**Commodities**”. Those other “**Commodities**” are then typically sold for [other] ‘‘‘**Monies**’’’, and those ‘‘‘**Monies**’’’ are then often used, again, to buy other “**Commodities**”. We have “**Commodities**”, whose place is taken by ‘‘‘**Monies**’’’, whose place is then taken again by “**Commodities**”, whose place is then taken by ‘‘‘**Monies**’’’, whose place is then taken again by “**Commodities**”, or: **… ⇋ C ⇋ M ⇋ C’ … C’’ ⇋ M’ ⇋ C’’’…**. The ‘‘‘***complex unity***’’’, or ‘‘‘***dialectical synthesis***’’’, of these ***opposite*** categories of “**Commodities**” and ‘‘‘**Monies**’’’ is the ***category*** which describes the ongoing society-reproductive process of monies-facilitated commodity exchange, ‘**The Monies-Mediated Circulations of Commodities**’.

Q4. ¿Do these three 'social relations of societal self-re-production' categories of "**Commodities**" ['«***arché***» **thesis**'], "**Monies**" ['**first *contra*-thesis**'], and "**Monies-Mediated Circulations of Commodities**" ['**first *uni*-thesis**'], taken together, exhaust our "raw" experience and fragmentary knowledge of our existing system of human-social life reproduction, capturing and comprehending all that is going on inside this system -- inside our own socio-economic lives -- i.e., all of its recurrent human-social phenomena**?** ¿Or, are there still other categories, so far left out of our explicit account, that are needed to provide a more complete picture/scientific explanation of this human-social [sub-]totality in which, and as which, we live; which we '''personify'''**?**

A4. These three categories taken together and mutually 'super[im]posed', and the analyses that unpack their implicit content, do further advance our detailed, scientific explanation of our experience of our present-day socio-economic life. However, they still leave out fundamental aspects of that experience, the experience of what is going on inside the present system of our social lives and of the daily social activity of those lives' reproduction. Indeed, they still leave out of explicit account the category of "**Capitals**" itself -- the category of those special "**Monies**" which are ***not*** just consumed, to buy those kinds of commodities that are used up to continue/reproduce the availability, and the ability to work, of worker-consumers, but of "**Monies**" which are "**invested**" so that they may "turn a [**money**-]**profit**"; "**Monies**" that "make ***more***" "**Monies**". Some of the "**Monies**" used to "**Circulate Commodities**" are '''**Monies**-***Capitals***''', ***not*** subsistence "**Monies**". Some of the "**Commodities**" that are "**Circulated**" by those '''**Monies**-***Capitals***''' are ***not*** subsistence "**Commodities**", but are '''***Commodities*-*Capitals***''', "***Commodities***" that may be sold for "**Money**-***Profits***", over and above what they cost to produce. Indeed, the bulk of our present-day global human socio-economic process of human-societal self-reproduction can be described as a usually-profit-yielding '''**Circulation of Commodities**-***Capitals* mediated by Monies**-***Capitals***''', an alternation of "**Commodities**" and "**Monies**" in the form of an alternation of '"**Commodities**-***Capitals***''' and '"**Monies**-***Capitals***'''. We must thus now engage the analysis of these ***Capitals***-category sub-categories as well: **....** The foregoing analyses of these "**Capitals**" sub-categories -- these categories named '"***Capitals*-*in*-*General***''', **&** '"***Commodities*-*Capitals***''', **&** '"***Monies*-*Capitals***''' and their overall socio-economic process category, named '''***The Self-Mediated Circulations of the Total Social Capital***''' -- have leaped our detailed grasp and scientific explanation of the socio-economic phenomena that we have recurrently experienced in the system of socio-economy that we live today, far ahead of where we began, with the relatively ***simple***, ***abstract*** category of "**Commodities**".

Q5. ¿OK**?** We have arrived at a 'qualitative sum', a '***non***-amalgamative ***cumulum***', a '***non***-reductionist super[im]position' of **7** social relations categories -- "**Commodities**", "**Monies**", '''**Monies-Mediated-Circulations-of-Commodities**''', '"**Capitals-in-General**''', '"**Commodities-Capitals**''', '"**Monies-Capitals**''', and '''**The Self-Circulations of the Total Social Capital**''', the latter as a categorial description of the socio-economic process of our contemporary global human societal self-re-production as a whole. ¿Do these seven categories amount to an adequate, complete account of the system of 'socio-[political-]economy' in which and as which we live**?** ¿Or (is)(are) there (an)other categor(y)(ies) that (is)(are) needed to contain/provide a fuller explanation**?**

A5. The '''***dialectical analysis***''' of these ***seven*** categories comprehends a great deal about our present system of human-societal self-reproduction. But these categories are still deficient in many ways, falling far short of a full theorization of the content of a '''synchronic slice''', or cross section "cut", through our society of today. For the present purpose, we will finish by focusing upon just one of these deficiencies, but one that constitutes, in a very important way, the ***final*** deficiency of a categorial progression that constitutes a '''***systematic*-*dialectical***''' theory-***presentation***: the transition from the self-reproduction to the 'self-revolutionization' and self-transcendence of such a system. On the contrary, in the Platonian and Hegelian ***ideologies*** of '"Systematic Dialectics''', such a transition has no place. After all, unless the '''synchronic slice''' in question, the 'time-width' of the '''slice''' that forms the locus of ***systematic presentation***, and our experience and knowledge of life within that '''slice''', happens to fully encompass the period of revolutionary transition from ***predecessor system*** to ***successor system***, that transition is outside the locus of explanation and ***presentation***, and the phenomena of that transition are outside of our "raw", lived experience and of our fragmentary knowledge. However, the Marxian tradition of '"Systematic Dialectics''' is founded upon a recognition of the transient, transitory, and transitional character of all natural formations, including of all 'human-natural', human-social formations. Any '''synchronic slice''', or "cross-section cut, through/from human history", is thus recognized, in the Marxian tradition of ***dialectic***, as merely a synchronic «***in vitro***» extraction/abstraction from the ***essential*** '''diachronicity''' and '''historicity''' of reality. An account of the predicted transition from the ***predecessor*** to the ***successor*** system must thus form the final phase for the Marxian ***method of presentation***. This final phase thus does, however, still involve historical ***extrapolation***, from ***present*** empirical experience and experiential knowledge, to beyond it, to a ***dialectically***-inferred, ***not***-[yet-]***present historical future***. It involves transitioning the ***presentation*** from the '***re***-construction' of its ***past*** up to its ***present*** form, to the '***pre***-construction' and to the '***present***[iz]-ation' of the ***future***' -- to scientific ***prediction***.

Marx described this 'proto-***historical-dialectical***', 'quasi-***diachronic***', final phase in his '***systematic/synchronic-dialectical method of theory-presentation***' in the following terms: **"**On the other side, much more important for us is that our method indicates the points where historical investigation must enter in, or where bourgeois economy as a merely historical form of the production process points beyond itself [M.D.: on this side, pointing in the ***direction*** of the history ***before itself***] to earlier historical modes of production. In order to develop the laws of bourgeois economy, therefore, it is not necessary to write the *real history of the relations of production*. But the correct observation and deduction of these laws, as having themselves become in history, always leads to primary equations -- like the empirical numbers, e.g. in natural science -- which point towards the past living behind this system. These indications, together with a correct grasp of the present, then also offer the key to the understanding of the past -- a work in its own right which, it is to be hoped, we shall be able to undertake as well. This correct view likewise leads at the same time to the points at which the suspension [M.D.: i.e., the '***self-«aufheben»-ation***'] of the present form of the production-relations gives signs of its becoming -- foreshadowings of the future [M.D.: thus, on this, opposite, side, pointing in the ***opposite direction***, that of the inferably ***impending*** history ***after itself***, i.e., via points whereat the ***present*** form of the [social-self-re-]production process points ***beyond itself*** on/to the/***its aft***[***er***] ***side***]. Just as, on the one side the pre-bourgeois phases appear as merely historical, i.e. suspended [M.D.: as '***self-«aufheben»-ated***'] presuppositions, so do contemporary conditions of production likewise appear as engaged in *suspending themselves* [M.D.: i.e., as engaged in '***self-«aufheben»-ating' themselves***] and hence in positing the *historic presuppositions* for a new state of society.**"** [K. Marx, ***Grundrisse***, M. Nicolaus, transl., Pelican [London: **1972**], pp. **460**-**461**, *emphasis* per original].

There are ways in which our experience of the existing system of socio-politico-economics points ***beyond itself***, points to ***after itself***, to ***beyond*** the existence of this ***existing*** system; foreshadows the ***possible*** self-induced breakdown of this system; its internal, immanent 'self-transitioning' *from* a 'self-organizing system' *to* a 'self-***dis***-organizing system', **&** *to* a 'self-***re***-organizing system'. Such foreshadowings include the recurring ***global economic crises***, of ever-increasing amplitude -- such as the ***global crisis*** that began in **~1907**, the *global* "***Great Depression***" ***crisis*** that began in **~1929**, the ***Global Wars*** -- **WWI** and **WWII** -- that followed the **1907** and **1929** ***global crises***, and the "***Great Recession***" ***global crisis***, that began in **~2008** -- and also the quiet global expansion of producers' cooperatives, i.e., of worker-owned industrial capital; of 'workers' capitalism', that may all point ***beyond*** the self-reproduction of the capital-based system of socio-political economy, to its gradual, and then sudden, self-***non***-reproduction; to the production, from out of it, of a new system, e.g., of a higher system, one that solves some of the problems of humanity-self-devastating ***global crises***, of ***global wars***, and of ***global de facto multi-genocides***, with which the present system ***inherently plagues itself***. Or, which may point to a self-induced breakdown of the existing global system that eventuates in a ***new global war*** **&**/or in a ***collapse*** into a ***New***, ***Final*** **Dark Age**: '''mutual ruin of the contending classes'''. [Karl Marx and Frederick Engels, ***The Communist Manifesto***.]. Thus we need to consider, finally, a category of "**Transition**", of the 'self-transitioning' of the ***present*** system into something else; a category of the ***self-critique*** of the category of the presently predominant social relation of production; a category of the theoretical ***and practical immanent***, ***or self-, critique*** of "***the Capital-relation***" [Marx].

Caveat. This dialogic model of the content of Marx's three volumes of «***Das Kapital***» is, like all models, no more than a '''homeomorphism'', a '''many-to-one''' abbreviation, or mapping, of the object that it models. In particular, this model addresses, explicitly, only the '''value-form(s)''' content of Marx's work, which includes Chapters ***I.***, ***III.***, **&** ***IV.*** of volume **I**, much of volume **II**, **&** only a little of volume **III**. It omits especially the production-centered parts of volumes **I**, **II**, **& III**, which require a deeper «***arché***» -- one even deeper than the "**Elementary or Accidental Form of Value**" «***arché***» addressed in the first '***meta-model***' of «***Das Kapital***» exposited herein, above. The present, dialogic model covers most of volume **I** up to Chapter ***V.***, entitled by Marx "Contradictions in the General Formula of Capital", which addresses the primary «***aporia***» of all modern political-economics, the paradox of, and the mystery of, the sustaining source of the profit on [industrial] capital, and Marx's solution to that «***aporia***», his ***theory of surplus-value***. This 'dialogue-model' does ***not*** address, in particular, in volume **I**, the Part on the production of absolute surplus-value, the Part on the production of relative surplus-value, the Part on the production of absolute and of relative surplus-value, or, in volume **III**, the Parts on the "lawful" fate of the capital-profit-rate, given its source in surplus-value, i.e., the parts entitled "The Conversion of Surplus-Value into Profit...", "The Conversion of Profit Into Average Profit", "The Law of the Tendency of the Rate of Profit to Fall", "The Division of Profit into Interest and Profit of Enterprise", and "Transformation of Surplus-Value into Ground-Rent". This abbreviated 'dialogue-model' of «***Das Kapital***», because of its very 'abbreviated-ness', is useful, given the familiarity of its categories, for illustrative purposes regarding this essay's main '***meta-model***', given its similarities to, as well as its contrasts with, that ''***meta-model***'. The 'derivation-presentation' of a far fuller ***dialectical*** '***meta-model***' of Marx's ***Capital***, flowing -- '''following''' by 'connotational entailment' -- from an «***arché***» far deeper than are either of those explored in this essay, is planned to be the focus of a forthcoming separate work.

$\vdash .\ {}^{2}_{h}\mathfrak{g}_{KMC}^{\text{«}\underline{K}\text{»}} = {}^{3}_{h}\mathfrak{g}_{maN}^{\#}$ [Asserting [$\vdash .$] an analogy [$=$] between the two terms indicated]: In a very remote, 'dialectical-structural' way, the **7**th term in the solution of the «***Das Kapital***», or «***K***», model-equation, as described above, is analogous to [' $=$ '] the term ${}^{3}_{h}\mathfrak{g}_{maN}^{\#}$. The ${}^{2}_{h}\mathfrak{g}_{KMC}^{\text{«}\underline{K}\text{»}}$ term represents the ***epitome*** of the '''*real subsumption*''' of all priorly-posited categories of exchange-value social-relations-of-production, by the «*K*apital» social-relation-of-production category, ${}^{2}_{h}\underline{K}_{\text{«}\underline{K}\text{»}}$, which, 'unioned'/blended/combined with all of the other exchange-value social-relations-of-production categories' meanings for all of the priorly-arisen terms, forms/expresses/defines the zenith of the global system of competing individual [industrial] capitals, and the completion of the development-to-organic-totality of that system, call it ${}^{2}_{h}\mathbf{K}_{\text{«}\underline{K}\text{»}}$. The ${}^{3}_{h}\mathfrak{g}_{maN}^{\#}$ term is also the **7**th term, the one which represents the '''*real subsumption*''' of all priorly-arisen number-kinds -- i.e., of kinds **a** and **N** -- by the "minus" number-kind, the **m**, in the solution of the systematic '*meta-model meta-equation*' for the "***Standard Arithmetics***", described in the sequel. That latter term, 'unioned'/blended/combined with all of the number-meanings of all of the other, priorly-arisen kind-of-number terms, represents the completion of the axioms-system of the *integers* arithmetic, ${}^{3}_{h}\underline{\mathbb{Z}}_{\#}$. This is admittedly a very distant analogy, one that crosses over the boundaries between '*Meta-Systematic-Dialectic*' and '*Systematic-Dialectic*', and between the "pure" '*ideo-dialectic*' of the **#** domain of discourse, and the far more 'socially-exo-empirically-grounded', 'ideo-*physio-dialectic*' of the «***K***» domain of discourse, i.e., between taxonomy level **3** and taxonomy level **2** of the *E.D.* Universal Taxonomy, for the **h** domain. The *general* nature of such analogies as this will be revealed in the paragraphs immediately following. For each **s**tage $\mathbf{s}_{\#} > \mathbf{0}$ of the '*Systematic-Dialectical*' *method of presentation* of our '"*Standard Numbers*"'-Systems domain of discourse, **#**, the new 'contra-thesis' category of number-kind that first emerges into 'explicitude', and into '''*formal subsumption*''' of all already-emerged kinds of number, in and by that **s**tage, has ordinality $\mathbf{2}^{\mathbf{s}_{\#}}$. That is, that 'contra-thesis' category is represented by the '$(\mathbf{2}^{\mathbf{s}_{\#}})$th' term of the progression, the term that *speci*fically ***interprets*** the *gene*ric **q**ualifier term, $\mathfrak{q}_{\mathbf{2}^{\mathbf{s}_{\#}}}$, for **s**tage $\mathbf{s}_{\#}$, for the *speci*fic meaning-realm of the domain of discourse being systematized /-taxonomized, and being explicated / theorized. In the case of this essay, the domain of discourse is denoted by **#**, and is that of the '"*Standard Numbers*"' and their '"*Standard Arithmetics*"'. Each such **s**tage thus begins the addition/accretion of a new "kind of number" epithet/determination, whose ordinal number is $\mathbf{2}^{\mathbf{s}_{\#}}$. For each **s**tage $\mathbf{s}_{\#} > \mathbf{1}$ of this '*Meta-Systematic-Dialectical*' *method of presentation* of our system of '"*Standard Numbers*"', that new epithet/determination term is always preceded by a 'grand synthesis' term, or 'full uni-thesis' term. That term combines all of the epithets/determinations emerged prior to **s**tage $\mathbf{s}_{\#}$ in its subscript. That term represents the fullness, the zenith term, of the '''*real subsumption*''', *by* the $(\mathbf{2}^{\mathbf{s}_{\#}-\mathbf{1}})$th term 'contra-thesis' number-kind '''predicate''', *of* all number-kind predicates that emerged prior to ***its*** emergence. The ordinal number of this 'full synthesis', '''*real subsumption*''', zenith term is $(\mathbf{2}^{\mathbf{s}_{\#}} - \mathbf{1})$. That term ***interprets*** the *gene*ric **q**ualifier term, $\mathfrak{q}_{\mathbf{2}^{\mathbf{s}_{\#}}-\mathbf{1}}$, for **s**tage $\mathbf{s}_{\#}$, for the *speci*fic meaning-realm being modeled, here **#**. We term this $(\mathbf{2}^{\mathbf{s}_{\#}} - \mathbf{1})$th term the "*epitome*", and the '*culminant*', of the expanded number-space, and of the typically net-expanded axioms-system, of the $(\mathbf{2}^{\mathbf{s}_{\#}-\mathbf{1}})$th kind of number, whose arithmetic's axioms-system *presentation* is completed in **s**tage $\mathbf{s}_{\#}$. So, for **s**tage $\mathbf{s}_{\#} = \mathbf{2}$ --

$${}^{3}_{h}\mathfrak{H}^{\#}_{\mathbf{2}} = \left({}^{3}_{h}\mathbf{N}_{\#}\right)^{2} = \left({}^{3}_{h}\mathbf{N}_{\#}\right)^{4} = {}^{3}_{h}\mathbf{N}_{\#} \oplus {}^{3}_{h}\mathbf{a}_{\#} \oplus {}^{3}_{h}\mathfrak{g}_{\mathbf{aN}}^{\#} \oplus {}^{3}_{h}\mathbf{m}_{\#}$$

-- the '$(2^{s_\#})$th' = '(2^{2})th' term -- the 4th term, $\overset{3}{h}\underline{m}_{\#}$ -- is the new '*contra-thesis*' term for the $s_\# = 2$ stage's emergent "minus" kind of number, and the '$(2^{s_\#} - 1)$th' = '$(2^{2} - 1)$th' term -- the 3rd term, $\overset{3}{h}\mathfrak{g}_{aN}^{\#}$ -- is the '*culminant*', or "*epitome*", term for the '*contra-thesis*' epithet a, just prior to that of m, and which combines, in its subscript(s), all of the [in this case, 2] kind-of-number epithet's already extant since stage 0, plus up through stage 1, just prior to stage 2, namely, in this case, the "Natural" numbers' epithet / determination / predicate, N, already extant since stage 0, and the 'aught' numbers' epithet / determination / predicate, a, already extant since stage 1. For stage $s_\# = 3$ --

$$\overset{3}{h}\underline{\mathfrak{H}}_{3}^{\#} = \left(\overset{3}{h}\underline{N}_{\#}\right)^{2^3} = \left(\overset{3}{h}\underline{N}_{\#}\right)^{8} =$$

$$\overset{3}{h}\underline{N}_{\#} \oplus \overset{3}{h}\underline{a}_{\#} \oplus \overset{3}{h}\mathfrak{g}_{aN}^{\#} \oplus \overset{3}{h}\underline{m}_{\#} \oplus \overset{3}{h}\mathfrak{g}_{mN}^{\#} \oplus \overset{3}{h}\mathfrak{g}_{ma}^{\#} \oplus \overset{3}{h}\mathfrak{g}_{maN}^{\#} \oplus \overset{3}{h}\underline{f}_{\#}$$

-- the '$(2^{s_\#})$th' = '(2^{3})th' term -- the 8th term, $\overset{3}{h}\underline{f}_{\#}$ -- is the new '*contra-thesis*' term for the $s_\# = 3$ stage's emergent "fractional" kind of number, and the '$(2^{s_\#} - 1)$th' = '$(2^{3} - 1)$th' term -- the 7th term, $\overset{3}{h}\mathfrak{g}_{maN}^{\#}$ -- is the '*culminant*', or "*epitome*", term for the '*contra-thesis*' epithet m, just prior to that of f. This term combines, in its subscript(s), all of the [in this case, 3] kind-of-number epithet's already extant since stages 0 and 1, plus up through stage 2, just prior to stage 3, namely, in this case, the "Natural" numbers' epithet / determination / predicate, N, already extant since stage 0, the 'aught' numbers' epithet / determination / predicate, a, already extant since stage 1, and the "minus" numbers' epithet / determination / predicate, m, already extant since stage 2.

These '$(2^{s_\#} - 1)$th' terms always contain, and end in, the subscripted epithet / determination / predicate N. This means that these terms all pertain to the original, «*arché*», '''core''', '''analytical-geometrical''' region of every successively-determined / 'explicitized' "standard" number-space, and arithmetical axioms-system, evoked and presented in our 'meta-model' for #. I.e., each '$(2^{s_\#} - 1)$th' term -- e.g., $\mathfrak{g}_{aN}^{\#}$ and $\mathfrak{g}_{maN}^{\#}$ -- connotes the way(s) in which the '$(2^{s_\# - 1})$th' 'contra-thesis' kind of number characteristic acts mentally to modify / redefine / add to the meaning of the core / '«*arché*»-ic' '''analytical-geometrical''' [sub-]domain of number space --

I II III

. . . ...

-- and thus portrays each successive "new" in the context of the old*est* and *most familiar* of the "old", thus characterizing and epitomizing the effect and the import of the "new", '$(2^{s_\# - 1})$th', 'ideo-ontology' of number overall. Each '$(2^{s_\#} - 1)$th' term registers the conceptual transformation(s) of the «*arché*» '''*ground*'''/core of the numbers-system, required by the $(2^{s_\# - 1})$th term 'contra-thesis' number-kind, i.e., by the immediately previous step's new concept of number, in order for that new concept of number to fully subsume the «*arché*» '''*ground*'''/core of ***all*** arithmetics. This transformation, that each new 'contra-thesis' element of new number 'ideo-ontology' etches into this core of 'numberdom', is a "litmus test", a 'character-ization', an "*epitome*", of everything that that new 'contra-thesis' means -- an "*epitome*" for the new 'ideo-ontology', and of the new 'ideo-technology', of the resulting new, qualitatively expanded 'number-space', for its new, qualitatively expanded axiomatic system of arithmetic, and for its new, qualitatively-expanded definition of [counting] number as a whole.

Overall, the Seldon Function here describes a '''relay''', from each given «***aporia***» to the next.

We therefore 'pre-solve' for all of the $(2^{s_\#}-1)$th terms, for $s_\# > 2$ for our # 'meta-model', to be presented in the core section of this essay, here, even before entering upon that core section, as follows below. Note: in the 'pre-solutions' given below, we use the '''number-space''', or "number-set", ***syntax*** of a given arithmetical axioms-sub-system, *gene*rically expressed by/as *lower*-case ${}^{3}_{h}x_\#$, to characterize the ***meaning*** of its corresponding *kind of number* and *kind of arithmetic* 'conceptual ontology', as specified by its axioms-system, connoted *gene*rically by *upper*-case ${}^{3}_{h}X_\#$. The symbol '|' is used below to replace the phrase "such that".

- $\mathbb{Q}_3 \in \exists\, {}^{3}_{h}\mathfrak{g}_{aN}^{\#} \equiv {}^{3}_{h}w_\# \mid w \equiv aN \equiv \{0...01, 0...02, 0...03, ...\}$, intimating the core of what ${}^{3}_{h}W_\#$ means, 'explicitizing' the number 0 as universal *placeholder* $[(2^{s_\#-1}) - 1 = (2^{3-1}) - 1 = (2^{2}) - 1 = 3]$;
- $\mathbb{Q}_7 \in \exists\, {}^{3}_{h}\mathfrak{g}_{maN}^{\#} \equiv {}^{3}_{h}z_\# \mid z \equiv maN \equiv \{+0...01, +0...02, +0...03, ...\}$, intimating the core of what ${}^{3}_{h}Z_\#$ means, 'explicitizing' "*sign*" ['+' or '–'], i.e., '*co-linear bi-directionality*' $[(2^{s_\#-1}) - 1 = (2^{4-1}) - 1 = (2^{3}) - 1 = 8 - 1 = 7]$;
- $\mathbb{Q}_{15} \in \exists\, {}^{3}_{h}\mathfrak{g}_{fmaN}^{\#} \equiv {}^{3}_{h}q_\# \mid q \equiv fmaN \equiv \{(+0...01/+0...01), (+0...02/+0...01), ...\}$, intimating the core of what ${}^{3}_{h}Q_\#$ means, re-conceptualizing and re-expressing ***all*** earlier-emerged number-kinds as **numerator/denominator fractions** $[(2^{s_\#-1}) - 1 = (2^{5-1}) - 1 = (2^{4}) - 1 = 16 - 1 = 15]$;
- $\mathbb{Q}_{31} \in \exists\, {}^{3}_{h}\mathfrak{g}_{dfmaN}^{\#} \equiv {}^{3}_{h}r_\# \mid r \equiv dfmaN \equiv \{(+0...01.0...), (+0...02.0...), ...\}$, intimating the core of what ${}^{3}_{h}R_\#$ means, re-conceptualizing and re-expressing ***all*** earlier-emerged number-kinds as decimal *fract*[*ion*][*al*]*s*, ***either*** with *potentially-infinitely-repeating* digits [including *potentially-infinitely-repeating* 0s] in the *decimal places* to the right of the *decimal point*, ***or*** with *potentially-infinitely-NON-repeating* digits in the *decimal places* to the right of the *decimal point*, with decimally-scaled self-similarity in *those fract*[*ion*][*al*] *decimal places* $[(2^{s_\#-1}) - 1 = (2^{6-1}) - 1 = (2^{5}) - 1 = 32 - 1 = 31]$;
- $\mathbb{Q}_{63} \in \exists\, {}^{3}_{h}\mathfrak{g}_{idfmaN}^{\#} \equiv {}^{3}_{h}c_\# \mid c \equiv idfmaN \equiv \{((+0...01.0...)r + (+0...0.0...)i), ...\}$, intimating the core of what ${}^{3}_{h}C_\#$ means, re-conceptualizing and re-expressing ***every number*** -- even the 'ideo-fossil' of the "**N**atural" numbers inside the "**R**eal" numbers' "number-*line*" -- as 're-placed', or as explicitly located, inside the **C** "number-*plane*", i.e., as '''re-contextualized''' to a context wherein every number has an **i**-dimension [as well as a "**r**eal"-dimension, "***perpendicular***" or "***orthogonal***" to that **i**-dimension], even if that **i**-component [and/or that **r**-component] is 'quantified' by 0; "zeroed-out"; "null" $[(2^{s_\#-1}) - 1 = (2^{7-1}) - 1 = (2^{6}) - 1 = 64 - 1 = 63]$.

Given the individual '*culminant*' term solutions/definitions stated above, we can now characterize each of the $(2^{s_\#-1})$th terms as distinct «*species*» of ***dialectical*** or «***aufheben***» ***operator***, as follows.

Note: The syntactics of the representation of these incremental number-kinds 'ideo-neo-ontologies', or 'conceptual neo-ontologies increments', including those of their sub-axioms-systems, axioms-sub-systems, or axioms-system-components, and those of their included number-sub-spaces, get to be a bit of a "bear", as can be seen already in the definitions of the later '***culminant epitomes***' above. But, if you "bear with" this exposition, you will come to see ***just how exacting*** an "*in*exact", "rule-of-thumb", "heuristic", '*in*tensional-*in*tuitional' -- if ***dialectical*** -- analysis / solution / delineation can really be, by using ${}^{3}_{h,N}\mathbf{Q}_{\#}$. Expressed as 'operatorial' elements, and, in particular, as 'determinate-*negator*'-*cum*-'*preservor*'-*cum*-'meta-fractal-scale-*raisor*' ['«***aufheben***»-***ator***'] operators, these 'contra-thesis' terms that produce the '***culminant epitome***' terms defined above, by acting upon / interacting with their predecessor-**s**tage's '***culminant epitome***' terms, can be characterized as follows --

- $\amalg_{2} \in\!\!\dashv {}^{3}_{h}\mathbf{a}_{\#} \equiv {}^{3}_{h}\mathcal{G}_{\mathbf{a}}^{\#}$, as in ${}^{3}_{h}\mathbf{a}_{\#} \otimes {}^{3}_{h}\mathbf{N}_{\#} \equiv {}^{3}_{h}\mathcal{G}_{\mathbf{a}}^{\#} \otimes {}^{3}_{h}\mathcal{G}_{\mathbf{N}}^{\#} = {}^{3}_{h}\mathcal{G}_{\mathbf{N}}^{\#} \oplus {}^{3}_{h}\mathcal{G}_{\mathbf{aN}}^{\#} \equiv {}^{3}_{h}\mathbf{N}_{\#} \vdash\!\!\oplus\!\!\dashv {}^{3}_{h}\mathbf{W}_{\#}$,

  is contained in ['$\sqsubset$'] ${}^{3}_{h}\mathcal{H}^{\#}_{1}$, and ever-after. It signifies the operation of '**a*ught*-*ization***' of all priorly-***presented*** arithmetical axioms-system-component terms, the explicit incorporation, and 'semantification', of '***self***-subtraction' operations into all priors by means of the '**a**ught numbers' idea, the 'explicitization' of the number '**0**' as "placeholder", and as a full number in its own right, in the conceptualization [«***begrifflichkeit***»], and in the expression/syntactic-representation, of all preceding concepts of number and of arithmetic signified by each term upon which this operator operates, as in the above example, where it '«***aufheben***»***s***' ${}^{3}_{h}\mathbf{N}_{\#}$, ***firstly***, by determinately-***negating*** it -- by ***negating*** its '***no*** non-counts as numbers' negative determination -- by adding determination **a**, **&**, ***secondly***, by ***elevating*** ${}^{3}_{h}\mathbf{N}_{\#}$ into a richer context/scale of number-meaning(s), while, ***thirdly***, also thereby ***preserving*** ${}^{3}_{h}\mathbf{N}_{\#}$ within that richer, higher context $[(2^{s_{\#}-1}) = (2^{2-1}) = (2^{1}) = 2]$;

- $\amalg_{4} \in\!\!\dashv {}^{3}_{h}\mathbf{m}_{\#} \equiv {}^{3}_{h}\mathcal{G}_{\mathbf{m}}^{\#}$, as in ${}^{3}_{h}\mathcal{G}_{\mathbf{m}}^{\#} \otimes {}^{3}_{h}\mathcal{G}_{\mathbf{aN}}^{\#} = {}^{3}_{h}\mathcal{G}_{\mathbf{aN}}^{\#} \oplus {}^{3}_{h}\mathcal{G}_{\mathbf{maN}}^{\#} \equiv {}^{3}_{h}\mathbf{W}_{\#} \vdash\!\!\oplus\!\!\dashv {}^{3}_{h}\mathbf{Z}_{\#}$,

  is $\sqsubset {}^{3}_{h}\mathcal{H}^{\#}_{2}$, and ever-after. It signifies the operation of '***sign-ization***', or of '***co-linear bi-directional-ization***, or of '***integer-ization*** , of all priorly- ***presented*** arithmetical axioms-system-component terms, the explicit incorporation and 'semantification' of the ***un***restricted subtraction operation into all priors, by means of the 'explicitization' of number-line [bi-]directionality, into the conceptualization, and into the expression/-syntactic-representation, of all preceding concepts of number and of arithmetic signified by each term upon which this operator operates, e.g., '«***aufheben***»***ing***' the '***cumulum***', ${}^{3}_{h}\mathbf{W}_{\#}$, the '**W**hole numbers' context, up into the **Integers**' context; from ${}^{3}_{h}\mathcal{H}^{\#}_{2}$ to ${}^{3}_{h}\mathcal{H}^{\#}_{3}$ $[(2^{s_{\#}-1}) = (2^{3-1}) = (2^{2}) = 4]$;

- $\amalg_{8} \in\!\!\dashv {}^{3}_{h}\mathbf{f}_{\#} \equiv {}^{3}_{h}\mathcal{G}_{\mathbf{f}}^{\#}$, as in ${}^{3}_{h}\mathcal{G}_{\mathbf{f}}^{\#} \otimes {}^{3}_{h}\mathcal{G}_{\mathbf{maN}}^{\#} = {}^{3}_{h}\mathcal{G}_{\mathbf{maN}}^{\#} \oplus {}^{3}_{h}\mathcal{G}_{\mathbf{fmaN}}^{\#} \equiv {}^{3}_{h}\mathbf{Z}_{\#} \vdash\!\!\oplus\!\!\dashv {}^{3}_{h}\mathbf{Q}_{\#}$,

  is $\sqsubset {}^{3}_{h}\mathcal{H}^{\#}_{3}$, and ever-after. It signifies the operation of '***fraction-ization***', or of '***ratio-ization***', of all priorly-***presented*** arithmetical axioms-system-component terms, the explicit incorporation and 'semantification' of the ***un***restricted division operation [*except* still always restricting from division by **0**] into all priors, by means of the 'explicitization' of '***division-ization***', into the conceptualization, and into the expression/-syntactic-representation, of all preceding concepts of number and of arithmetic signified by each term upon which this operator operates, thus '«***aufheben***»***ing***' each such term from the context of the **Integers** into the context of the '"***Ratio***-nal"' numbers; from ${}^{3}_{h}\mathcal{H}^{\#}_{3}$ into ${}^{3}_{h}\mathcal{H}^{\#}_{4}$ $[(2^{s_{\#}-1}) = (2^{4-1}) = (2^{3}) = 8]$;

• $\sqsubset_{16} \hookleftarrow {}^{3}_{h}\mathbf{d}_{\#} \equiv {}^{3}_{h}\mathbf{q}_{\mathbf{d}}^{\#}$, as in ${}^{3}_{h}\mathbf{q}_{\mathbf{d}}^{\#} \otimes {}^{3}_{h}\mathbf{q}_{\mathbf{fmaN}}^{\#} = {}^{3}_{h}\mathbf{q}_{\mathbf{fmaN}}^{\#} \oplus {}^{3}_{h}\mathbf{q}_{\mathbf{dfmaN}}^{\#} \equiv {}^{3}_{h}\mathbf{q}_{\#} \vdash\!\oplus\!\dashv {}^{3}_{h}\mathbf{r}_{\#}$'

is $\sqsubset {}^{3}_{h}\underline{)\!\!H\!\!(}^{\#}_{4}$, and ever-after. It signifies the operation of '***sometimes approximate indefinite decimal-ization***', or of '***sometimes approximate indefinite decimal representation***', of all priorly-*presented* arithmetical axioms-system-component terms, the explicit incorporation and 'semantification' of the ***un***restricted root-extraction [***except*** still restricting root extractions for values of **d** numbers < **0**], and of **S**tern-**B**rocot **T**ree approximations of selected "transcendental **R**eal numbers", into all priors, by means of the 'explicitization' of '*ir-ratio*[*n*][*-al*]*-ization*', the explicit incorporation and 'semantification', for both repeating and ***non***-repeating **d**ecimal representations, of ***potentially***-infinite right-of-**d**ecimal-point digit-sequences of **d**ecimally-scaled self-similar, '*fract*[ion]*al*-parts' into the conceptualization, and into the expression/syntactic-representation, of all preceding concepts of number and of arithmetic signified by each term upon which this operator operates, thus '«***aufheben***»***ing***' each such term from the **Rational** numbers context up into the "**Real**" numbers context; from ${}^{3}_{h}\underline{)\!\!H\!\!(}^{\#}_{4}$ into ${}^{3}_{h}\underline{)\!\!H\!\!(}^{\#}_{5}$ $[(2^{s_{\#}-1}) = (2^{5-1}) = (2^{4}) = 16]$;

$\sqsubset_{32} \hookleftarrow {}^{3}_{h}\mathbf{i}_{\#} \equiv {}^{3}_{h}\mathbf{q}_{\mathbf{i}}^{\#}$, as in ${}^{3}_{h}\mathbf{q}_{\mathbf{i}}^{\#} \otimes {}^{3}_{h}\mathbf{q}_{\mathbf{dfmaN}}^{\#} = {}^{3}_{h}\mathbf{q}_{\mathbf{dfmaN}}^{\#} \oplus {}^{3}_{h}\mathbf{q}_{\mathbf{idfmaN}}^{\#} \equiv {}^{3}_{h}\mathbf{r}_{\#} \vdash\!\oplus\!\dashv {}^{3}_{h}\mathbf{c}_{\#}$'

is $\sqsubset {}^{3}_{h}\underline{)\!\!H\!\!(}^{\#}_{5}$, and ever-after. It signifies the operation of '***complex-ization***', or of '***i-ization***', or of $\sqrt{-1}$ -based '**2-D-*ization***', or of the $\sqrt{-1}$-based '***plane-ization***' -- the «***aufheben***» '''re-contextualization''', of all extant kinds of number and kinds of arithmetic, from the $\mathbf{R}^{1}$ "number-*line*" context to the $\mathbf{C}^{1}$ '''number-*plane*''' context -- of all priorly-*presented* arithmetical axioms-system-component terms, the explicit incorporation and 'semantification' of ***un***restricted square-root extraction from ***negative*** "**Real**" numbers, into all priors, via 'explicitization' of **i** 'convolute qualifier' components, in the conceptualization, and in the expression/syntactic-representation, of all preceding concepts of number and of arithmetic signified by each term upon which this operator operates, thus '«***aufheben***»***-ing***' each such term from the "**Real**" numbers context up into the "**Complex**" numbers context; from ${}^{3}_{h}\underline{)\!\!H\!\!(}^{\#}_{5}$ into ${}^{3}_{h}\underline{)\!\!H\!\!(}^{\#}_{6}$ $[(2^{s_{\#}-1}) = (2^{6-1}) = (2^{5}) = 32]$.

Inventory of Knowns. As of our adding-in/registering/taking into account the considerations presented above, we have already, in this essay so far, and even before entering into its core section, solved our ${}^{3}_{h}\mathbf{Q}^{\mathbf{N}}_{\#}$-algebraic '***dialectical meta-model meta-equation***' -- our "purely"-qualitative '***meta-model***' -- of the ***progression*** of the [axioms-]systems of the "***Standard Arithmetics***", for the following **12** of the **64** terms that it generates up to its **s**tep/**s**tage **6** --

(1.) For the **1**st, or «***arché***», term, ${}^{3}_{h}\mathbf{N}_{\#} \mapsto \sqsubset_{1}$, of ordinal number $2^{0} = 1$: ***given*** / 'pre-stipulated';

(2.) For all of the **6** 'contra-thesis' terms, for **s**teps $s_{\#} = 1$ through $s_{\#} = 6$, ordinal numbers $2^{s_{\#} > 0}$:

(2.a) ${}^{3}_{h}\mathbf{a}_{\#} \mapsto \sqsubset_{2}$, the '**a**ught numbers', accreting ***zero*** as a number, ordinal number $2^{1} = 2$;

(2.b) ${}^{3}_{h}\mathbf{m}_{\#} \mapsto \sqsubset_{4}$, the '**m**inus numbers', accreting ***signed directionality*** of numbers, ordinal number $2^{2} = 4$;

(2.c) ${}^{3}_{h}\mathbf{f}_{\#} \mapsto \sqsubset_{8}$, the '**f**ractional numbers', accreting ***exact parts*** of integral numbers, ordinal number $2^{3} = 8$;

(2.d) ${}^{3}_{h}\mathbf{d}_{\#} \mapsto \sqsubset_{16}$, '**SBT** approximators' of '''**d**iagonal, *line*-eal numbers''' ["irrational numbers", "algebraic" & "transcendental"], accreting ***never-finitely-fully-specifiable/approximated fragment-parts*** of integral/whole numbers, ordinal number $2^{4} = 16$;

(2.e) ${}^{3}_{h}\mathbf{i}_{\#} \mapsto \sqsubset_{32}$, '**2-*dimensional***, ***plane-ular numbers***', adding a **2**nd number-dimension, ordinality $2^{5} = 32$;

(2.f) ${}^{3}_{h}\mathbf{h}_{\#} \mapsto \sqsubset_{64}$, '**4-D**, ***hyper-cube-al numbers***', adding **3**rd & **4**th number-dimensions, ordinality $2^{6} = 64$;

(3.) For all of the 5 '*epitomic culminant*' terms, for steps $s_\# = 2$ through $s_\# = 5$, ordinal numbers $(2^{s_\#} - 1)$:

(3.a) ${}^3_h\mathcal{Q}_{\mathbf{aN}}^{\#} \equiv {}^3_h\underline{\mathbf{w}}_\# \ \text{E}\!\!\rightarrow\!\!\ni \ \mathbb{I}_{\llcorner 3}$, ordinal number $(2^2 - 1) = 3$;

(3.b) ${}^3_h\mathcal{Q}_{\mathbf{maN}}^{\#} \equiv {}^3_h\underline{\mathbf{z}}_\# \ \text{E}\!\!\rightarrow\!\!\ni \ \mathbb{I}_{\llcorner 7}$, ordinal number $(2^3 - 1) = 7$;

(3.c) ${}^3_h\mathcal{Q}_{\mathbf{fmaN}}^{\#} \equiv {}^3_h\underline{\mathbf{q}}_\# \ \text{E}\!\!\rightarrow\!\!\ni \ \mathbb{I}_{\llcorner 15}$, ordinal number $(2^4 - 1) = 15$;

(3.d) ${}^3_h\mathcal{Q}_{\mathbf{dfmaN}}^{\#} \equiv {}^3_h\underline{\mathbf{r}}_\# \ \text{E}\!\!\rightarrow\!\!\ni \ \mathbb{I}_{\llcorner 31}$, ordinal number $(2^5 - 1) = 31$;

(3.e) ${}^3_h\mathcal{Q}_{\mathbf{idfmaN}}^{\#} \equiv {}^3_h\underline{\mathbf{c}}_\# \ \text{E}\!\!\rightarrow\!\!\ni \ \mathbb{I}_{\llcorner 63}$, ordinal number $(2^6 - 1) = 63$;

Count of Remaining Unknowns. Since we have already solved-for -- since we know the 'this-meta-model-specific' meanings of -- the '$(2^{s_\#})$th' and '$(2^{s_\#} - 1)$th' terms, for steps $s_\# = 0$ through $s_\# = 6$, the remaining task of solution to be accomplished in the next -- core -- section of this essay can be characterized, quantitatively, as follows. In each stage/step $s_\# \mid s_\# \in \mathbf{W}$, we have a maximum of $2^{s_\#} - 2^{s_\#-1}$ terms to solve for. Because $2^{-1} = 1/2^{+1} = ½$, we have also that $2^{s_\#-1} = 2^{s_\#} \times 2^{-1} = 2^{s_\#} \times ½ = 2^{s_\#}/2$. Thus, $2^{s_\#} - 2^{s_\#-1} = 2^{s_\#} - 2^{s_\#}/2 = 2^{s_\#}/2 = 2^{s_\#-1}$. So, we have, maximally, $2^{s_\#-1}$ new terms to solve for in stage/step $s_\#$ of our '*meta-model*'. However, in this essay so far, we have already been given the meanings of, or have already solved for the meanings of, two kinds of terms, the '$(2^{s_\#})$th' and '$(2^{s_\#} - 1)$th' kinds of terms, for each stage, $s_\#$. Thus, we have only $(2^{s_\#-1} - 2)$ terms to solve for in each stage/step $s_\#$. Therefore, taking stock of our remaining solutions-task sage-by-stage, we have, in --

- stage **0**. $2^{0-1} - 2 = 2^{-1} - 2 = ½ - 2 = -3/2 \notin \mathbf{W}$. Therefore we have **no** terms yet to solve for in stage **0**.
- stage **1**. $2^{1-1} - 2 = 2^{0} - 2 = 1 - 2 = -1 \notin \mathbf{W}$. Therefore we have **no** terms yet to solve for in stage **1**.
- stage **2**. $2^{2-1} - 2 = 2^{1} - 2 = 2 - 2 = 0 \in \mathbf{W}$. Therefore we have **0** terms yet to solve for in stage **2**.
- stage **3**. $2^{3-1} - 2 = 2^{2} - 2 = 4 - 2 = 2 \in \mathbf{W}$. Therefore we have **2** terms yet to solve for in stage **3**.
- stage **4**. $2^{4-1} - 2 = 2^{3} - 2 = 8 - 2 = 6 \in \mathbf{W}$. Therefore we have **6** terms yet to solve for in stage **4**.
- stage **5**. $2^{5-1} - 2 = 2^{4} - 2 = 16 - 2 = 14 \in \mathbf{W}$. Therefore we have **14** terms yet to solve for in stage **5**.
- stage **6**. $2^{6-1} - 2 = 2^{5} - 2 = 32 - 2 = 30 \in \mathbf{W}$. Therefore we have **30** terms yet to solve for in stage **6**.

Thus, most of our solution-effort -- most of the applications of the '***Organonic Algebraic Method***', the procedure specified earlier in the present, background, section -- will be concentrated, in the next, core, section, in the last stages of the presentation -- in stages **6**, **5**, **4**, **&** **3**, and, especially, most of all, in stage **6**.

B.μ. The *Systematic Dialectic* -- or 'Trans-Platonian' «*Arithmos Eidetikos*» -- of the *Kinds* of *Opposition*, according to Musès and Seldon. Both the '''***Systematic Dialectics***''', and the '***Meta-Systematic Dialectics***', of ***dialectical theory-presentation*** are about ***opposition***, and its ***resolution***. ¿But about what ***kind*** of ***opposition*****?** About this question, we observe that much confusion prevails in public discourse. The purpose of this, final section of the background section, Part **I**., is to assist the reader, if afflicted with any part of this prevailing public confusion, to clear it up for herself or for himself.

Public discourse seems to fix more emphasis upon a kind of '***necessarily co-existent***', 'polar-***complementary***', '***mutually***-completing', and "symbiotic" '***opposite-ness***', such as female-vs.-male, plant-vs.-animal, or North-Pole-vs.-South-Pole of a bar magnet.
Dr. Charles Musès, one of the early mentors of **The Foundation**, had called attention also to a conceptually-neglected, and, indeed, '"***opposite***"', category of "***opposition***", in these terms: "...Whereas before, we have a multitude of natural and mutually complementing pairs like female/male, day/night, finite/infinite, white/black, *et al*., now we have the additional possibility of pathological, host/parasite pairs like good/evil, honesty/deception, health/sickness, in which we have no longer two self-completing entities, both of which are needed in the scheme of things. Rather, we now have pairs of which only one is needed for well-being, the other being parasitic (*not* symbiotic) and actually inimical to it. The Pythagoreans, misunderstanding their Egyptian teachers, placed the host/parasite duality of good and evil (hence also sickness) on the same footing as the quite different class of benign, self-complementary duals of finite/infinite, male/female, *et al*. And later philosophers, both oriental and occidental (e.g., Carl Jung), have repeated that fundamental error, stemming from inaccurate perception that failed to make the distinction between two radically different kinds of opposites: those which are wave-like and mutually complementary; and the later, pathological variety. . .where one of the pair parasitizes on the other and, attacking it, attempts to destroy it permanently. The grip of the ancient error in the human mind is evidenced by the fact that this fundamental distinction was taught in no university philosophy course of the twentieth century as of 1983. Indeed, Jung's confused "coincidence of opposites" continues to be parroted." [Charles Musès, ***Destiny and Control in Human Systems***: ***Studies in the Interactive Connectedness of Time*** (***Chronotopology***), Kluwer-Nijhoff Publishing [Boston: **1985**], pp. **136-137**].

The examples of matter vs. anti-matter, and of opposing armies joining battle, provide further example cases of this ***other*** «***species***», this '***contra***' ***kind***, of ***opposition*** -- "***contra***" to the ***second*** «***species***», the one called '***complementary opposition***'-- the ***first*** «***species***» that Dr. Seldon came to name '***annihilatory opposition***'.

In early dialogues with Dr. Musès, Dr. Seldon insisted upon the salient existence of yet a ***third*** basic «***species***» of the «***genos***» of '***oppositenesses***', a ***dialectical synthesis*** of the previous two, which Dr. Seldon named '***supplementary opposition***', and '***progressive opposition***'. ***Supplementary-opposite*** pairs are, ***initially at least***, diachronically-related, with one both pre-existing, and giving birth to, the other, even if they become, *later*, after that birth, and after some maturation, ***mutually*** symbiotic and ***co-existent***, and are typically also connected together by an «***aufheben***» process.

Example cases of '***supplementary opposition***' include those of the ***kind*** of ***opposition*** that exists between temporal, historical ***predecessor*** and ***successor*** systems, and between other kinds of ***predecessor***/***successor*** pairs, e.g., between an «***arithmos***» made up out of «***monads***» and its successor '***meta***-«***arithmos***»' made up out of the '***meta***-«***monads***»' of those «***monads***», successions in which the ***successor*** '***quanto-qualitatively***' ***exceeds*** and ***supersedes*** its ***predecessor*** -- e.g., parents vs. children, ***atom units*** vs. ***multi-atom molecule units***; ***city-state units*** vs. ***multi-city-state empire units***; ancient and medieval, pre-capitalist, "antediluvian" [Marx] forms of ***Monies***, vs. ***Money-Capitals***, and the human-social system of ***capitalism*** vs. its **F.*E*.*D*.**-predicted ***successor***-system, that of '***political-economic democracy***', etc.

The 'micro-temporal', 'ordinally-sequentially-related', '***Qualo***-Peanically-related', «***aufheben***»-related '***consecua***' of the presentational progressions of '''***Systematic Dialectics***''', and of '***Meta-Systematic Dialectics***', also inhere in this ***third*** category of ***opposition***.

Supplementary opposition is the «***species***» for '***meta-monadologically***'-related, consecutive «***arithmoi***», that is, for '***meta-fractally***'-related, similarity-linked consecutive '***quanto-qualitative scales***' of 'ideo-ontology'/'physio-ontology'.

Perhaps needless to say, after the definitional descriptions and exemplifications set forth immediately above, all of the '***aporial***' ***oppositions*** encountered in our core '***meta-systematic dialectical***' exposition of '***The Gödelian Dialectic of the Standard Arithmetics***', commencing next, are cases of this ***third*** «***species***»; of '***supplementary opposition***'.

Thus, the system $\mathbf{W}_{\#}$ is a '***supplementary opposite***' ['$\vdash\!\oplus\!\dashv$'] of $\mathbf{N}_{\#}$, $\mathbf{Z}_{\#}$ a '***supplementary opposite***' ['$\vdash\!\oplus\!\dashv$'] of

$\mathbf{W}_{\#}$, $\mathbf{Q}_{\#}$ a '***supplementary opposite***' ['$\vdash\!\oplus\!\dashv$'] of $\mathbf{Z}_{\#}$, $\mathbf{R}_{\#}$ '$\vdash\!\oplus\!\dashv$' $\mathbf{Q}_{\#}$, $\mathbf{C}_{\#}$ $\vdash\!\oplus\!\dashv$ $\mathbf{R}_{\#}$, and $\mathbf{H}_{\#}$ $\vdash\!\oplus\!\dashv$ $\mathbf{C}_{\#}$, ...

Note, in the sequel, that the axioms-systems given explicitly for these successive/progressive ***Standard Arithmetics*** are ***mostly*** «***aufheben***»-***conserved*** in the transition from ***predecessor*** to ***successor*** axioms-system, also with 'ideo-ontologically' new, '''***supplementary***''' ***axioms*** being added, to create that ***successor system***. But note, also, that ***some*** of the ***predecessor system***'s axioms are "crossed out" in these representations, because they no longer exist or apply at the new 'ideo-ontological level' of the '***supplementary system***'.

Likewise, in the human-social domain, one would expect a ***progressive*** capitalist system to «***aufheben***»-carry-forward/subsume many features of its predecessor human-social system, such as the merchant-capital and usurers' capital 'antediluvian' forms of capital, such as the "monies" socio-ontology, and the "commodities" human 'socio-ontology', etc., but ***not*** serf labor and slave labor.

This latter observation, together with the former, will afford you a sense of the '''***complex unity***''' of '***annihilatory opposition***' and '***complementary, non-annihilatory opposition***', that constitutes '***supplementary opposition***'.

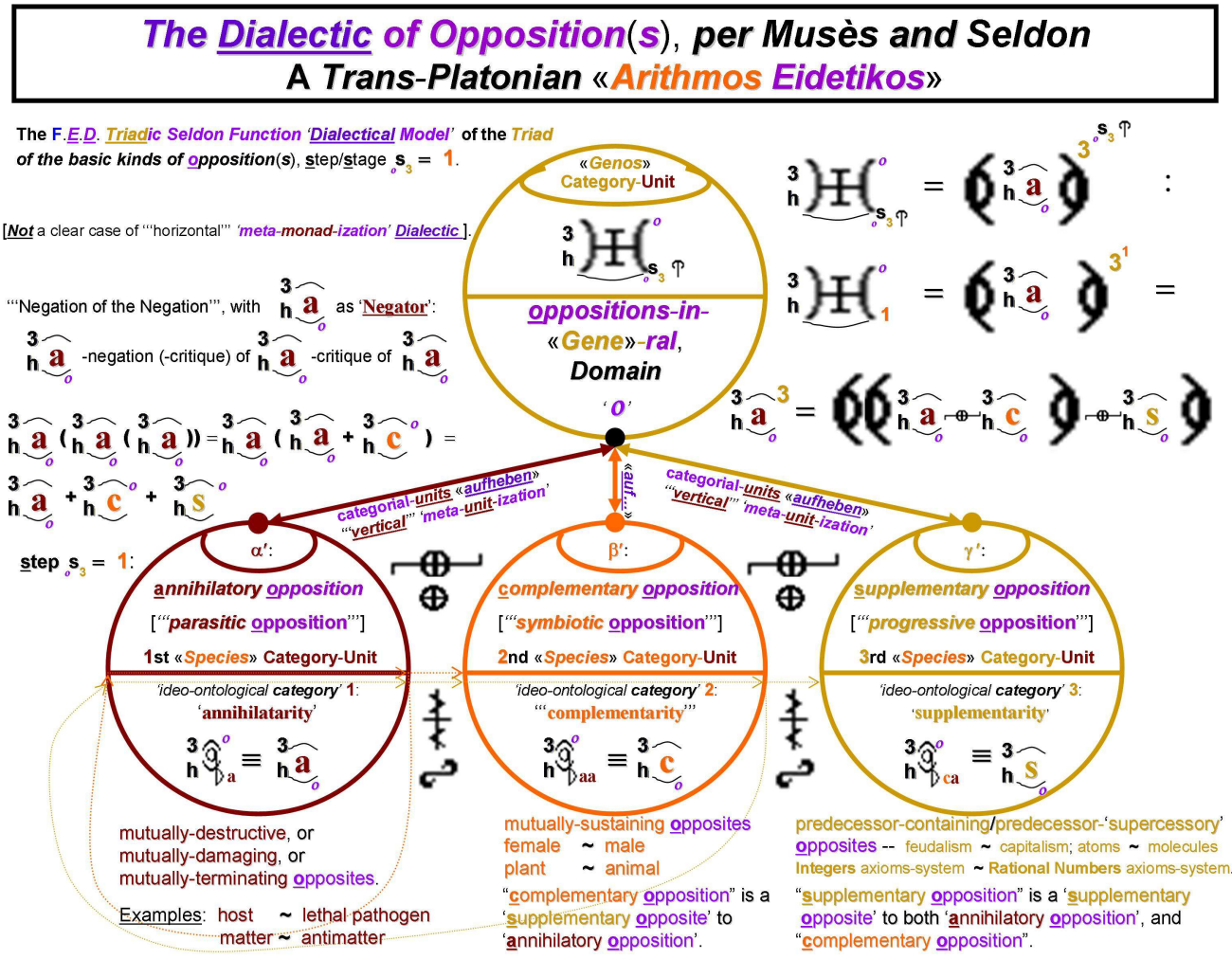
The diagram below is set forth to summarize the above-narrated, ***four***-«***eide***»-«***monads***» ***assemblage*** [«***arithmos***»], or trans-Platonian ***four-fold*** «***arithmos eide-tikos***», that summarizes our '***systematic dialectic of oppositions***' -- both 'formulaically', using the '***Triadic Seldon Function***', as of $s_0 = 1$, and '[picto]graphically' -- as yet a further example of '***dialectical*** [***ideo***-]***systematics***', or of '***dialectical ideo-taxonomics***'.

Three '''extra credit''' questions --

¿In what «***species***» does the '''***opposition***''' of '***annihilatory opposition***' vs. '***complementary opposition***' inhere**?**

¿The '''***opposition***''' of '***annihilatory opposition***' vs. '***supplementary opposition***'**?**

¿The '''***opposition***''' of '***supplementary opposition***' vs. '***complementary opposition***'**?**



* Although ***not*** emphasized in this text, F.E.D's approach to arithmetics, and to analysis, is a '"practical"' and '"pragmatic"' -- 'practice-able' -- one; is what we term '''Realistic''', and 'Meta-Finitistic'. That is, while we admit qualitative -- 'ideo-ontological' -- differences, and qualitative -- 'ideo-ontological' -- changes, into our accounts of arithmetics and of analysis, we do ***not***, in accord with our '***Principle of Metafinity***', admit "**infinities**", as they are '''***non***-constructible''' and '***contra***-empirical' -- '''***un***-Realistic'''.

Thus, while the cardinal number of "**N**atural numbers" may be ***potentially*** infinite, in something like Aristotle's sense, the set **N** never actually constitutes a constructed, presently "actually infinite" set. We use the symbol $\pitchfork$ to represent, e.g., the largest "**N**atural" number that can be represented by the digital computer that we are using as a key instrumentality in a given project.

Thus, for example, the function '$\mathbf{SBT}_{\pitchfork}(x)$' calls for the best approximation of the **S**tern-**B**rocot **T**ree "best rational approximations" of the "irrational" number **x** that can be expressed by / within the digital computer in question, given its $\pitchfork$ limitation.

Likewise, '$\mathbf{AP}_{\pitchfork}(x)$' calls for any "**A**lgebraic **P**olynomial" function of **x** -- with rational coefficients and "**N**atural" number exponents -- that can be represented within the same $\pitchfork$ limitation, e.g., given the limits which that limitation imposes upon the values of any such polynomial function's coefficients, exponents, and number of terms.

Our '''Realistic''' definition of the "irrational" "**R**eal" numbers involves all of the computer's '''representable''' approximations of the **S**tern-**B**rocot **T**ree's "best rational approximations" of the "algebraic irrational numbers" that can be constructed by the digital computer in question, within its $\pitchfork$ limitation, "unioned" with the '$\pitchfork$-representable' versions of the **S**tern-**B**rocot **T**ree "best rational approximations" for a ***finite selection*** of just the project-***needed*** "transcendental irrational numbers", e.g., those for **e** and **π**.

The expression '$x \notin \aleph^{w}C$' means that **x** is ***not*** an element of the set of the "**C**omplex" numbers, **C**, [i.e., when the exponent of $\aleph$ is $0 \in W$], or of any 'ideo-ontologically' higher **Standard** number-set [i.e., when the exponent of $\aleph$ is $0 < w \in W$] -- that **x** is ***not*** a "transcendental number" in any such 'trans-**Real numbers**' sense.