F.E.D. Preface to E.D. Brief #7, on the $_{Z}Q$, by Guest Author "J2Y"

by *Hermes de Nemores*, General Secretary to the F.E.D. General Council

Commentary on E.D. Brief #7. Our new guest author, known pseudonymously as "Joy-to-YoU", and whom I shall reference herein, using the nickname with which he often references himself in our correspondence -- "J2Y" -- has provided to you, our readers, *a new and highly-accessible* «***entrée***» into the ***third stage*** within Q, the 'meta-system' of the F.E.D. '***First Dialectical Arithmetics***': namely, into the $_{Z}Q$ axioms-system of ***dialectical arithmetic***, with its *core* set, or space, of ***dialectical***, '*Integer-based, or* **Z***-numbers-based, purely-qualitative* ***meta-numbers***' --

$$_{Z}Q \equiv \{\ldots, q_{-3}, q_{-2}, q_{-1}, q_{\pm 0}, q_{+1}, q_{+2}, q_{+3}, \ldots\}.$$

This new Brief, ***E.D. Brief #7***, caps a *trilogy* of Briefs prepared for you by J2Y, since late June **2012**, on the $_{N}Q$, the $_{W}Q$, and the $_{Z}Q$ ***dialectical arithmetics***, **&** their *exotic* arithmetical/algebraic '*ideo-ontology*' and '*ideo-phenomenology*'.

In each Brief of this trilogy, J2Y has alluded to the rising degree of "definiteness" -- of '''determinate-ness''', or of 'features-richness' -- expected to grow with every transition from term to '***Qualo***-Peanic' successor term in a ***dialectical categorial progression***, including in a ***dialectical*** [axioms-]***systems progression***, such as the one that J2Y has presented for you in his last three Briefs. We think the contents of these Briefs themselves provide specific "*self* evidence" of -- i.e., in themselves provide instantiation of, **&** data supporting -- this expectation regarding ***dialectical progression*** in general.

J2Y's ***Brief #5***, on the $_{N}Q$ system of ***dialectical arithmetic***, required **7** pages of text to achieve a satisfactory degree of specificity regarding that ***first*** system. His ***Brief #6***, on the $_{W}Q$ system, required **8** pages of content to satisfactorily cover the new features -- the $\triangle_{N}Q = {}_{A}Q$ incremental new '*ideo-ontology*' -- of that ***second*** system. His ***Brief #7***, on the $_{Z}Q$ system, took **19** pages of text to adequately address the new '*ideo-phenomena*' -- the $\triangle[{}_{N}Q \oplus {}_{A}Q]$ incremental new '*ideo-ontology*' -- of that ***third*** system. The escalation from **7** units to **8** units to **19** units -- using page units of expository text as a crude proxy for the 'features-richness' being exposited thereby and therein -- exhibits the kind of acceleration of "definiteness" to which J2Y often alluded therein.

What J2Y has accomplished for you, in ***E.D. Brief #7***, is to develop a *single* new '''*idea-object*''', denoted C_{Z}, with which he shows how to *co-generate*, in a coordinated way, key new features of the $_{Z}Q$ axioms-system, which are not ["yet"] extant in the $_{N}Q$ axioms-system, or even in the $_{W}Q$ axioms-system. He does so by way of subsuming, into a "pure-**q**ualifiers" arithmetic, the "*purely-quantitative*" arithmetic of the Standard Integers, the new kind of ["*signed*"] numbers contained in the set, or space --

$$Z \equiv \{\ldots -3, -2, -1, \pm 0, +1, +2, +3, \ldots\}$$

-- *vis-à-vis* the **W** and the **N** number-spaces, showing how to ***unify*** some of the *amazingly* novel characteristics of the $_{Z}Q$ axioms-system of "purely-**q**ualitative", ***dialectical arithmetic***.

These novel features of $_{Z}\underline{Q}$, *vis-à-vis* $_{W}\underline{Q}$, and $_{N}\underline{Q}$, as well as *vis-à-vis* other, "standard", arithmetics, include --

1. Continuation of the "identity" of the ***additive*** *identity element* with the ***multiplicative*** *identity element*, which first emerged, as q_0, in $_{W}\underline{Q}$, now in the form of $q_{\pm 0}$, in $_{Z}\underline{Q}$: $\underline{q}_z + q_{\pm 0} = \underline{q}_z = \underline{q}_z \times q_{\pm 0} = \underline{q}_z + q_{\pm 0} + \underline{q}_{z \pm 0}$ $= \underline{q}_z + \underline{q}_z = \underline{q}_z$ [using the **F**.*E*.*D*. ***'meta-genealogical evolute product'*** rule for $_{Z}\underline{Q}$ multiplication]; ...

2. Now with the added twist, in $_{Z}\underline{Q}$, for the first time, that ***additive*** *inverses* and ***multiplicative*** *inverses* are equal as well:

$$\underline{q}_{+z} + \underline{q}_{-z} = q_{\pm 0} = \underline{q}_{+z} \times \underline{q}_{-z} = \underline{q}_{+z} + \underline{q}_{-z} + \underline{q}_{(+z)+(-z)} = q_{\pm 0} + q_{\pm 0} = q_{\pm 0};$$

3. Equivalent expressions of $_{Z}\underline{Q}$, generated by "revolving" signs around the $\underline{q}$ symbol as center, e.g., counter-clockwise:

$$-\underline{q}_{+z}^{+1} = +\underline{q}_{-z}^{+1} = +\underline{q}_{+z}^{-1};$$

4. The emergence, in $_{Z}\underline{Q}$, for the first time, of what might have been expected to "wait" until $_{Q}\underline{Q}$, namely, of '$\underline{q}$ualifier fractions' with both '$\underline{q}$ualifier numerators' and of '$\underline{q}$ualifier denominators', and thus also of '$\underline{q}$ualifier ratios', and of the '$\underline{q}$ualifier division' operation, as a partial inverse operation of the $_{Z}\underline{Q}$ '$\underline{q}$ualifier multiplication' operation, viz. --

- for all **z** in **Z**: $\underline{q}_{+z} = \underline{q}_{+z}/q_{\pm 0} = q_{\pm 0}/\underline{q}_{-z}$; $\underline{q}_{-z} = \underline{q}_{-z}/q_{\pm 0} = q_{\pm 0}/\underline{q}_{+z}$, including $-q_{\pm 0} = +q_{\pm 0} = \pm q_{\pm 0}/\pm q_{\pm 0}$;
- for all **z** in **Z**: $\underline{q}_z/\underline{q}_z = [\underline{q}_z]^{+1} \times [\underline{q}_z]^{-1} = [\underline{q}_z]^{-1} \times [\underline{q}_z]^{+1} = [\underline{q}_z]^{\pm 0} = q_{\pm 0}$, including $[q_{\pm 0}]^{\pm 0} = q_{\pm 0}$;
- for all **z** in **Z**: $[\underline{q}_{+z}/q_{\pm 0}] \times [q_{\pm 0}/\underline{q}_{+z}] = [\underline{q}_{+z}/\underline{q}_{+z}] = [\underline{q}_z]^{+1-1} = [\underline{q}_z]^{-1+1} = [\underline{q}_z]^{\pm 0} = q_{\pm 0}$;
- for all **j**, **k** in **Z**: $[\underline{q}_k/\underline{q}_j]^{-1} = [\underline{q}_j/\underline{q}_k]^{+1} = \underline{q}_{+j} + \underline{q}_{-k} + \underline{q}_{+j-k}$; $[\underline{q}_j/\underline{q}_k]^{-1} = [\underline{q}_k/\underline{q}_j]^{+1} = \underline{q}_{+k} + \underline{q}_{-j} + \underline{q}_{+k-j}$;
- for all **z** in **Z**: $-1 \times \underline{q}_{+z} = \underline{q}_{-z}$; $-1 \times \underline{q}_{-z} = \underline{q}_{+z}$; $+1 \times \underline{q}_{+z} = \underline{q}_{+z}$, & $+1 \times \underline{q}_{-z} = \underline{q}_{-z}$;
- for all **z** in **Z**: $\pm 0 \times \underline{q}_z = q_{\pm 0}$, so $0\underline{q}_z = \underline{q}_z^{\,0} = q_0$.

For the **F**.*E*.*D*. research collective, this ***dialectical arithmetic***, $_{Z}\underline{Q}$, the ***third*** step in the *'meta-systematic meta-evolution'* ***within*** the **F**.*E*.*D*. ***'First Dialectical Arithmetics'***, $\underline{Q}$, has always -- ever since it first emerged in our research -- held, for us, a feeling of particularly acute ***irony*** for the progression ***inside*** $\underline{Q}$. On one hand, the $_{Z}\underline{Q}$ arithmetic presents some of the most astounding arithmetical *'ideo-phenomena'* we had ever encountered, as glossed above. On the other, because it models especially the ***second*** «***species***» in the «***species***»-***dialectic*** ***inside*** the «***genos***» category of "***opposition***" --

complementary opposition $\rightarrow$ ***annihilatory opposition*** $\rightarrow$ ***supplementary opposition***

-- namely, the ***annihilatory*** kind, all of those *astounding* features "go to waste" for most ***'meta-modeling'*** uses. That is, assigning the «***arché***» ontological category of a ***dialectical categorial progression*** to either $\underline{q}_{-1}$ or $\underline{q}_{+1}$, in a Seldon Function, generates two equivalent progressions, one in which all of the generic $\underline{q}$ualifiers in the generic progression have positive signs, the other in which all of the generic $\underline{q}$ualifiers have negative signs. One thus might as well stay with $_{W}\underline{Q}$ for model building, as using $_{Z}\underline{Q}$ in this way offers no enrichment over $_{W}\underline{Q}$ modeling. Combining both «***arché***», as --

$$\underline{|-|-|}_h = [\underline{q}_{-1} + \underline{q}_{+1}]^{2^h} = [q_{\pm 0}]^{2^h} = q_{\pm 0}$$

-- in the generic Seldon Function produces something even worse: the value of the Seldon Function for all epoc$\underline{h}$s, **h**, is the same, namely $q_{\pm 0}$, signifying a total '''*de*-manifestation''' of *all* ontology for *all* time. This yields only '''nihilist''' 'meta-models' of the universe, and of its sub-universes, for which we have little use. That's where J2Y's new, alternative version of $_{Z}\underline{Q}$, which he notates by $_{Z^*}\underline{Q}$, may come in. Its 'contra-axiomatization' of $\vdash . \underline{q}_{*+z} + \underline{q}_{*-z} \ddagger q_{*0}$, in place of our $\vdash . \underline{q}_{+z} + \underline{q}_{-z} = q_{\pm 0}$, may avert the "mutually annihilatory" propensity of our $\underline{q}_z$ in his $\underline{q}_{*z}$, making the later more suitable for the formulation of more useful ***dialectical*** ***'meta-models'***. We are investigating this possibility, with J2Y, right now.

Background for ***E.D.*** ***Brief #7***. **F.*E*.*D***. presents the systems-progression of the ‘*Gödelian Dialectic*’ of the axioms-systems of the standard arithmetics, in their first-order-and-higher-logics’ axiomatizations, in accord with a Dyadic Seldon Function ‘*meta-model*’, which describes -- ideographically, and “purely-*qualitatively*” -- a ‘*Meta-Systematic Dialectical*’ *order-of-presentation*, and *dialectical method-of-presentation*, of those successive systems of arithmetic. Using the notational convention that, if **X** denotes the standard number-space, or number-set, of a given kind of standard number, then that symbol, with a single underscore, $\underline{\mathbf{X}}$, will be used to denote its first-*and-higher-order*-logic axiomatization, we have that this **F.*E*.*D***. *order-of-presentation* can be expressed as follows, using $\underline{\#}$ as a tag for the total «*genos*» of the standard arithmetics, comprehending all of its «*species*», in the following, progressive ordering --

$$\underline{\mathbf{N}}_{\underline{\#}} \ni \underline{\mathbf{W}}_{\underline{\#}} \ni \underline{\mathbf{Z}}_{\underline{\#}} \ni \underline{\mathbf{Q}}_{\underline{\#}} \ni \underline{\mathbf{R}}_{\underline{\#}} \ni \underline{\mathbf{C}}_{\underline{\#}} \ni \ldots$$

-- and the Dyadic Seldon Function-based ‘*dialectical meta-model*’ which generates that progression is --

$$\underline{\mathrm{H}}_{\underline{\mathrm{S}}_{\underline{\#}}} \uparrow = \left(\underline{\mathbf{N}}_{\underline{\#}}\right)^{2^{\underline{\mathrm{S}}_{\underline{\#}}}} \uparrow .$$

Connected with the above-rendered *order-of-presentation*, **F.*E*.*D***. presents the *dialectical progression* of the particular «*species*» of *first*-order-logic-*only* axiomatized *dialectical arithmetics* [denoted generically by $\underline{\mathbf{X}}$], that reside ‘‘‘inside’’’ the «*genos*» of **F.*E*.*D***.’s $\underline{\mathbf{Q}}$ ‘*First Dialectical Arithmetics* meta-system’, in a corresponding order --

$${}_{\mathrm{N}}\underline{\mathbf{Q}}_{\underline{\#}} \ni {}_{\mathrm{W}}\underline{\mathbf{Q}}_{\underline{\#}} \ni {}_{\mathrm{Z}}\underline{\mathbf{Q}}_{\underline{\#}} \ni {}_{\mathrm{Q}}\underline{\mathbf{Q}}_{\underline{\#}} \ni {}_{\mathrm{R}}\underline{\mathbf{Q}}_{\underline{\#}} \ni {}_{\mathrm{C}}\underline{\mathbf{Q}}_{\underline{\#}} \ni \ldots$$

-- using $\underline{\#}$ as a tag for the total «*genos*» of the **F.*E*.*D***. ***non***-standard, *Dialectical Arithmetics*. The Dyadic Seldon Function-based ‘*dialectical meta-model*’ which generates that progression is --

$$\underline{\mathrm{H}}_{\underline{\mathrm{S}}_{\underline{\#}}} \uparrow = \left({}_{\mathrm{N}}\underline{\mathbf{Q}}_{\underline{\#}}\right)^{2^{\underline{\mathrm{S}}_{\underline{\#}}}} \uparrow .$$

In ***E.D.*** ***Brief #5***, J2Y gave you his able **&** novel derivation of the ${}_{\mathrm{N}}\underline{\mathbf{Q}}$, basing the *first* ***Stage*** of the *dialectic* within $\underline{\mathbf{Q}}$!
In ***E.D.*** ***Brief #6***, he provided his innovative derivation of the ${}_{\mathrm{W}}\underline{\mathbf{Q}}$, basing the *second* ***Stage*** of the *dialectic* inside $\underline{\mathbf{Q}}$!!
In ***E.D.*** ***Brief #7***, he now presents for you a pathway to the ${}_{\mathrm{Z}}\underline{\mathbf{Q}}$, basing the *third* ***Stage*** of the *dialectic* of the $\underline{\mathbf{Q}}$!!!
What J2Y has done is to illuminate a first **3** steps of the *vast* ***E.D.*** ‘*double-dialectic*’ / ‘*bi-directional dialectic*’ --

$$\begin{array}{ccccccccccccc}
\underline{\mathbf{N}}_{\underline{\#}} \ni & \underline{\mathbf{N}}_{\underline{\#}} \oplus {}_{\mathrm{N}}\underline{\mathbf{Q}}_{\underline{\#}} \ni & \underline{\mathbf{N}}_{\underline{\#}} \oplus {}_{\mathrm{N}}\underline{\mathbf{Q}}_{\underline{\#}} \oplus {}_{\mathrm{N}}\underline{\mathbf{U}}_{\underline{\#}} \oplus {}_{\mathrm{N}}\underline{\mathbf{M}}_{\underline{\#}} \ni & \underline{\mathbf{N}}_{\underline{\#}} \oplus {}_{\mathrm{N}}\underline{\mathbf{Q}}_{\underline{\#}} \oplus {}_{\mathrm{N}}\underline{\mathbf{U}}_{\underline{\#}} \oplus {}_{\mathrm{N}}\underline{\mathbf{M}}_{\underline{\#}} \oplus {}_{\mathrm{N}}\underline{\mathbf{q}}_{\mathrm{MN}}^{\underline{\#}} \oplus \ldots \ni \ldots \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\
\underline{\mathbf{W}}_{\underline{\#}} \ni & \underline{\mathbf{W}}_{\underline{\#}} \oplus {}_{\mathrm{W}}\underline{\mathbf{Q}}_{\underline{\#}} \ni & \underline{\mathbf{W}}_{\underline{\#}} \oplus {}_{\mathrm{W}}\underline{\mathbf{Q}}_{\underline{\#}} \oplus {}_{\mathrm{W}}\underline{\mathbf{U}}_{\underline{\#}} \oplus {}_{\mathrm{W}}\underline{\mathbf{M}}_{\underline{\#}} \ni & \underline{\mathbf{W}}_{\underline{\#}} \oplus {}_{\mathrm{W}}\underline{\mathbf{Q}}_{\underline{\#}} \oplus {}_{\mathrm{W}}\underline{\mathbf{U}}_{\underline{\#}} \oplus {}_{\mathrm{W}}\underline{\mathbf{M}}_{\underline{\#}} \oplus {}_{\mathrm{W}}\underline{\mathbf{q}}_{\mathrm{MW}}^{\underline{\#}} \oplus \ldots \ni \ldots \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\
\underline{\mathbf{Z}}_{\underline{\#}} \ni & \underline{\mathbf{Z}}_{\underline{\#}} \oplus {}_{\mathrm{Z}}\underline{\mathbf{Q}}_{\underline{\#}} \ni & \underline{\mathbf{Z}}_{\underline{\#}} \oplus {}_{\mathrm{Z}}\underline{\mathbf{Q}}_{\underline{\#}} \oplus {}_{\mathrm{Z}}\underline{\mathbf{U}}_{\underline{\#}} \oplus {}_{\mathrm{Z}}\underline{\mathbf{M}}_{\underline{\#}} \ni & \underline{\mathbf{Z}}_{\underline{\#}} \oplus {}_{\mathrm{Z}}\underline{\mathbf{Q}}_{\underline{\#}} \oplus {}_{\mathrm{Z}}\underline{\mathbf{U}}_{\underline{\#}} \oplus {}_{\mathrm{Z}}\underline{\mathbf{M}}_{\underline{\#}} \oplus {}_{\mathrm{Z}}\underline{\mathbf{q}}_{\mathrm{MZ}}^{\underline{\#}} \oplus \ldots \ni \ldots \\
\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \ldots \ni \ldots \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot &
\end{array}$$

The axioms of *the core axioms sub-set* of the **F**.*E*.*D*. $_{Z}\underline{Q}$ axioms-system for ***dialectical arithmetic*** are as follows --

(§1) $[\forall z \in \mathbf{Z}][\underline{q}_z \in {}_{Z}\underline{Q}]$ [the axiom of «***aufheben***» *connexion*, or of *subsumption* [of the *subsumption* of the **Z** by the $_{Z}\underline{Q}$]].

(§2) $[\forall z \in \mathbf{Z}][[\underline{q}_z \in {}_{Z}\underline{Q}] \Rightarrow [\underline{s}\underline{q}_z = \underline{q}_{z+1} \in {}_{Z}\underline{Q}]]$ [axiom of inclusion of $_{Z}\underline{Q}$ **q**ualifiers' ontological successors].

(§3) $[\forall j, k \in \mathbf{Z}][[[[\underline{q}_j, \underline{q}_k \in {}_{Z}\underline{Q}] \,\&\, [\underline{q}_j \neq \underline{q}_k]] \Rightarrow [\underline{s}\underline{q}_j \neq \underline{s}\underline{q}_k]]]$ [axiom of $_{Z}\underline{Q}$ successor ***uniqueness***].

(§4) $[\forall j, k \in \mathbf{Z}][[j \gtrless k] \Rightarrow [\underline{q}_j \neq \underline{q}_k]]$ [axiom of the ***qualitative uniqueness*** of distinct **Z**-based ontological ***qualifiers***].

(§5) $[\forall z \in \mathbf{Z}][\underline{q}_z + \underline{q}_z = \underline{q}_z]$ [axiom of $_{Z}\underline{Q}$ *idempotent addition*; of ***ontological category*** [ontological **q**ualifier] '***unquantifiability***'].

(§6) $[\forall i, j, k \in \mathbf{Z} - \{\pm 0\}][[j \gtrless \pm k] \Rightarrow [\underline{q}_j \pm \underline{q}_k \neq \underline{q}_i]]$ [axiom of ***irreducibility*** for **Z**-based ***qualitative sums***].

(§7) $[\forall j, k \in \mathbf{Z}][\underline{q}_j \times \underline{q}_k = \underline{q}_j + \underline{q}_k + \underline{q}_{j+k}]$ [axiom of '*the meta-genealogical evolute product rule*' for $_{Z}\underline{Q}$ **q**ualifier $\times$].

(§8) $[\forall j, k \in \mathbf{Z}][\underline{q}_j + \underline{q}_k = \underline{q}_j + \underline{q}_k]$ [axiom of **+***commutativity* of **Z**-based ***qualitative*** / ***qualifier sums***].

(§9) $[\forall z \in \mathbf{Z}][\underline{q}_z + q_{\pm 0} = q_{\pm 0} + \underline{q}_z = \underline{q}_z]$ [axiom of the **+***identity element* in $_{Z}\underline{Q}$].

(§10) $[\forall z \in \mathbf{Z}][\underline{q}_z + [-\underline{q}_{+z}]^{+1} \equiv [\underline{q}_{+z}/q_{\pm 0}] + [q_{\pm 0}/\underline{q}_{+z}] = \underline{q}_{+z} + \underline{q}_{-z} = q_{\pm 0}]$ [axiom of the **+***inverse elements* in $_{Z}\underline{Q}$].

(§11) $[\forall z \in \mathbf{Z}][\exists[-\underline{q}_{+z}]^{+1} \equiv +\underline{q}_{-z}^{+1} \equiv +\underline{q}_{+z}^{-1} \equiv q_{\pm 0}/\underline{q}_{+z} \mid \underline{q}_{+z} \times \underline{q}_{-z} = q_{\pm 0}]$ [axiom of the **×***inverse elements* in $_{Z}\underline{Q}$].

-- wherein **s** denotes the "Peano **s**uccessor operator", $s(z) = z + 1$, and wherein $\underline{s}$ denotes the $_{Z}\underline{Q}$ version of that **s**uccessor function, $\underline{s}[\underline{q}_z] = \underline{q}_{s(z)} = \underline{q}_{z+1}$.

Each successor-system in the '***Gödelian Dialectic***' of the **F**.*E*.*D*. axioms-systems progression --

$$_{N}\underline{Q}_{\#} \rightarrow {}_{W}\underline{Q}_{\#} \rightarrow {}_{Z}\underline{Q}_{\#} \ldots$$

is *more complex*, more '''[*thought*-]*concrete*''', and more "*definite*" -- richer in "*determinations*", in "*features*", in '*ideo-ontology*' -- than are its predecessor-systems, and hence is also richer in ideographical-linguistic expressive and descriptive capabilities and facilities than they are. Each successor-system also «***aufheben***»-'"*contains*"', «***aufheben***»-'"*elevates*"', and «***aufheben***»-*transforms*/-'"*negates*"' ***all*** of its predecessor-systems, and constitutes a '"conservative extension"' of its *immediate* predecessor-system. Corresponding to the first three **s**tages of **F**.*E*.*D*.'s ***dialectical presentation*** of the progression ***within*** $\underline{Q}$, expressed above, is that, to **s**tage $s_{\#} = 3$, of **F**.*E*.*D*.'s ***dialectical presentation*** of the standard systems of arithmetic:

$$\text{)H(}_{s_{\#}=0} = (\underline{N}_{\#})^{2^0} = \underline{N}_{\#}$$; with '$\vdash\!\oplus\!\dashv$' signing '*antagonistic addition*'/'*summings of* ***opposite*** **q***ualities*' --

$$\text{)H(}_{s_{\#}=1} = (\underline{N}_{\#})^{2^1} = \underline{N}_{\#} \vdash\!\oplus\!\dashv \underline{A}_{\#}$$, wherein **A** denotes the "**A**ught"-numbers, $\mathbf{A} \equiv \{[\forall n \in \mathbf{N}][n - n]\}$;

$$\text{)H(}_{s_{\#}=2} = (\underline{N}_{\#})^{2^2} = \underline{N}_{\#} \oplus \underline{A}_{\#} \oplus \underline{q}_{AN}^{\#} \vdash\!\oplus\!\dashv \underline{M}_{\#} \equiv \underline{W}_{\#} \vdash\!\oplus\!\dashv \underline{M}_{\#}$$; $\underline{q}_{AN}^{\#}$ unifying $\underline{A}_{\#}$ & $\underline{N}_{\#}$;

M ≡ the "**M**inus" numbers $\equiv \{[\forall n > 1 \in \mathbf{N}][1 - n]\}$;

$$\text{)H(}_{s_{\#}=3} = (\underline{N}_{\#})^{2^3} = \underline{N}_{\#} \oplus \underline{A}_{\#} \oplus \underline{q}_{AN}^{\#} \oplus \underline{M}_{\#} \oplus \underline{q}_{MN}^{\#} \oplus \underline{q}_{MA}^{\#} \oplus \underline{q}_{MAN}^{\#} \vdash\!\oplus\!\dashv \underline{F}_{\#} \equiv$$

$\underline{Z}_{\#} \vdash\!\oplus\!\dashv \underline{F}_{\#}$, wherein $\underline{q}_{MN}^{\#} \oplus \underline{q}_{MA}^{\#} \oplus \underline{q}_{MAN}^{\#}$ reconcile $\underline{M}_{\#}$ with $\underline{N}_{\#} \oplus \underline{A}_{\#} \oplus \underline{q}_{AN}^{\#}$, & with **F** ≡ the **F**ractional Numbers,

$\{[\forall z_j \leq z_k \neq \pm 0 \in \mathbf{Z}][z_j/z_k]\}$.